MATHEMATICS

(CLASS–VIII)

ARITHMETIC, ALGEBRA & GEOMETRY

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MATHEMATICS

(CLASS VIII)

ARITHMETIC, ALGEBRA & GEOMETRY

By
West Bengal Board of Secondary Education

Only approved Textbook in Mathematics for Class VIII for use in all recognised Secondary Schools under West Bengal Board of Secondary Education.

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FOREWORD

The revised syllabus in Mathematics was introduced in classes VI, VII and VIII in 198 , 1982 and 1983. This year the English version is being introduced in class VIII. Teachers from secondary schools, colleges, teachers, training colleges, universities and some renowned mathematicians helped us to frame the syllabus While the syllabus was being framed. there was a general feeling among the experts that initiative should be taken by the Board to write textbooks on mathematics. Realizing the importance of the opinion expressed by the experts, the Board decided to take the initiative to write the books. This decision of the Board received the approval of the Government of West Bengal. As a result of allthis, the Board entrusted the writing of the books to experts who are connected with the teaching of mathematics at different levels.

The authors have made an attempt to give logical explanations of different topics of the syllabus in a simple manner. They have kept in mind the fact that working out sums should be interesting and not be a mechanical process. They have also made an attempt to relate the topics to different life situations. As a result of this, the students, interest in mathematics will increase. While writing the book, the mental advancement of the students has also been kept in mind. The Board is of the opinion that this book also reflects the basic objectives of the syllabus as the books for classes VI and VII do.

The Board would like to request teachers of mathematics, educationists and those interested in education to bring to the notice of the Board of shortcomings of the book and send us their constructive suggestions.

The free distribution of the Mathematics textbook for class VI by the Government of West Bengal is the beginning of an admirable endeavour for distribution of textbooks at secondary level. This initiative of the Government of West Bengal deserves praise. Keeping in mind the economic constraints of the students in general, a moderate price has been fixed, as has been done in the case of the book for class VII.

Finally, on behalf of the Board, I should like to convey our gratitude to all the associations of teachers and to mathematicians who have helped the Board to frame the syllabus and prepare the manuscripts of the books.

We hope that these textbooks in mathematics, published by the Board will create an interest in the subject among the students and improve the standard of education in this discipline.

Calcutta
2nd January, 1984

Shri Bhabesh Moitra
President
West Bengal Board of
Secondary Education

SECOND EDITION

A reprint of the mathematics textbook for class VIII is published. It has been noticed with pleasure that the series of textbooks on mathematics published by the Board has received appreciation from the teachers in general. The Board has also received letters of praise for the textbook for class VIII from some teachers. After the publication of the first edition certain misprints and errors came to the notice of the Board and attempts have been made to remove those in this reprint edition. For further improvement of the textbooks published by the Board teachers and educationists are requested to send in their comments to us.

Calcutta
12.12.88

Ranju Gopal Mukherjee
President
West Bengal Board of Secondary Education

Revised Syllabus in Mathematics for Class VIII (vide West Bengal Board of Secondary Education Notification No. Syll./79/19, dated 23.11.79 & Syll./80/9, dated 12.8.82)

CLASS VIII—1984

Arithmetic 30 Marks

1. Revision of previous works only through exercise on topics introduced in earlier classes.

2. Average ; Applications to simple problems based on experience of daily life of the pupil.

3. Extraction of Square root of fractions and decimals. Applications to simple problems.

4. Applications of Unitary Method to problems relating to Time and Distance, and Time and Work.

(Problems should be direct and properly graded)

Algebra 40 Marks

1. Revision of previous works through examples.

2. Multiplication of Polynomials each with more than two terms. Division of Polynomials with a divisor having more than one term.

3. The following formulae and their applications :

$(a+b)^3=a^3+3a^2b+3ab^2+b^3=a^3+b^3+3ab(a+b)$, $(a-b)^3=a^3-3a^2b+3ab^2-b^3=a^3-b^3-3ab(a-b)$, $(a+b)(a^2-ab+b^2)=a^3+b^3$, $(a-b)(a^2+ab+b^2)=a^3-b^3$.

Physical representation of first two of the above formulae using blocks.

4. Factorisation using the above formulae.

5. Factorisation of a quadratic expression by breaking the middle term.

6. H.C.F. and L.C.M. of simple expressions by factoriaation and by division.

7. Fractions, and the four basic operations on them.

8. Solution of general linear equations involving one variable.

Geometry 30 Marks

The aim of teaching Geometry at this stage is to make the pupils familiar with logical deductive reasoning. Any form of logical reasoning is allowed. The following properties obtained through activities in previous classes should be taken as axioms.

(i) When a straight line cuts two other straight lines those other two straight lines are parallel, if a pair of corresponding angles are equal. (ii) Two intersecting straight lines cannot both be parallel to a third straight line. (iii) If the two sides and their included angle in a triangle are respectively equal to the two sides and their included angle in another triangle. the two triangles are congruent. iv) If the two angles and the side common to them in one triangle are respectively equal to the two angles and the side common to them in another triangle the two triangles are congruent.

1. To establish the following propositions :

(a) If a line segment stands on another straight line, the sum of the two angles so formed is equal to two right angles. (b) If the sum of two adjacent angles is equal to two right angles, exterior arms of the angles are on the same straight line. (c) If two straight lines intersect, the vertically opposite angles are equal. (d) When a straight line cuts two other straight lines those other two straight lines are parallel if, either, (i) a pair of alternate angles are equal, or, (ii) a pair of interior angles on the same side of the cutting line are together equal to two right angles. (e) If a straight line cuts two parallel straight lines, then (i) the corresponding angles are equal. (ii) the alternative angles are equal, (iii) the interior angles on the same side of the cutting line are together equal to two right angles. (f) The sum of the angles of a triangle is equal to two right angles. (g) If one side of a triangle be produced, the exterior angle so formed is equal to the sum of two interior opposite angles. (h) The sum of the interior angles of a polygon of n sides is equal to $2(n-2)$ right angles. (i) If two sides of a triangle are equal, the angles opposite to them are also equal, and conversely. Congruence of two triangles—SSS. (k) Congruence of two right-angled triangles. (l) If two sides of a triangle are unequal, the angle opposite to the greater side is greater than the angle opposite to the less, and conversely (m) Any two sides of a triangle are together greater than the third side. (n) Of all line segments that can be drawn to a given straight line from a given point outside it, the perpendicular is the shortest.

2. Constructions with proof :

(i) To draw a straight line through a given point parallel to a given straight line. (ii) To divide a line segment into a number of equal segments.

3. Simple problems based on the above propositions, and constructions.

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ARITHMETIC

Chapter 1

Revision Exercises on Previous Lessons

Exercise 1(a)

1. Express in vulgar fractions of lowest form :

   $\cdot125$, $1\cdot025$, $\cdot00512$, $10\cdot124$, $\cdot00625$, $\cdot3125$, $\frac{\cdot56}{\cdot08}$, $\frac{7\cdot02}{1\cdot25}$.

2. Arrange in descending order of magnitude :
   (a) $\cdot001$, $\cdot0009$, $\cdot0011$, $\cdot0101$, $\cdot00098$.
   (b) $\frac{1}{2}$, $\cdot499$, $\frac{1}{3}$, $\cdot51$, $\cdot299$.
   (c) $\frac{1}{\cdot1}$, $\frac{1}{\cdot01}$, $\frac{\cdot01}{2}$, $\frac{\cdot001}{5}$, $\frac{\cdot1}{\cdot0001}$.
3. Express in vulgar fractions of lowest form :
   (a) $\cdot0\dot{8}\dot{1}$, $\cdot\dot{8}\dot{1}$, $\cdot00\dot{8}\dot{1}$, $2\cdot0\dot{0}8\dot{1}$, $2\cdot21\dot{8}\dot{1}$.
   (b) $\cdot00\dot{7}2\dot{9}$, $3\cdot10\dot{7}\dot{6}$, $\cdot\dot{1}4285\dot{7}$, $\cdot057142\dot{8}$, $14\cdot01\dot{3}\dot{1}$.
4. Simplify and give the answer in vulgar fraction of lowest form :
   (a) $10\cdot6\dot{2}\dot{8} - 3\cdot\dot{7}23\dot{0} - 2\cdot2\dot{3}\dot{8}$.
   (b) $\frac{\cdot1\dot{2}0\dot{3}}{3\cdot6\dot{4}\dot{2}} \div \frac{\cdot\dot{7}1428\dot{5}}{\cdot\dot{8}5714\dot{2}} \times \frac{\cdot0\dot{3}0\dot{7}}{\cdot3\dot{4}\dot{5}}$.
5. (a) If 1 metre=$3\cdot281$ ft., express $5\cdot37$ ft. in metres. (find the value correct upto two places of decimals.)
   (b) If 1km.=$\cdot6214$ mile, express $15\cdot05$ miles in kilometres. (Give the answer correct upto one place of decimal.)
   (c) If 1 gallon=$4\cdot536$ litres, express $10\frac{1}{2}$ litres in gallons correct upto two places of decimals.

   If a petrol dealer wants to give nearest gallon for $10\frac{1}{2}$ litres of petrol, how much would he give ? In that case find correct upto two decimal places of a gallon how much more or less he would give.

Exercise 1(b)

1. Fill up the blanks :

(a) A factor of a fraction $= \dfrac{\text{a factor of the numerator}}{\text{a ....of the denominator}}$

(b) A common factor of some fractions

$= \dfrac{\text{a common...... of the numerators}}{\text{a common multiple of the denominators}}$

(c) H.C.F. of some fractions $= \dfrac{\text{H.C.F. of the numerators}}{\text{........ of the denominators}}$

(d) A multiple of a fraction $= \dfrac{\text{a ......of the numerator}}{\text{a factor of the denominator}}$

(e) L.C.M. of some fractions $= \dfrac{\text{......of the numerators}}{\text{......of the denominators}}$

(f) H.C.F. of two fractions × their L.C.M.

$= \dfrac{\text{product of two numerators}}{\text{.......of two denominators}}$

2. Find two smallest common multiples of the following fractions :

(a) $\dfrac{3}{5}, \dfrac{9}{10}$

(b) $\dfrac{5}{6}, \dfrac{7}{8}, 1\dfrac{1}{3}$

3. Find L.C.M. :

(a) $\dfrac{4}{5}, \dfrac{2}{15}, 1\dfrac{1}{3}$ (b) $1\dfrac{5}{7}, 3\dfrac{1}{5}, \dfrac{9}{14}$ (c) $20, 3\dfrac{3}{4}, 6\dfrac{2}{3}$

(d) 4·3, 1·29, 21·5 (e) ·75, 12·5, 6·25, ·005

4. Find H.C.F. :

(a) $3\dfrac{1}{2}, 5\dfrac{1}{4}$

(b) $3, 4\dfrac{1}{2}, 10\dfrac{1}{4}$ (c) $2\dfrac{13}{16}, 1\dfrac{23}{40}, 3\dfrac{3}{32}$

(d) 2·8, ·25, 20 (e) ·24, ·6, ·96

Exercise 1(c)

1. (a) What is the square of square root of 5 ?
   (b) What are the square roots of square roots of the following numbers ?
   1, 81, 2401, 10556001.
   (c) What will be the number of digits in the square root of a number of 8 digits ?
   (d) By what smallest whole number should 98 be multiplied to make the product a perfect square ?
   (c) By what smallest whole number should 192 be divided to make the quotient a perfect square ?

2. Find square root :
   (a) 9216, 99225, 10201, 651249, 53824
   (b) 1002001, 1192464, 375769, 105625, 57214096.

3. Find number of digits in the square roots of the following numbers without actually finding square roots. What is or are possible digits in the unit's place of these square roots ?

15129, 1000014129, 4967284

4. What smallest whole number should be subtracted from 14669 to make the difference a perfect square ?

5. Find greatest and smallest numbers of 5 digits which are perfect squares.

6. Each person of a group contributed twice as many 25 paise as the number of persons in the group and the total contribution was Rs.84·50. How many persons were in the group ?

7. The games-master of a school arranged 876 students in a solid square in the play ground. What was the maximum number of rows in which the students were arranged and how many students were left out in that case ?

8. What is the minimum number of soldiers who should join 9778 soldiers so that the total number of soldiers may be arranged in a square ?

9. 4726 soldiers were divided in two groups. In arranging the solid soldiers of each group in a solid square it was found that there were some excess soldiers in the second group. Number of rows in the

first group was 32. What was the number of rows in the second group ? What was the minimum number of soldiers in excess in that group ?

10. Product of two numbers is 2500. One number is 4 times the other. What are the numbers ?

Exercise 1 (d)

1. In a sprint competition Sumana ran 100 metres in 20 seconds; in the same time Sutapa ran 95 metres and Sutanuka ran 92 metres. What time would Sutapa take to run 100 metres at the same speed ? What time would Sutanuka take to run 100 metres ? At the same rate what will be the speeds of Sumana, Sutapa and Sutanuka in kilometres per hour ?

2. Rahul travels a distance of $1\frac{1}{2}$ km in 25 minutes. If he travels half the distance at double speed, what time will he take to travel the total distance ?

3. Kamalesh walks at a speed of 5 km/hr. By cycle his speed is 15 km/hr. What time will he save if he travels a distance of 14 km by bicycle instead of walking ?

4. Nurul covers 20 metres in 60 steps. He takes 5 seconds for 6 steps. What will be his speed in kilometre per hour at the same rate ?

5. Satish walks 25 metres in 70 seconds. Suresh walks 17 metres in 45 seconds. What is the difference in their speeds ?

6. What time will a train, 80 metres long, running at a speed of 50 km/hr. take to cross a post ? What time will the train take to cross a platform, 200 metres long ?

7. Sound travels 342 m/second. What is the speed of sound in kilometres per hour (in nearest kilometre) ?

8. Anil and Pradip walk at speeds of 12 km/hr. and 5 miles/hr. respectively. In a walking competition of 2 km Anil takes rest for 5 minutes on the way and Pradip does not take any rest. Will they reach the destination at the same time or one after another ? If one reaches after another,who will finish first ? How much earlier will he reach ? (1 mile = 1·6 km)

Exercise 1 (e)

1. What is the interest of Rs.880 in $3\frac{1}{4}$ years at the rate of $6\frac{1}{2}$% per annum ?

2. What will be the amount for Rs.1240 in $7\frac{1}{2}$ years at the rate of $7\frac{1}{2}\%$ per annum ?

3. If Rs. 525 amounts to Rs. 682·50 in 5 years, what is the rate percent of interest per year ?

4. At what rate percent of interest per year will a principal be doubled in 6 years ?

5. In how many years will a principal be doubled at the rate of 4% interest per annum ?

6. Rs. 4000 was deposited with a bank at 4% interest per annum. In what time will the interest be Rs.400 ?

7. Binod Babu built a house by taking a loan of Rs.12000 from a bank at 6% interest per annum. He took 2 months to build the house. He then let the house at Rs.180 per month. After what time of letting the house will he be able to repay the bank-loan along with interest accumulating the rent of the house ?

8. What will be the interest on Rs.3375 for 4 years at 6% interest per annum ?

9. What principal will amount to Rs. 1417 in 5 years at 6% interest per annum ?

10. What principal will amount to Rs.899·50 in 6 years at $4\frac{3}{4}\%$ interest per annum ?

11. A bank pays an interest of Rs.130 on Rs.3250 for every 6 months. What is the rate of interest per annum ?

Exercise 1 (f)

1. Zamal has a plot of land of length 5 m and breadth 4 m. Sunil has a square plot of land of length 4 m. What are the differences of the perimetres and also of the areas of their plots ?

2. The inside of a room is 6 m long, 5 m 50 cm broad and 3 m 25 cm high. What is the area of the four inner walls of the room ? What will be the cost of painting these walls at Rs.2·25 per square metre ?

3. The length, breadth and height of the interior of a room are 5·1 m, 3·2 m and 3 m respectively. In that room there are one door of length 2·4 m and breadth. 9 m and two windows each of length 2 m and breadth 1·1 m. What will be the cost of whitewashing the inner walls of the room at the rate of 60 paise per square metre ?

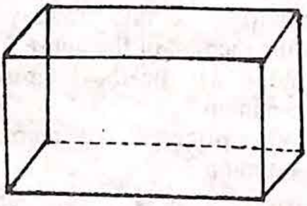
4. The length of the floor of a room is twice its breadth. Total cost of cementing that floor at the rate of Rs.10·25 per sq. m is Rs.1312. Find the length and breadth of the room.

5. Lengths of two squares are 6 m and 8 m respectively. Find the length and perimeter of a square whose area is equal to the sum of the areas of the above two squares.

6. Around the inside of a field of length 50 m and breadth 42 m there is a path of width 3 m. What is the area of the path ? What will be the total cost of making that path at Rs.1·50 per square metre?

7. Along the middle of a square garden of length 20 m there are two paths of widths $1\frac{1}{2}$ m and $2\frac{1}{2}$ m. What is the area of paths ? What is the area of the garden excluding the area of paths ?

8. Length, breadth and height of the outer side of a box are 2 m, $1\frac{1}{2}$ m and 1 m respectively. Find the area of its six faces. What will be the cost of painting six outer faces of the box at Rs. 4·50 per square metre ?



Chapter 2

Average

The word 'average' originated from the Latin word 'avaragium'. This implies giving something from one to another so as to bring both to the same level. By this word we generally convey the idea of the value in the middle.

You have got some general idea about average in class IV. You have been seen there that to compare two or more different magnitudes of same kind, we add the magnitudes and divide the sum by the total number of quantities to get the average value or the mid value. Afterwards we compare weightage of different values with this.

For example, let the marks obtained in mathematics examination by three friends. Ram, Shyam and Jadu be 75, 78 and 84 respectively. Let us suppose that we will have to compare their performances.

Had there been two candidates we could have compared by ordinary subtraction and we could say that Ram had obtained 3 marks less than Shyam. Since they are three in number it is inconvenient to compare their performances by ordinary subtraction. Hence we need a yardstick with respect to which the marks of all three of them may be compared. Generally, we take the average of the given magnitudes as that yard stick. you know that the average value is calculated by the following formula :

Average value

$$= \frac{\text{sum of the magnitudes of the given quantities}}{\text{total number of the given quantities}}$$

Accordingly, the average of the marks obtained by three of them will be given by

$$\text{Average marks} = \frac{75 + 78 + 84}{3} = 79$$

Now comparing their individual marks with this average marks we find that Ram has obtained 4 marks less. Shyam has obtained 1 mark less and Jadu has obtained 5 marks more.

The idea of average is used in different activities of real life. For example, average yields per acre, average income, average annual increase of population, etc. In practical work also the idea of average helps in different manner. For instance when a tailor prepares different sizes of readymade garments, he considers the average height of children of different age groups. We can give many such examples. Try to find some such examples.

Algebraic formula for finding average value :

In class VII you have learnt how to express unknown and indefinite numbers by algebraic symbols and how to express different arithmetical processes in the form of a formula by using algebraic symbols. Now let us see how the formula for finding average may be expressed algebraically. Let us suppose that the magnitudes of some quantities of the same kind are as follows :

Magnitude of the 1st quantity is a_1
" " " 2nd " " a_2
" " " 3rd " " a_3

....

....

" " " (n-1) th ,, ,, a_{n-1}
,, " " n th " " a_n

Total number of quantities is n . Sum of the magnitudes of the quantities :

You know that average value $a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n$

$$= \frac{\text{sum of the magnitudes of the quantities}}{\text{total number of the quantities}}$$

Hence, taking average value as A, the formula obtained from the above discussion will be as follows :

$$A = \frac{a_1+a_2+a_3+\cdots+a_{n-1}+a_n}{n}$$

Let us now see how this formula can be used in actual problem.

Ages of Mina and her two friends are as follows. Find their average age.

Mina	Kamala	Sulekha
14 yrs 6 months	14 yrs 11 months	15 yrs 7 months

Here number of quantities n = 3 and the magnitudes are :

a_1 = 14 yrs 6 months
a_2 = 14 yrs 11 months
a_3 = 15 yrs 7 months

Hence required average

$$= \frac{a_1+a_2+a_3}{3}$$

$$= \frac{14 \text{ yrs } 6 \text{ mths} + 14 \text{ yrs } 11 \text{ mths} + 15 \text{ yrs } 7 \text{ mths}}{3}$$

$$= \frac{45}{3} \text{ yrs} = 15 \text{ yrs.}$$

Hence their average age is 15 years.

Some corollaries :

you know : **average value**

$$= \frac{\textbf{sum of the magnitudes of the quantities}}{\textbf{total number of the quantities}}$$

We get two more relations from this :

(1) Sum of the magnitudes of the quantities = average value × total number of the quantities

(2) Total number of the quantities

$$= \frac{\textbf{sum of the magnitudes of the quantities}}{\textbf{average value}}$$

Ex. 1. The yields of 7 coconut trees in your garden are 62, 75, 68, 70, 83 74, and 65 coconuts respectively. What is the average yield of coconut per tree ?

The question is to find out the number of coconuts per tree if the total yield remains same and assuming that each tree yields same number of coconuts.

Now, total number of trees = 7 and total number of coconuts = 62+75+68+70+83+74+65=497.

Hence required average $= \frac{497}{7} = 71.$

[**N.B.** It should be remembered that average value is a probable value which may not tally with any actual value. In the above example average value is 71, but actually there is no tree yielding 71 coconuts.]

Ex.2. In a cricket match average run of 11 players of your school team is 14. What is the total run of your school team ?

You know : sum of the magnitudes of the quantities

= average value × number of the quantities

Here, average run = 14 and number of the players = 11.

∴ required total run = $14 \times 11 = 154$.

∴ total run of your school team is 154.

Ex. 3. Total weekly emoluments of the workers of a factory is Rs. 1416. Average weekly emolument of each worker is Rs. 118. How many workers are there in the factory ?

You know : number of the quantities

$$= \frac{\textbf{sum of the magnitudes of the quantities}}{\textbf{average value}}$$

Here, average emolument = Rs. 118 and total emoluments = Rs. 1416.

∴ required number of workers $= \frac{1416}{118} = 12$.

Hence there are 12 workers in the factory.

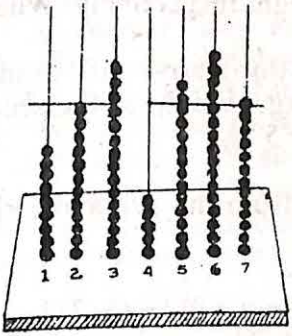


Fig No. 1.

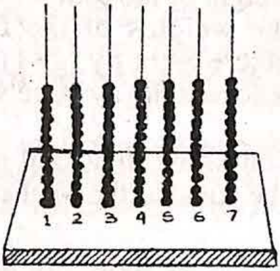


Fig No. 2.

Fig 1 is a frame like an abacus having 7 vertical iron rods. The rods contain 9, 12, 15, 5, 14, 16 and 13 movable balls respectively. The average of the number of balls will be

$\frac{9+12+15+5+14+16+13}{7} = \frac{84}{7} = 12$. See that in the figure there are 12 balls in rod no. 2. Now a thread is tied horizontally from the first to the seventh rod just above the last ball of rod no. 2. It is found that the number of balls in some of the rods is less than the average and in some other rods the number is more than the average. For example, there are 3 and 7 balls less in rod nos. 1 and 4 respectively, whereas there are 3, 2,4, and 1 balls more in rod nos. 3, 5, 6 and 7 respectively.

That is in 2 rods there are altogether 3+7 = 10 balls less and in other 4 rods there are altogether 3+2+4+1 = 10 balls more.

If the excess balls are removed from the rods containing them and placed according to requirement in the rods which have less balls, the number of balls in each rod will be the same as the average value (see Fig. 2). From this we may conclude that the shortage from the average value is equal to the excess over the average value.

Ex. 4 Average weight of 4 boys is 42 kg. The weights of the first, second and third boys are 2 kg less, 5 kg more and 1 kg less than the average weight respectively. What is the weight of the fourth boy ?

The weights of the first and third boys fall short of average weight by (2+1) kg = 3 kg. Weight of the second boy exceeds the average weight by 5 kg.

You know that total shortage from the average weight will be equal to the total excess.

Hence the weight of the fourth boy will be (5–3) kg = 2 kg less than the average weight. Thus the weight of the fourth boy will be (42–2) kg = 40 kg.

Exercise 2

1. In a school there are 72, 64 and 53 students in classes V, VI and VII respectively. What is the average number of students in those three classes ?

2. Your school cricket team consists of 11 players. Your team has scored 176 runs in the first innings. What is the average run of the players ?

3. Yield of paddy per bigha from 5 bigha land of Mahim was 4·25 quintals, 5 quintals, 5·25 quintals, 4·75 quintals and 4·50 quintals respectively. What is the average yield of paddy per bigha ?

4. Six almirahs out of seven of Raghunath Das library contain 135, 180, 122, 167, 155 and 132 books respectively. The average number of books per almirah of the seven almirahs is 148. How many books are there in the seventh almirah ?

5. A weaver's factory produces on average 15 pieces of cloth per day from Monday to Saturday of a week. What is total produce during those days ?

6. In a school there are 90, 87, 82, 64, 52 students in classes V, VI, VII, IX, X respectively. If the average number of students in the six classes of the school be 77, what is the number of students in class VIII ?

7. There are 1988 books in 14 book cases of your school library. Some new books are brought. If the new books are arranged in those book cases the average number of books per case will be 156. How many new books are purchased ?

8. In a family the average age of father and mother is 35 years. But the average age of father, mother and their only son is 27 years. What is the age of the son ?

9. In a cricket match the average run of the first three players of a team is 50 and the total run of 11 players is 342. What is the average run of the remaining 8 players of the team ?

10. There are 25 students in a class. Average age of first 10 students is 14·4 years and that of last 8 is 14·8 years. What should be the average age of 7 students in the middle so that the average age of all students of the class will be 14·5 years ?

11. There were 984 coconuts in 12 trees of a coconut garden. What will be the average number of coconuts per tree after taking down 26 coconuts in average from 5 trees and 38 coconuts in average from 7 trees ?

12. In a week the average rainfall per day was ·25 cm. There was no rainfall on Sunday. Rainfall on Monday, Tuesday, Wednesday, Thursday and Friday was ·4 cm, ·02 cm, ·45 cm, ·28 cm and ·58 cm respectively. What was the rainfall on Saturday ?

13. Average height of 30 students of class VIII of a school is 147 cm. Average height of 14 students is 145 cm and that of another 6 is 152 cm. What is the average height of the rest 10 students of the class ?

14. 52000 saplings were planted in 325 rows of a plot of land in a tea garden. After one month it was observed that 3575 saplings were dead. What is the present average of sapling in the above rows ?

15. Average mid-day temperature of a week starting from Monday was 73·9°. Average mid-day temperature of the first three days was 73·6° and that of last three days was 73·7°. What was the mid-day temperature on Wednesday ?

16. Average of three numbers is 77. First number is twice the second and the second number is twice the third. Find the three numbers.

17. Population of a village was 12500 in 1961. In 1981 the population increased to 13460. What was the average increase of population per year ?

18. Average age of 15 boys of a class in a school is 10 years. What will be the average age of the boys if 3 new boys of ages 14 years, 15 years and 19 years join that class ?

19. Last year a cultivator got in an average 3·8 quintals paddy per bigha from his 15 bighas land. This year he planted high yielding paddy in 6 bigha land and got in average 5·5 quintals paddy per bigha. He got a total of 40·5 quintals paddy from the rest of the land. Was the average yield of this year more or less than that of the last year and by how much ?

20. 100, 120, 130, 125 and 150 machine parts were produced by a factory in the first five months of a year respectively. What was the average production during these five months ? What will be the annual production of the factory at the above rate ?

21. Average income of A and B is Rs. 505, that of B and C is Rs. 535 and of A and C is Rs. 520. What are the actual incomes of A, B and C ?

22. Monthly rainfall from June to September of a certain year was 12·5 cm, 27·04 cm. 20·05 cm and 6·29 cm respectively. What was average daily rainfall during those four months ?

23. Average age of Ram, Hari and Manju is 16 years; average age of Hari, Manju and Namita is 12 years. If the age of Ram is 15 years, what is the age of Namita ?

24. Average run of a batsman is increased by 5 on his scoring 100 runs in the eleventh innings. What was his average run after the eleventh innings ?

25. Each of A and B got 8 wickets for 28 runs. Afterwards A got one wicket for 35 runs and B got 4 wickets for 56 runs. Whose average of getting wicket was better ?

26. A train goes from Calcutta to Madhupur at an average speed of 45 km per hour and comes back at an average speed of 36 km per hour. What is average speed of the train for both journeys.

Weighted Average :

What was discussed about average so far, each quantity had different values. But there may be cases where we have more than one quantity having same value and different quantities having different values. In these cases the average value will depend on weightage of the value and the number of quantities. Let us consider the following example.

You have purchased for your village library 14 books at Rs. 5 each, 22 books at Rs. 4 each and 64 books at Rs. 3 each. What is the average cost price of each book ?

From the prices of books it may appear that average cost price will be Rs. $(5+4+3) \div 3$ = Rs. 4. But actually average cost price will not be this.

Since the number of books having cost price of Rs. 3 is much more, average cost price will be nearer to Rs. 3 instead of being Rs. 4.

Let us see how it is so :

Cost price of 14 books at Rs. 5 each = Rs.(5×14) = Rs. 70

Cost price of 22 books at Rs. 4 each = Rs.(4×22) = Rs. 88

Cost price of 64 books at Rs. 3 each = Rs.(3×64) = Rs.192

Total number of books = 100 Total cost price = Rs.350

Hence average cost price of each book =Rs. $\frac{350}{100}$ = Rs. 3·50

Thus, in this case, we find that the average value depends not only on the values of the quantities but also on the number of quantities of each value. The number of quantities of a particular value is called its weight. Hence we may say that average value depends on the weight of each value. In these cases the average is called **weighted average.**

Algebraic Formula :

Like ordinary average, weighted average can also be expressed by algebraic formula. Following is a table of values and number of quantities having those values. It is required to find out the average value.

Value	a_1	a_2	a_3	$a_4 \cdots$	a_{n-1}	a_n
Weight	x_1	x_2	x_3	$x_4 \cdots$	x_{n-1}	x_n

From the above table we get

Total value of x_1 quantities is a_1x_1
" " " x_2 " " a_2x_2
" " " x_3 " " a_3x_3
....
" " " x_{n-1} " " $a_{n-1}x_{n-1}$
" " " x_n " " a_nx_n

Total number of quantities $= (x_1 + x_2 + \cdots + x_n)$ Total value $= (a_1x_1 + a_2x_2 + \cdots + a_nx_n)$

Hence taking A as average value.

$$A = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n}{x_1 + x_2 + x_3 + \cdots + x_n}$$

Let us now see how this algebraic formula can be used in actual problem.

Following is the particulars of height of 40 boys of class VIII in a school. Find the average height of the boys in that class.

Particulars :

No. of boys = 7, height of each = 1 m 43 cm = 1·43 m
No. of boys = 11, height of each = 1 m 45 cm = 1·45 m
No. of boys = 16, height of each = 1 m 47 cm = 1·47 m
No. of boys = 4, height of each = 1 m 48 cm = 1·48 m
No. of boys = 2, height of each = 1 m 50 cm = 1·50 m

Above particulars may be expressed in algebraic formula in the following manner.

No. of boys $x_1 = 7$, height of each $a_1 = 1{\cdot}43$ m
No. of boys $x_2 = 11$, height of each $a_2 = 1{\cdot}45$ m
No. of boys $x_3 = 16$, height of each $a_3 = 1{\cdot}47$ m
No. of boys $x_4 = 4$, height of each $a_4 = 1{\cdot}48$ m
No. of boys $x_5 = 2$, height of each $a_5 = 1{\cdot}50$ m

Hence, total height of 7 boys of 1·43 m is
$= a_1x_1 = (1{\cdot}43 \times 7)\text{m} = 10{\cdot}01$ m
Total height of 11 boys of 1·45 m is
$= a_2x_2 = (1{\cdot}45 \times 11)\text{m} = 15{\cdot}95$ m
total height of 16 boys of 1·47 m is
$= a_3x_3 = (1{\cdot}47 \times 16)\text{m} = 23{\cdot}52$ m
total height of 4 boys of 1·48 m is
$= a_4x_4 = (1{\cdot}48 \times 4)\text{m} = 5{\cdot}92$ m
Total height of 2 boys of 1·50 m is
$= a_5x_5 = (1{\cdot}50 \times 2)\text{m} = 3{\cdot}00$ m

$x_1+x_2+x_3+x_4+x_5 = 40$ and
$a_1x_1+a_2x_2+a_3x_3+a_4x_4+a_5x_5 = 58{\cdot}40$ m

Required average height

$$= \frac{a_1x_1+a_2x_2+a_3x_3+a_4x_4+a_5x_5}{x_1+x_2+x_3+x_4+x_5}$$

$$= \frac{58{\cdot}40\text{m}}{40} = 1{\cdot}46 \text{ m.}$$

Ex. 1. A businessman bought 15 kg of rice at Rs. 2·90 per kg, 12 kg of rice at Rs. 3·10 per kg and 7 kg of rice at Rs. 3·48 kg What is the average cost price per kg ?

Total price of 15 Kg at Rs. 2·90 is Rs. (2·90 × 15) = Rs. 43·50
Total price of 12 Kg at Rs. 3·10 is Rs. (3·10 × 12) = Rs. 37·20
Total price of 7 Kg at Rs. 3·48 is Rs. (3·48 × 7) = Rs. 24·36
Total price of 34 Kg = Rs. 105·06

Average cost price = $\frac{\text{Rs. } 105{\cdot}06}{34}$ = Rs. 3·09

Exercise 3

1. In a mango garden there are 520 mangoes in each of 7 trees, 484 in each of 11 trees and 566 in each of 6 trees. What is the average yield of mango per tree ?

2. Each student of classes V, VI, and VII of a school contributed Rs. 1·20, Rs. 1·30 and Rs. 1·58 respectively for flood relief. Find average contribution of the students of those three classes if the number of students in classes V, VI and VII are 77, 68 and 65 respectively.

3. Your school was open for 23 days on a certain month. Presence of boys in your class was as follows : 43 boys for each of 4 days, 47 for each of 6 days, 51 for each of 12 days and 38 for 1 day. What was the average of the number of boys present per day ?

4. 90, 48 and 30 books were purchased for Friend's Library at prices Rs. 5·80, Rs. 7·00 and Rs. 12·00 per book respectively. What was the average price of each book ?

5. In a certain month the average rainfall for the first 7 days was 1·70 cm, for the next 15 days it was 1·30 cm and for the last 8 days it was 0·65 cm. What was the average rainfall per day during that month ?

6. A train started from Howrah and reached Burdwan after travelling for the first 10 minutes at an average speed of 400 m per minute, next 1 hr. 20 minutes at an average speed of 1 km per minute and last 50 minutes at an average speed of 630 m per minute. What was the average speed of the train per minute for the whole journey ?

7. A cultivator got 3·33 qunital of paddy per bighas from his 17 bigha land. If the average yield for 8 bighas of his land was 2·97 quintal per bighas and for 7 bighas it was 3·35 quintal per bighas, what was the total yield for the rest of the land ?

8. Marks in mathematics obtained by the boys of class VIII of Haranath Nirodasundari Ghosh Vidyamandir are given in the following table. Find the average marks obtained by each boy.

No. of boys	15	18	5	2
Marks obtained by each	30	50	60	80

9. Temperatures of different days in a week are given in the following table. Find the average temperature per day.

Day	Monday and Tuesday	Wednesday to Friday	Saturday and Sunday
Temperature	28°C	31°C	27°C

10. The population of a village increased at the rate of 105 for the first two years, at the rate of 100 for the next 9 years and at the rate of 110 for the last 9 years. What was the average increase of population of the village per year ?

11. Following is the details of runs of the players of two cricket teams. Average runs which team was more and by how much ?

Team 'A'	No. of players	3	2	5	1
	Runs of Each player	8	25	29	102
Team 'B'	No. of players	2	3	1	5
	Runs of each player	16	21	35	80

12. Salary of a man was Rs.530 for the first year, for the next 4 years it was at the rate of Rs.550 per year and for the last 5 years it was Rs.600 per year. What is the average per year increase of his salary ?

13. Marks obtained by 40 students of class VIII of a school in Mathematics and English are as follows. In which subject is the average mark higher and by how much ?

Mathematics	No. of students	11	20	8	1
	Marks obtained by eaach	34	55	70	86
English	No. of students	14	20	4	2
	Marks obtained by each	38	46	57	80

14. Heights of 15 boys of class VIII of a school are as follows. What is the average height of the boys ?

No. of boys	3	9	2	1
Height of each	157 cm	157 cm	160 cm	163 cm

15. Following is the particulars of age of 25 students of class VIII of a school. What is their average age ?

No. of students	7	3	6	9
Age of each	13·7 yrs	14·1 yrs	14·5 yrs	15·2 yrs

16. Following is the rate of incentives given to each worker per ton for daily production over 88 tons in a factory :

Production	88-less than 95 tons	95-less than 100 tons	100 tons and more than that
Incentive	Rs.1·25	Rs.1·50	Rs.1·75

Average daily production for three months are as follows :

First month	Second month	Third month
94 ton	96 ton	105 ton

(i) What is the average daily incentive in each month ?

(ii) What will be the average incentive per worker for the above three months ?

[Take 1 month = 30 days. Incentive is excess payment for extra work.]

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Chapter 3

Square root of vulgar fraction and decimal fractions.

In classes VI and VII you have learnt how to find square roots of whole numbers.

In class VI you have seen in arithmetic :

square of $2 = 2^2 = 2 \times 2 = 4$, hence $\sqrt{4} = 2$

square of $5 = 5^2 = 5 \times 5 = 25$, hence $\sqrt{25} = 5$

Again, in class VII you have seen in algebra :

square of $x = x^2 = x \times x$, hence $\sqrt{x^2} = x$

square of $p = p^2 = p \times p$, hence $\sqrt{p^2} = p$

Now, let us see the squares and square roots of vulgar and decimal fractions.

In case of vulgar fractions :

square of $\frac{1}{2} = \left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, hence $\sqrt{\frac{1}{4}} = \frac{1}{2}$

square of $\frac{2}{3} = \left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, hence $\sqrt{\frac{4}{9}} = \frac{2}{3}$

square of $\frac{5}{4} = \left(\frac{5}{4}\right)^2 = \frac{5}{4} \times \frac{5}{4} = \frac{25}{16}$, hence $\sqrt{\frac{25}{16}} = \frac{5}{4}$

In case of decimal fractions :

square of $\cdot 1 = (\cdot 1)^2 = \cdot 1 \times \cdot 1 = \cdot 01$, hence $\sqrt{\cdot 01} = \cdot 1$

square of $\cdot 5 = (\cdot 5)^2 = \cdot 5 \times \cdot 5 = \cdot 25$, hence $\sqrt{\cdot 25} = \cdot 5$

square of $1\cdot 2 = (1\cdot 2)^2 = 1\cdot 2 \times 1\cdot 2 = 1\cdot 44$, hence

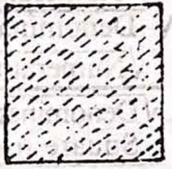
$\sqrt{1\cdot 44} = 1\cdot 2$

From the above discussion you get some idea about the square root of vulgar and decimal fractions. Now let us see how to find square roots of the above fractions.

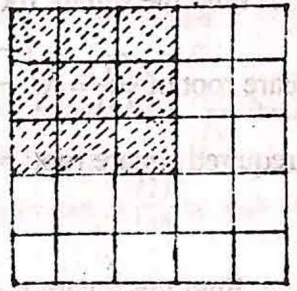
Square root of vulgar fraction :

In class VI you have found square roots of small whole numbers by factorising them. It is easier to find the square root of a whole number by factorising it,since the number of its factors is limited and definite. But in class VII you have learnt that a fraction can have innumerable factors. Hence it is not easy to find square root of a vulgur fraction by factorising it. It is also not possible to find square root of a vulgar fraction by the method of division. Let us try to find a method for finding square root of vulgar fraction.

In geometry and sums on area in class VII you have seen that a^2 means area of a square on side of length a (see the adjoining figure). Here if we find square root of a^2 we will get a. Thus we find that the square root of area of a square is the length of its side.



In the adjoining figure a square is drawn on a side of 5 cm. This square has been divided into 25 squares by drawing 4 horizontal and 4 vertical lines at a distance of 1 cm from each other. Area of each small square is 1 sq. cm and the area of the whole square is 25 sq. cm.



Now see that there are 9 small squares in the shaded portion. Hence area of this portion is 9 sq. cm and this portion is $\frac{9}{25}$ of the original square.

Now what shall we get if we find square root of $\frac{9}{25}$?

We shall get the length of a side of the square of area $\frac{9}{25}$.

From the above figure we find that the length of a side of the square $\frac{9}{25}$ is $\frac{3}{5}$ of a side of the original square.

Thus we may say $\sqrt{\frac{9}{25}}=\frac{3}{5}$.

How can we get this result ?

You know that square root of 9 is 3 and square root of 25 is 5.

Thus we may write $\sqrt{\frac{9}{25}}=\frac{\sqrt{9}}{\sqrt{25}}=\frac{\sqrt{3x3}}{\sqrt{5x5}}=\frac{3}{5}$

Hence we get the following rule for finding square root of a vulgar fraction :

square root of vulgar fraction

$$=\sqrt{\frac{\text{Numerator}}{\text{Denominator}}}$$

$$=\frac{\sqrt{\text{Numerator}}}{\sqrt{\text{Denominator}}}$$

$$=\frac{\text{Square root of numerator}}{\text{Square root of denominator}}$$

Ex. 1. Find the square root of $\frac{81}{121}$

Square root of $\frac{81}{121}=\sqrt{\frac{81}{121}}=\frac{\sqrt{81}}{\sqrt{121}}=\frac{9}{11}$

$\therefore$ required square root $=\frac{9}{11}$

Ex. 2. Find the square root of $\frac{196}{5625}$

Square root of $\frac{196}{5625}=\sqrt{\frac{196}{5625}}=\frac{\sqrt{196}}{\sqrt{5625}}=\frac{14}{75}$

For

$$\begin{array}{r|r|l} & 1\overline{96} & 14 \\ & 1 & \\ \hline 24 & 96 & \\ & 96 & \\ \cline{2-2} \end{array} \qquad \begin{array}{r|r|l} & \overline{56}\,\overline{25} & 75 \\ & 49 & \\ \hline 145 & 725 & \\ & 725 & \\ \cline{2-2} \end{array}$$

$\therefore$ required square root $=\frac{14}{75}$.

Example 4

1. Find square roots of the following fractions :

(i) $\frac{225}{256}$ (ii) $\frac{169}{529}$ (iii) $\frac{841}{900}$ (iv) $1\frac{496}{729}$

(v) $1\frac{539}{1225}$ (vi) $6\frac{433}{676}$ (vii) $\frac{7921}{9216}$ (viii) $\frac{13456}{50625}$

(ix) $\frac{16384}{21316}$ (x) $1\frac{2355}{5041}$ (xi) $15\frac{385}{576}$ (xii) $82\frac{66}{169}$

(xiii) $109\frac{628}{729}$ (xiv) $\frac{4225}{127449}$ (xv) $14\frac{835}{6241}$ (xvi) $\frac{6889}{11025}$

(xvii) $\frac{2907025}{3171961}$ (xviii) $163\frac{7821}{30625}$ (xix) $\frac{2211169}{29811600}$

(xx) $25840\frac{9}{16}$

2. Which fraction multiplied by itself will give the product $4\frac{285}{529}$?

3. Area of the square 'A' is $\frac{121}{256}$th of the area of the square 'B'. What fraction of the side of 'B' is the side of 'A' ?

4. Find the fraction by which $\frac{35}{42}$ should be multiplied so that the square root of the product is 1.

5. Find the smallest whole number by which $\frac{49}{27}$ should be multiplied so that the product is a fraction which is a perfect square.

6. By which number should the square root of $\frac{121}{144}$ be multiplied so that the product is 1 ?

7. AB = 12 units; AC = $\frac{3}{12}$th of AB.

How many times will the square drawn on AC be square drawn on AB ? Show this by drawing a figure.

Square root of decimal fraction :

At the beginning of this chapter you have seen that square of $\cdot 1 = (\cdot 1)^2 = \cdot 1 \times \cdot 1 = \cdot 01$, hence $\sqrt{\cdot 01} = \cdot 1$ and square of $1{\cdot}2 = (1{\cdot}2)^2 = 1{\cdot}2 \times 1{\cdot}2 = 1{\cdot}44$, hence $\sqrt{1{\cdot}44} = 1{\cdot}2$.

Do you get any idea about the method of finding square root of decimal fraction from the above discussion ?

Let us suppose that it is required to find square roots of ·01, ·81, 1·21 and ·0009.

You know how to express a decimal fraction as a vulgar fraction. You have also learnt in this chapter how to find square root of a vulgar fraction. Hence you may find the square roots of the above decimal fractions after expressing them as vulgar fractions and then express the result as decimal fractions. Thus

square root of ·01 = $\sqrt{\cdot 01} = \sqrt{\frac{1}{100}} = \frac{\sqrt{1}}{\sqrt{100}} = \frac{1}{10} = \cdot 1$

square root of ·81 = $\sqrt{\cdot 81} = \sqrt{\frac{81}{100}} = \frac{\sqrt{81}}{\sqrt{100}} = \frac{9}{10} = \cdot 9$

square root of 1·21 = $\sqrt{1\cdot 21} = \sqrt{\frac{121}{100}} = \frac{\sqrt{121}}{\sqrt{100}} = \frac{11}{10} = 1\cdot 1$

square root of ·0009 = $\sqrt{\cdot 0009} = \sqrt{\frac{9}{10000}} = \frac{\sqrt{9}}{\sqrt{10000}} = \frac{3}{100}$

$= \cdot 03$

Ex. 1. Find square roots of ·0064 and ·0196

Square root of ·0064 = $\sqrt{\cdot 0064} = \sqrt{\frac{64}{10000}} = \frac{\sqrt{64}}{\sqrt{10000}}$

$= \frac{8}{100} = \cdot 08$

Square root of ·0196 = $\sqrt{\cdot 0196} = \sqrt{\frac{196}{10000}} = \frac{\sqrt{196}}{\sqrt{10000}}$

$= \frac{14}{100} = \cdot 14$

Ex. 2. Find square roots of 2·25, 13·69 and 1·4641

Square root of 2·25 = $\sqrt{2\cdot 25} = \sqrt{\frac{225}{100}} = \frac{\sqrt{225}}{\sqrt{100}} = \frac{15}{10} = 1\cdot 5$

Square root of $13{\cdot}69 = \sqrt{13{\cdot}69} = \sqrt{\frac{1369}{100}} = \frac{\sqrt{1369}}{\sqrt{100}}$

$= \frac{37}{10} = 3{\cdot}7$

Square root of $1{\cdot}4641 = \sqrt{1{\cdot}4641} = \sqrt{\frac{14641}{10000}} = \frac{\sqrt{14641}}{\sqrt{10000}} = \frac{121}{100}$

$= 1{\cdot}21$

This method of finding square root is inconvenient and more time consuming if the number is quite large. Hence finding square root of decimal fraction by method of division is more convinient .

Square root of decimal fraction by method of division :

In class VII you have found square roots of whole numbers which are perfect squares by the method of division. Here we have to find square roots of decimal fractions which are perfect squares by the method of division. If the square root of a decimal fraction is a terminating decimal fraction then it is called a perfect square decimal fraction.

Note the following examples :

$({\cdot}1)^2 = {\cdot}1 \times {\cdot}1 = {\cdot}01$, hence $\sqrt{{\cdot}01} = {\cdot}1$

$({\cdot}3)^2 = {\cdot}3 \times {\cdot}3 = {\cdot}09$, hence $\sqrt{{\cdot}09} = {\cdot}3$

$({\cdot}9)^2 = {\cdot}9 \times {\cdot}9 = {\cdot}81$, hence $\sqrt{{\cdot}81} = {\cdot}9$

$({\cdot}12)^2 = {\cdot}12 \times {\cdot}12 = {\cdot}0144$, hence $\sqrt{{\cdot}0144} = {\cdot}12$

$({\cdot}36)^2 = {\cdot}36 \times {\cdot}36 = {\cdot}1296$, hence $\sqrt{{\cdot}1296} = {\cdot}36$

$({\cdot}013)^2 = {\cdot}013 \times {\cdot}013 = {\cdot}000169$, hence $\sqrt{{\cdot}000169} = {\cdot}013$

$({\cdot}112)^2 = {\cdot}112 \times {\cdot}112 = {\cdot}012544$, hence $\sqrt{{\cdot}012544} = {\cdot}112$

We find the following in the above examples :

(1) In every square decimal fraction, the number of digits after decimal point is even.

N.B. Try to find the reason for the above from the rule of multiplication of decimal fraction.

(2) Number of digits after the decimal point of the square root is half the number of digits after the decimal point of the original number.

Thus at the time of finding square root by the method of division for each pair of digits after decimal point in the given number there will be one digit after decimal point in the square root. Hence the digits after decimal point should be grouped from left. Of course, if there is a whole number before decimal point of the given number, then for this portion the grouping should be to the left starting from the digit in the unit's place as before.

Along with this, another rule is to be followed. In continuing the process of division decimal point should be placed in the quotient as soon as the first pair after decimal point is taken into consideration.

Ex. 1. Find the square root of ·020736.

$$\begin{array}{r|l} & \cdot 02\ 07\ 36\ \big(\ \cdot 144 \\ & \underline{1} \\ 24 & 107 \\ & \underline{96} \\ 284 & 1136 \\ & \underline{1136} \end{array}$$

required square root = ·144.

Ex. 2. Find the square root of 170·485249.

$$
\begin{array}{r|l}
 & 170\cdot48\,52\,49\;\big(13\cdot057 \\
 & \underline{1} \\
23 & 70 \\
 & \underline{69} \\
2605 & 14852 \\
 & \underline{13025} \\
26107 & 182749 \\
 & \underline{182749}
\end{array}
$$

∴ required square root = 13·057

Exercise 5

1. Find the square roots of the following decimal fractions :

(i) 0·4
(ii) 0·0144
(iii) ·0081
(iv) 0·3136
(v) 0·7569
(vi) 0·8836
(vii) 12·25
(viii) 32·49
(ix) 55·0564
(x) 3·4596
(xi) 0·000169
(xii) 0·050625
(xiii) 121·4404
(xiv) 186·5956
(xv) 76·195441
(xvi) 273·5716
(xvii) 7468·4164
(xviii) 781·2025

2. Find the square root of the product of 0·032 and ·2.

3. Which decimal fraction multiplied by itself will give a product 1·1025 ?

4. What is the length of a side of a square whose area is 4·1209 sq. metres ?

5. Find the length of a side of a square whose area is equal to the sum of the areas of two rectangles, whose areas are 3·24 sq. m and 2·52 sq. m.

6. What decimal fraction should be added to ·75 so that the square root of the sum is 1 ?

7. Which number should be subtracted from 48·03 so that the square root of the difference is 5·7 ?

8. Find the difference of length of the sides of squares whose areas are 1·4641 sq. m and 1·0609 sq. m.

Square root of a non-perfect square number :

In classes VI and VII you have learnt how to find square root of whole numbers which are perfect squares. In Class VII you have also learnt how to find square roots of vulgar and decimal fractions which are perfect squares . Now let us see how to find square root of a number, which is not a perfect square.

You know one-digit numbers 2,3,5,6,7,8 are not perfect squares.

Again, you have seen in class VII that a perfect square number of two or more digits should have any one of the digits 0,1,4,5,6,9, in its unit's place. This, however, does not mean that any number having the above digits in the unit's place will be a perfect square. Thus any such number may or may not be a perfect square. But it is certain that a number having 2,3,7,8, in its unit's place will never be a perfect square. Thus we find that there are innumerable numbers which are not perfect squares. We can get some idea about the square roots of such numbers from their approximate values. We discuss this below.

Let us suppose that we have to find square root of 2.

Since 2 is not a perfect square, we cannot get any definite number whose square is 2. You know that in any whole number there is an unwritten decimal point on the right side of the digit in the unit's place and we may have any number of 0 (zero) to the right of this decimal point. Hence the square root of any non-perfect square number may be found, by the method of division, upto any number of decimal places. Let us now see how this method may be applied in finding square root of 2.

```
          2·000000 ( 1·414
          1
      ---------------
 24 )  100
        96
      ---------------
281 )   400
        281
      ---------------
2824 )  11900
        11296
      ---------------
          604
```

Hence $\sqrt{2} = 1{\cdot}414$

N.B. Note that square root of 2 has been found upto three places of decimals. Actually this square root is a non-terminating decimal fraction. In class VII you have learnt how to find approximate values of non-terminating decimal fractions. In the same way we may find value of the present square root correct upto two, three, four etc. decimal places.

Ex. 1. Find square root of 3 upto three decimal places.

```
          3·000000 (1·732
          1
      ---------------
 27 )  200
       189
      ---------------
343 )  1100
       1029
      ---------------
3462 )  7100
        6924
      ---------------
         176
```

Hence $\sqrt{3} = 1{\cdot}732$

Now try to find square roots of 5,6,7, etc.

Ex. 2 Find square root of 257 upto three decimal places.

$$\begin{array}{r|l} & 257{\cdot}000000\ \big(\,16{\cdot}031 \\ & 1 \\ \hline 26 & 157 \\ & \underline{156} \\ \hline 3203 & 10000 \\ & \underline{9609} \\ \hline 32061 & 39100 \\ & \underline{32061} \\ & 7039 \end{array}$$

Hence required square root = 16·031

Square root of non-perfect square vulgar fraction :

Let us suppose that we are to find square root of $\frac{3}{7}$ correct upto three decimal places. You surely understand that $\frac{3}{7}$ is not a perfect square fraction. Square roots of such numbers can be determined in two ways.

First method : First express $\frac{3}{7}$ as a decimal fraction upto certain number of places after decimal. Then find the square root of the decimal fraction and take approximate value as required.

N.B. Number of places after decimal in the square root should be one more than the number of decimal places upto which square root is to be corrected. Hence the given vulgar fraction has to be expressed as decimal fraction with its decimal places equal to twice the number of decimal places required in the square root. In this case, the square root is to be found correct upto three decimal places. Hence $\frac{3}{7}$ is to be expressed as a decimal fraction with 8 decimal places.

Square root of $\frac{3}{7} = \sqrt{\frac{3}{7}} = \sqrt{{\cdot}42857142} = {\cdot}6546$

$= {\cdot}655$

Because

$$\begin{array}{r|l} & {\cdot}42857142\,(\,{\cdot}6546 \\ & 36 \\ 125 & 685 \\ & 625 \\ 1304 & 6071 \\ & 5216 \\ 13086 & 85542 \\ & 78516 \\ & 7026 \end{array}$$

Second Method : If the denominator of $\frac{3}{7}$ is multiplied by 7 it will be a perfect square. But to keep the value of the fraction unchanged if the denominator is multiplied by any number the numerator has also to be multiplied by the same number. Thus the number will be $\frac{21}{49}$

Now, square root of $\frac{21}{49} = \sqrt{\frac{21}{49}} = \sqrt{\frac{21}{7}}$

But $\sqrt{21} = 4{\cdot}5825$ because

$$\begin{array}{r|l} & 21\,(\,4{\cdot}5825 \\ & 16 \\ 85 & 500 \\ & 425 \\ 908 & 7500 \\ & 7264 \\ 9162 & 23600 \\ & 18324 \\ 91645 & 527600 \\ & 458225 \\ & 69375 \end{array}$$

$\therefore$ square root of $\frac{3}{7} = \frac{4{\cdot}5825....}{7} = {\cdot}6546 .... = {\cdot}655$

N.B. For this method the denominator should always be made a perfect square.

Exercise 6

1. Find square roots of the following numbers (correct upto three decimal places) :

(i) 5 *(ii)* 7 *(iii)* 12 *(iv)* 37
(v) 86 *(vi)* 115 *(vii)* 257 *(viii)* 709
(ix) 1133 *(x)* 5055 *(xi)* 6297 *(xii)* 9029
(xiii) 11523

2. Find square roots of the following fractions (correct upto three decimal places) :

(i) $\frac{11}{23}$ *(ii)* $\frac{1}{7}$ *(iii)* $\frac{2}{3}$

(iv) $5\frac{2}{7}$ *(v)* $8\frac{12}{13}$ *(vi)* $\frac{337}{455}$
(vii) $\frac{1106}{1323}$ *(viii)* $70\frac{22}{27}$ *(ix)* $\frac{3}{5}$

3. Find square roots of the following decimal fractions (correct upto two decimal places) :

(i) 0·4 *(ii)* 2·5 *(iii)* 0·223
(iv) 0·2327 *(v)* 0·19941 *(vi)* 14·21021
(vii) 0·00123 *(viii)* 10·4752 *(ix)* 101·02027
(x) 115·23697 *(xi)* 279·007302

Chapter 4

Time and distance

In class VII you have worked out sums on time and distance. There you have also got some idea about speed. This speed is of two types — uniform speed and non-uniform speed. In problems on time and distance you came across the following :

(1) Speed : There are two things in the idea of speed —

(i) Unit time, (ii) distance travelled in unit time.

That is, when it is said that the speed of a train is 60 km per hour, it is meant that the train travels a distance of 60 km in 1 hour.

(2) Total time : In any particular problem time for which a moving body traverses is called total time of the problem.

(3) Total distance travelled : In any particular problem the distance travelled by the moving body during total time is called total distance travelled.

Again, you have got three formulae depending on the mutual relationship of the above three items :

$$(1)\ \text{Speed} = \frac{\text{Total distance travelled}}{\text{Total time}}$$

$$(2)\ \text{Total distance travelled} = \text{Speed} \times \text{Total time}$$

$$(3)\ \text{Total time} = \frac{\text{Total distance travelled}}{\text{Speed}}$$

Now some problems on the basis of the above formulae are discussed.

Ex. 1. A goods train started to reach the destination at a distance of 324 km in 9 hours. After travelling for 5 hours some trouble developed in the engine and the speed is reduced by 12 km per hour. How many hours after the scheduled time will the train reach the destination ?

Analysing the problem we get the following 5 parts :

(i) In 9 hours the train is scheduled to travel 324 km — from this the scheduled speed is to be determined.

(ii) Total distance travelled at this speed in 5 hours is to be found.

(iii) Total distance left after travelling for 5 hours is to be found.

(iv) Time required to travel distance left at a speed 12 km less than the original speed is to be determined.

(v) Scheduled time was 9 hours. Now what time has actually been spent and its difference from the scheduled time are to be determined.

Let us now work out the sum step by step, part by part :

(i) In 9 hrs the train was scheduled to travel 324 km
∴ " 1 hr " " " " " " $\frac{324}{9}$ km
= 36 km.

Hence the original speed was 36 km per hour.

(ii) In 1 hr the train travels 36 km
∴ " 5 hrs " " " (36×5) km = 180 km.

Hence before the trouble developed in the engine the train covered 180 km in 5 hours.

(iii) Thus to reach the destination the train will have to travel (324 − 180) km i.e. 144 km more.

(iv) Because of the trouble the train will run with a speed of (36 − 12) km per hour = 24 km per hour.

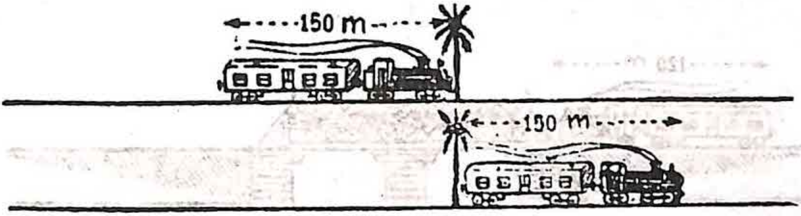
The train will go 24 km in 1 hr.
∴ " " " " 1 km " $\frac{1}{24}$ "
∴ " " " " 144 km " $(\frac{1}{24} \times 144)$ hrs = 6 hrs.

(v) Scheduled time to reach the destination was 9 hrs
But actual time taken is (5+6) hrs = 11 hrs.

The train will reach the destination (11 –9) hrs i.e., 2 hrs after the scheduled time.

Let us now see what happens when the moving body has a length of its own and has to cross a point or another body having some length.

Ex. 2. There is a palm tree by the side of the railway track. What time will a train of length 150 m, travelling at a speed of 45 km/hr, take to cross the palm tree ?



The process of crossing starts when the front of the engine reaches the palm tree as shown in the upper portion of the above figure. The process of crossing ends when the end of the train just leaves the tree as shown in the lower portion of the figure. During this time the train has to cover a distance equal to its own length i.e., 150 m. (Here the horizontal length of the tree is not taken into consideration. When the horizontal length of a vertical object is not considered it is taken as a point.)

Hence we may say that in order to cross a point a moving object will have to cover its own length. Here the time may be calculated by unitary method.

To cover 45 km or 45000 m the train takes 1 hr

or (60×60) sec

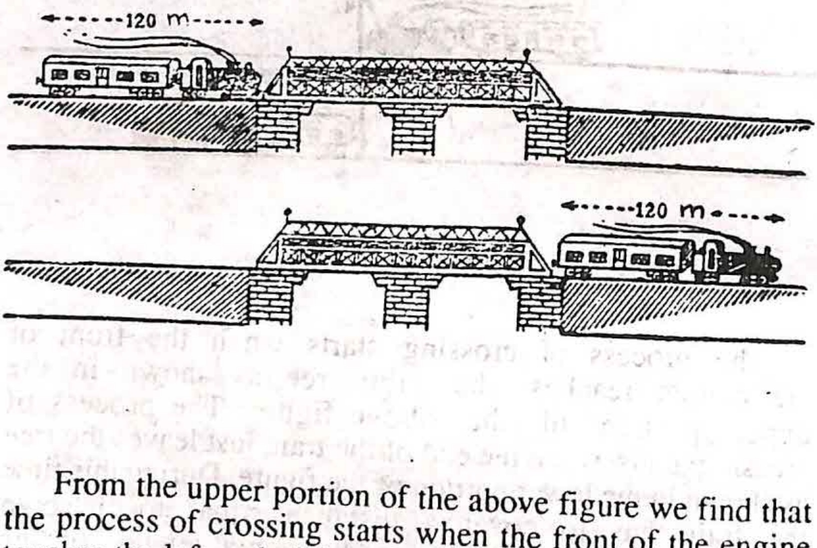
$\therefore$ to cover 1 m the train will take $\dfrac{60 \times 60}{45000}$ sec

$\therefore$ to cover 150 m the train will take $\dfrac{60 \times 60}{45000} \times 150$ sec

$= 12$ sec

$\therefore$ the train will take 12 sec to cross the palm tree.

Ex. 3. Find the time taken by a train of length 120 m moving with a speed of 60 km/hr to cross a bridge of length 330 m.



From the upper portion of the above figure we find that the process of crossing starts when the front of the engine touches the left end of the bridge. From the lower portion of the figure we find that the process of crossing ends when the end of the train just leaves the right end of the bridge. From the figure we find that to do the above the train has to cross the length of the bridge and its own length i.e., it crosses (330+120) m or 450 m.

Thus we may say that if a moving body of given length has to cross a static body of given length, then it crosses a distance equal to the sum of its own length and the length of the static body.

Now we work out the problem by unitary method.

For going 60 km or 60000 m the train takes 1 hr

or (60×60) sec

For going 1 m the train takes $\frac{60 \times 60}{60000}$ sec

" " 450 m " " " $\frac{60 \times 60 \times 450}{60000}$ sec

$= 27$ sec

$\therefore$ the train will take 27 sec to cross the bridge.

Exercise 7

1. In 50 minutes Binny goes from one place to another by a taxi moving at a speed of 48 km/hr. He comes back from the second place to the first by bicycle in 2 hr 40 minutes. Find the distance between the two places and the speed of the bicycle.

2. The speed of a motor-cycle is one and a half times that of a truck. What time will the motor-cycle take to cover a distance which is covered by the truck in 1 hr. 20 minutes ?

3. Mr. Hossain started at 12.40 p.m. to reach a place, 100 km away, at 6.30 p.m. At 4.00 p.m. he finds that he has covered only half the distance. By how much should he increase his speed per hour so that he can reach the destination at scheduled time ?

4. In a 12 km race a 'tanga' defeats a cart by 12 minutes. If the speed of tanga be 15 km/hr, what is the speed of the cart ?

5. Standing on the side of a railway track you find that a train of length 280 m crosses you in 14 seconds. What is the speed of the train ?

6. Length of a bridge is 250 m. What time will a cart of length 5 m, moving at a speed of 4·5 km/hr. take to cross the bridge ?

7. A train running at a speed of 48 km/hr. passes a telegraph post in 15 seconds. What time will it take to cross a bridge of length 240 m ?

8. A passenger of an express train found that he crossed a platform in 15 seconds. If the speed of the train was 60 km/hr, what was the length of the platform ?

9. A train passes a man standing on the platform in 10 seconds, and also clears the platform which is 150 m long in 22 seconds. Find the length and speed of the train.

10. A train running at a speed of 60 km/hr. takes 36 seconds to cross a bridge, 350 m long. Another train of length 100 m less than that of the first crosses the bridge at a speed of 45 km/hr. What are the lengths of the two trains ? What time does the second train take to cross the bridge ?

11. A man saw a train standing at the platform from a distance of 225 m that and started running at a speed at 6 km/hr. to board the train. Find the minimum time for which the train should be at the platform to enable the man to board it.

12. A train, 150 m long, takes 30 seconds to cross a bridge, 250 m long. What time will the train take to cross a platform, 130 m long ?

13. A man standing at a distance of 250 m from the bus stand saw a bus at a distance of 500 m from the stand and started running towards the bus stand. The bus was moving at that time at a speed of 30 km/hr and it stopped at the stand for 40 seconds. The man was just able to board the bus immediately before it started from the stand. What was the speed of the man ?

14. A train crosses a telegraph post in 8 seconds and a bridge 264 m long in 20 seconds. Find the length and speed of the train.

15. A train crosses two bridges of lengths 210 m and 122 m in 25 seconds and 17 seconds respectively. Find the length and speed of the train.

16. A boat takes $1\frac{1}{2}$ hrs. to go from one end of the Hooghly Chinsurah municipality to the other end along the river. If the speed of the boat is 5 km/hr, what is the length of the municipality along the river ? What time does the boat take to cover 21 km distance from Bandel Church to Imambarah on the way ?

Relative Speed :

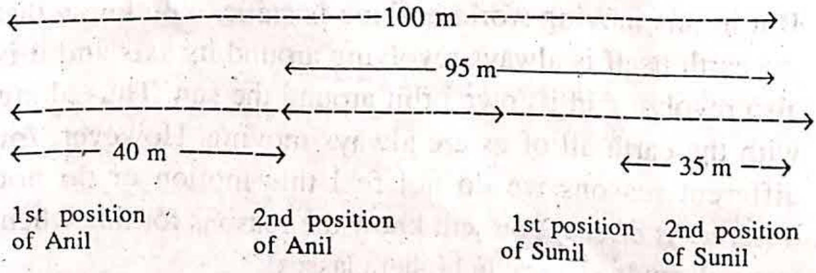
So far in the discussion about moving bodies, we assumed that bodies around the moving one are all static. But in this moving world nothing is static. You know that the earth itself is always revolving around its axis and it is also revolving in its own orbit around the sun. Thus along with the earth all of us are always moving. However, for different reasons we do not feel this motion or do not observe its effect. You will know the reasons for this when you will study science in higher classes.

In life situation we, however, come across a number of moving bodies and we are required to solve different problems connected with those moving bodies. These aspects are discussed, in short, below.

In a flat race you notice, inspite of starting from the same place, the distances between the competitors keep on changing, because their speeds are different. If two persons walk along a straight path in the same direction with different speeds or on opposite directions with any speed the distance between them also keeps on changing. Sometimes this distance increases and sometimes it decreases. Let us suppose that Anil and Sunil are walking along the same direction and their speeds are 40 m per minute and 35 m per minute respectively. Sunil is ahead of Anil by 100 metres.

(1) At the start Sunil is 100 m ahead of Anil. Now they start walking in the same direction. After 1 minute it will be seen that Anil has walked 40 m and Sunil has walked 35 m. So the distance between them will be 95 m. Thus we find

that the distance between them is reduced by (100 – 95) m = 5 m.



Similarly, at the end of the second minute the distance between them will be reduced by 5 m more. That is, in 2 minutes the distance between them will be reduced by 10 m. In this way we find that the rate of decrease of the distance between them is 5 m per minute, which is the difference of their speeds. Thus the time required for the distance between them to be reduced by 100 m is (100 ÷ 5) minutes = 20 minutes. That is, after 20 minutes they will meet together. After this Anil will be ahead of Sunil and after one minute the distance between them will be 5 m and after the next minute it will be 10 m. Thus we find that after they meet each other the distance between them goes on increasing and the rate of this increase is 5 m per minute. If they keep on walking in the same manner try to answer the following questions : (i) How many minutes after starting will Anil be 50 m behind Sunil ? (ii) How many minutes after starting will Anil be 50 m ahead of Sunil ? (iii) What is the maximum number of times that the distance between Anil and Sunil will be 50 m ? (iv) What is the maximum number of times that the distance between Anil and Sunil may be 110 m and after how many minutes ?

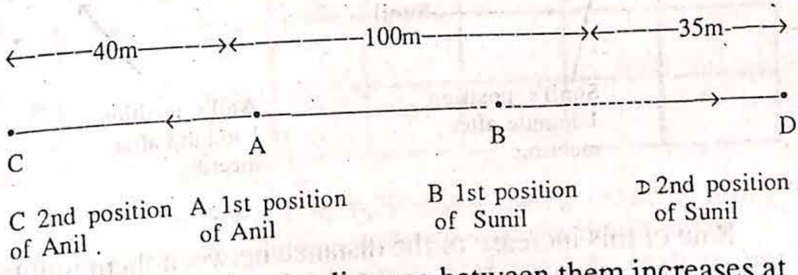
(ii) W

nce If the speed of Sunil is more than that of Anil, then the distance between them will never reduce. Let us suppose that the speed of Anil is same (i.e., 40m/min.), but the speed of Sunil is changed to 50m/min. In this case the distance between them will go on increasing at the rate of 10 m per minute. If they keep on walking in the same direction they will never meet each other. Now try to answer the following questions : (i) After how many minutes will the distance between them be 50 m more than that at the start ? (ii) What will be the actual distance between them at that time ?

(3) In the above two cases it was assumed that Anil and Sunil were walking in the same direction. Let us now see what happens if they walk in opposite directions. As before, let us suppose that the speeds of Anil and Sunil are 40 m/min. and 35 m/min. respectively and they are 100 m apart.

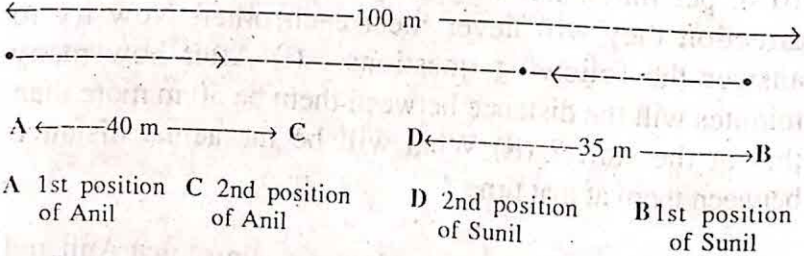
(a) If they walk in opposite directions and away from each other, the distance between them after 1 minute will be

(40+100+35) m = 175 m.

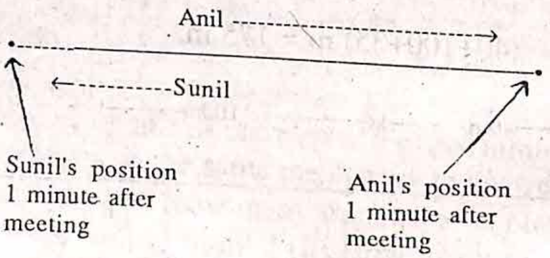


Thus we find that the distance between them increases at a rate of (175 – 100) m = 75 m per minute. Note that this rate of increase per minute is equal to the sum of the two speeds per minute. Hence, in 10 minutes the distance between them will increase by (75×10) m = 750 m. In that case total distance between them will be $(750+100) = 850$ m.

(b) If Anil and Sunil walk towards each other, then also they walk in opposite directions.



In this case in 1 minute the distance between them decreases by $(40+35)$ m = 75 m. Actual distance between them after 1 minute will be $(100 - 75)$ m = 25 m. total time required for the distance to decrease by 100 m will be $(100 \div 75)$ min $= 1\frac{1}{3}$ minutes i.e., 1 minute 20 seconds. Thus they will meet each other after 1 minute 20 seconds. After they meet, the distance between them will gradually increase.


Anil
Sunil
Sunil's position 1 minute after meeting
Anil's position 1 minute after meeting

Rate of this increase of the distance between them will be 75 m per minute. Now try to answer the following questions :

(i) After what time from the starting point will the distance between them be 50 m ?

(ii) What is the maximum number of times that the distance between them may be 50 m and after what time ?

(iii) After what time from the starting point will the distance between them be 1 km ?

Let us now suppose that the distance between Anil and Sunil and their speeds are changing at different positions. Their speeds at some positions and initial distances between them are given in the following table. Supposing that Anil walks towards Sunil and both of them walk in the same direction, fill in the gaps in the following table.

Position	Speed of Anil per minute (m) what time (min)	Speed of Sunil per minute (m)	Initial distance between Anil and Sunil (m)	Increase or decrease of the distance after 1 min. (m)	Increase of decrease of the distance after 10 minute (m)	Increase or decrease of the distance after 20 minute (m)	Will they meet ? If so after what time (min)
1st position	30	25	20				
2nd position	35	25	25				
3rd position	18	20	30				
4th position	20	35	30				
5th position	40	30	60				

For each of the three following cases make a table similar to the above and fill in the gaps of the three tables.

(a) Sunil is walking towards Anil and both of them are walking in the same directions ;

(b) Anil and Sunil are walking towards each other ;

(c) Anil and Sunil are walking away from each other in opposite directions.

Now observe the following general remarks :

(1) If two moving bodies, situated at a definite distance from each other, move in the **same direction** the distance between them may increase or decrease. If the first body moves towards the second and if its speed is more than that of the second, the distance between them will decrease at the first instance and at one stage the two bodies will meet each other. Then the distance between them will gradually increase and the first body will be ahead of the second.

(2) If the speed of the second body is more than that of the first the distance between them will go on increasing.

In both the cases the rate of increase or decrease of the distance between the bodies will be equal to the difference of their speeds.

(3) If two moving bodies move away from each other the distance between them gradually increases and the rate of this increase is equal to the sum of the speeds of the two bodies.

(4) If two moving bodies move towards each other the distance between them decreases at the beginning. After some time both of them reach the same position. Then the distance between them gradually increases.

The rate of this decrease or increase of distance is the sum of the speeds of the two bodies.

Generally speaking,

(i) if the movement is in the same direction,

the rate of increase or decrease of the distance between two moving bodies = difference of their speeds.

(ii) if the movement is in the opposite directions.

the rate of increase or decrease of the distance between two moving bodies = sum of their speeds.

Suppose that you are at a distance of 50 m from your friend, who is at rest. You walk towards him in a straight path at an uniform speed then your friend will experience a speed equal to your actual speed.

Suppose your friend is 50 m ahead.

(1) Now if both of you walk in the same direction with same speed the distance between both of you will remain same. You will have the feeling that your friend is stationary. If you walk faster than your friend the distance between both of you will gradually decrease and the rate of this decrease of distance is equal to the difference of your speeds. It will appear to your friend that you are approaching him at the above reduced speed. On the other hand, it will appear to you that your friend is receding from you at the above reduced speed.

Again if your speed is less than that of your friend, it will appear to you that your friend is going away from you. The rate at which the distance between both of your is increasing is equal to the difference of speeds of your friend and yourself. It will appear to your friend that you are moving back at a speed equal to the above reduced speed.

This type of apparent speed is called the relative speed of one with respect to the other. From the above discussion we may generally say that when two moving bodies move in the same direction, relative speed of one with respect to the other is equal to the difference of their actual speeds.

(2) Again, if you and your friend were walking in opposite directions right from the beginning then it would have appeared to both of you that you were going away from each other at an increased speed. Actually the rate of increase of the distance between both of you is equal to the sum of your speeds. In this case also the apparent speed is called the relative speed of one with respect to the other. In general we may say that if two moving bodies move away from each other in opposite directions, the relative speed of one with respect to the other will be equal to the sum of their actual speeds.

Observe that the time you will take to reach your friend when both of you walk in the same direction is more than the time you would have taken to reach him had he been stationary. Again if both of you walk towards each other the time required to meet each other would have been much less.

In case of movement in the same direction relative speed is less and so time required will be more. Again for movement towards each other, relative speed is more and so the time required will be less.

In general, we may say :

(1) **time required to cover the distance between two bodies**

$$= \frac{\textbf{That distance}}{\textbf{relative speed}}$$

(2) **relative speed × time required to cover distance between two bodies = That distance.**

(3) relative speed

$$= \frac{\text{distance between two bodies}}{\text{time required to cover that distance}}$$

Remember that if you and your friend walk away from each other in opposite directions then the distance between both of you will increase at the rate of relative speed. In that case you will never meet.

Example 1. Two cyclists start at the same time from Howrah and Asansol respectively and proceed towards each other. Their speeds are 8 km/hr and 10 km/hr respectively. If the distance between Howrah and Asansol is 200 km what will be the distance between them after 7 hrs ?

Since they are approaching each other from opposite directions their relative speed will be (8+10) km/hr = 18 km/hr, i.e., in 1 hr they come closer to each other by 18 km. Hence in 7 hrs they come closer to each other by (18×7) km = 126 km.

After 7 hrs the distance between them will be (200–126) km =74 km.

Example 2. In a flat race Meena and Anjumana ran at speeds of 140 m and 145 m per minute respectively. In the race of 580 m who won and by how many metres?

Since the speed of Anjumana was more she won the race.

She covered 145 m distance in 1 minute.
" " 1 " " " $\frac{1}{145}$ "
" " 580 " " " $\frac{1}{145}\times580$ "
= 4 minutes.

Again, since they ran in the same direction, their relative speed was (145–140) m = 5 m. Since the speed of Anjumana was more she went 5 m more per munute.

In 1 minute Anjumana went 5 m more

" 4 " " " (5 ×4) m = 20 m more.

Hence in the flat race Anjumana won by 20 m.

Exercise 8

1. At the same time two cyclists start moving towards each other from Burdwan and Howrah respectively. The distance between Burdwan and Howrah is 90 km. If their speeds are 12 km/hr and 10·5 km/hr respectively. At what time after starting will they meet each other? What is the distance of that place from Howrah?

2. Two loaded trucks start at the same time from the godown towards a destination which is 72 km away. Their speeds are 45 km/hr and 48 km/hr respectively.

   (i) After 10 minutes which truck will be ahead and by what distance?

   (ii) When one reaches the destination how far away will be the other from the destination?

   (iii) What time before the second will the first truck reach the destination ?

3. Two places A and B are 480 km apart. A truck loaded with coal starts from A towards B at a speed of 45 km/hr and 2 hours later a tempo loaded with wheat starts from B towards A at a speed of 30 km/hr. At what time after starting of tempo and at what distance from B will the truck and the tempo meet?

4. Muniya's village is at a distance of 20 km from the railway station. At 10 a.m. With an intention to go to Calcutta Muniya's father started by a cart for the railway station. After an hour Muniya got a message through a letter and started at 11·30 a.m. by a bicycle to reach that message to his father. The speeds of the cart and the bicycle are 4·8 km/hr and 12·8 km/hr respectively. At what time and at what distance from the house will Muniya be able to pass the message to his father?

5. Two buses started at the same time from Calcutta for Siliguri at a speed of 45 km/hr. At Barasat one bus had a tyre puncture. The other bus kept on moving. After half an hour the first bus got the tyre changed and started moving at a speed of 60 km/hr. At what distance from Barasat would this bus meet the other?

6. Two trains of length 200 m and 240 m approach each other at speeds 42.5 km/hr and 36.7 km/hr respectively. At what time after they meet will they cross each other?

7. A goods train, 250 m long, is running at a speed of 33 km/hr. A mail train, 200 m long, is approaching the goods train from behind at a speed of 60 km/hr. At what time after they meet will the mail train cross the goods train?

8. The distance between Dakshineswar and Diamondharbour along the river is 75 km. One motor boat starts from Dakshineswar towards Diamondharbour at 12 noon at a speed of 24 km/hr. Another motor boat starts after 2·5 minute at a speed of 27 km/hr. When will it meet the first boat?

9. Distance between two places A and B is 60 km. A man starts from A by bicycle and 3 hrs. after that his friend starts from the same place by motor cycle. If their speeds are 15 km/hr and 60 km/hr respectively, where will they meet?

10. Two boats, Ganga and Jamuna, take part in 800 m rowing competition in a lake. Moving at a speed of 5 m/sec the boat Jamuna defeats Ganga by 16 m. What is the speed of Ganga ?

11. You three brothers started walking towards the railway station at a speed of 3 km/hr. After 15 minutes your father started by rickshaw and reached station in 5 minutes along with you. What was the distance of the railway station from your home and what was the speed of the rickshaw?

12. The circumference of a circular track is 1500 m. A and B start running at the same time from the same place of the track. By the time A completes 6 rounds of the track B falls behind by one round. If the speed of B is 5 m/sec what is the speed of A?

13. One electric train and another diesel train were on two parallel tracks at a station. Their speeds were 72 km/hr and 60 km/hr respectively. They started at the same time, but the electric train had to stop for some time on the way for want of electricity. After 2 hrs

48 minutes they reached the next station together. How long did the electric train stop?

14. In a 12 km bicycle race Abhra defeats Kajal by 5 seconds. If the speed of Kajal be 18 km/hr. what is the speed of Abhra?

15. Madan and Mojammel start from the same place of 1650 m long circular track and move in opposite directions at speeds 5 m/sec and 6 m/sec respectively. When will they meet for the first time? How far will their meeting point for the second time be from their starting point?

16. In a flat race one has to touch an article at a distance of 475m and come back to the starting point. A and B start running together at speeds 15 km/hr and 13·5 km/hr respectively. When and where will A meet B on his way back after touching the article?

17. Buses start from a depot at an interval of 15 minutes at speed 16 km/hr. A man walking from the opposite direction crosses the buses at an interval of 12 minutes. What is the speed of the man?

18. Gun was fired twice at an interval of 21 minutes from a particular place. A man coming towards that place heard the sound at an interval of 20 minutes 15 seconds. If the speed of sound is 360 m/sec, what is the speed of the man in kilometres per hour?

19. Standing on the side of a railway track a man found that two trains of same length crossed him in opposite directions in 18 seconds and 12 seconds respectively. What time will the trains take to cross each other?

Displacement of moving body under the current of a river :

When a boat moves in a river with current there are two moving bodies—(1) boat with oars (2) current of the river. But there is a difference between the movements of two trains and a boat in the river. Two trains move separately with their own speeds and as a result it is easy to find the relative speed of one with respect to the other. But in case of a boat two different speeds act on the same body at the same time. Thus it is not possible to solve this type of problem by the formula for relative speed. These problems are to be solved by the formula for joint effective speed.

The things to be considered here are :

(i) Actual speed of the boat; that is the distance covered by a boat in unit time in still water under the influence of oars.

(2) Speed of the river current; that is, the distance covered by water in unit time because of high or low tide or normal flow.

(3) Joint effective speed; that is the distance covered by the boat in unit time when both its own speed and the speed of the current are simultaneously working on it.

When a boat moves in a river we get two different relations if the speeds of the current and the boat are in the same or opposite directions.

(i) When the boat moves along the current (downstream) the speed of the river-current increases the effective speed.

∴ joint effective speed= actual speed of the boat + speed of the river – current.

(ii) When the boat moves against the current (upstream) it has to proceed against the speed of the current and hence the speed of the river – current and the speed of the river current decreases the effective speed.

∴ joint effective speed=actual speed of the boat – speed of the river current.

Example 1. In still water the actual speed of a boat carried by oars is 7 km/hr and the speed of the river – current is 3 km/hr. (1) What time will the boat take to cover a distance of 20 km along the current? (2) What time will it take to come back the same distance against the current?

(1) We know that when the boat goes along the current the joint effective speed will be (7+3) km/hr or, 10 km/hr.

Hence the time required to cover a distance of 20 km along the current will be $\frac{20}{10}$ hrs or, 2 hrs.

(2) We know that when the boat goes against the current the joint effective speed will be (7 – 3) km/hr or, 4 km/hr.

Hence the time required to cover a distance of 20 km will be $\frac{20}{4}$ hrs or 5 hrs.

Exercise 9

1. In still water the speed of a boat carried by oars is 10 km/hr and the speed of the current of the river is 4 km/hr.

   (i) What time will the boat take to cover a distance of 35 km along the current?

   (ii) What time will be boat take to cover a distance of 30 km against the current?

2. The speed of the current of a river is 3 km/hr and actual speed of a boat in still water is 9 km/hr. How far will that boat go in that river in

   (i) moving down stream for 3 hrs 20 min?

   and (ii) moving upstream for 8.5 hrs ?

3. A boat covers a distance of 40 km in 4 hrs along the current. If the speed of that boat in still water be 8 km/hr what is the speed of the river-current?

4. A boat takes 5 hrs to go 20 km against the current. If the speed of the river-current be 3 km/hr, what is the actual speed of the boat?

5. A boat whose actual speed is 7 km/hr takes 3 hrs to cover a distance of 12 km downstream. What time will it take to come back to the starting point?

6. Distance of A and B along a river is 48 km. A boat with actual speed of 12 km/hr takes 3 hrs to cover that distance downstream. What distanee will it cover in 3 hrs against the current?

7. In still water a man can row 10 km in 1 hr. But upstream he takes 5 times that time to cover the same distance. What is the speed of the river-current?

8. A man can travel by a boat 9 km/hr along the current and 5 km/hr against the current. Find the actual speed of the boat and that of the river current.

9. With the help of oar a boat can travel 6 km/hr in still water. Against the current it takes thrice the time to travel 6 km. What time will it take to travel 50 km along current?

10. A boat travels 36 km along the current in 3 hrs. To come back it takes 6 hrs. What time will it take to travel the same distance in still water?

11. A swimmer can swim 5 km in one hour in still water. During low tide he swims in the Ganges from Ahiritola to Kidderpore,12 km apart, in 1 hr 12 min. What is the speed of the low tide?

12. A boat can travel 7 km/hr in still water. That boat takes 4 hrs to go from Diamondharbour to Birlapur at the time of high tide. But it takes 3 hrs 45 minutes to come back to Diamondharbour during low tide. If the distance between Diamondharbour and Birlapur be 48 km, find the speeds of the high and low tides.

13. A motor boat can travel in still water at a speed of 35 km/hr. At the time of high tide it goes 50 km against the tide in 1 hr 40 minutes. What time would it have taken to cover the same distance had it gone along the tide?

14. An anchored boat on a quay snapped the rope because of the high tide and was carried away to a distance of 6.30 km in 1 hr 12 min. Afterwards the boatman rowed it back to that quay in 2 hr 48 min. What was the actual speed of the boat when rowed?

15. Speeds of two boats A and B in still water are 10 km/hr and 12 km/hr respectively. At 10 a.m. A starts along the current and B starts against the current from the places C and D respectively. If the distance between C and D be 121 km when will A and B meet?

16. A boat takes 10 hrs to travel a distance along the current. It takes 20 hrs to travel the same distnace aginst the current. If the speed of the boat in still water be 9 km/hr, what is the distance?

17. Speeds of a boat along and against the current are 7 km/hr and 5 km/hr respectively. Had the speed of the boat been 1 km less and

that of the current been 1 km more, what more or less time would the boat have taken to travel 15 km against the current? What would have happened had the boat travelled along the current?

18. Speeds of the two boats A and B downstream are 9 km/hr and 8 km/hr respectively. If the speed of A be 7 km/hr upstream what more time will be taken by which boat to travel a distance 112 km in still water?

Chapter 5

Time and Work

Things connected with problems on time and work are: (1) Span of time (2) amount of work (3) number of agents doing work.

Let us first discuss their mutual relationships.

(1) What will be the change of time in finishing the work when the number of persons is kept unchanged and the amount of work is increased or decreased?

Ex. 1. A cultivator can plough 3 'bighas' of land in 5 days. In how many days will he plough 12 'bighas' of land?

the problem is : amount of land number of days (in bighas)

3	5
12	?

By unitary method :

Time taken to plough 3 bighas of land is 5 days

∴ " " " " 1 " " " " $\frac{5}{3}$ "

∴ " " " " 1 " " " " $\frac{5}{3} \times 12$ days

= 20 days

Hence 20 days will be required to plough 12 bighas of land.

Thus we find that there is a direct relationship between amount of work and span of time, that is more time will be required if the amount of work increases and less time will be required if the amount of work decreases. Moreover, changes in their values are proportional.

(2) What will be the change in the amount of work when the number of persons is increased or decreased keeping the time for finishing the work unchanged?

Ex. 2. 9 workers can produce 6 bicycles in one week. In a bicycle factory there are 72 workers. On being asked how many bicycles will be produced in that factory in a week, Basantika replied, "50". Is Basantika right?

The problem is :

Number of workers	amount of work, that is, no. of bicycles produced
9	6
72	?

By unitary method :

In one week 9 workers produce 6 bicycles

" " " 1 worker " $\frac{6}{9}$ "

" " " 72 workers " $\frac{6}{9} \times 72$ "

$= 48$ bicycles. 4

∴ In one week 72 workers will produce 48 bicycles. Hence Basantika's reply was not right.

Thus we find that there is a direct relationship between number of persons doing the work and the amount of work, that is, if the number of persons is increased the amount of work will also increase and if the number is decreased the amount of work will decrease. In this case also changes in their values are proportional.

(3) What will be the change in time when the number of persons is increased or decreased keeping the amount of work unchanged?

Ex. 3. Irrigation canal of a village can be cleaned by 24 villagers in 12 days. If 36 villagers do the work, in how many days will the canal be cleaned?

The problem is :

number of persons	time (in days)
24	12
36	?

By unitary method :

Time required by 24 persons is 12 days

" " " 1 person " 12×24 days

[In the above case the work of 24 persons has to be done by 1 person and so more time will be required].

$\therefore$ time required by 36 persons is $\dfrac{12 \times 24}{36}$ days

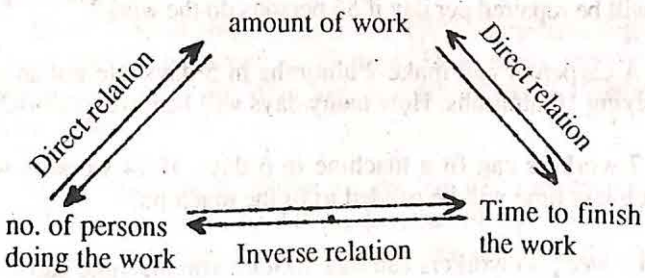
$= 8$ days

[Because more persons will require less time.]

$\therefore$ the work will be finished in 8 days if 36 villagers do the work.

Thus we find that there is an inverse relationship between number of persons and time required; that is less time will be required if the number of persons is increased and more time will be required if the number of persons is decreased. Here it is to be observed that the rate of increase of one is same as the rate of decrase of the other.

The above relationships are expressed in the following diagram.



working. What is the total number of days required to plough the total land of the farm?

15. The quantity of goods carried by a buffalo cart in a certain time is $\frac{1}{10}$th of that carried by a truck in the same time. If 3 trucks and 5 buffalo carts, working 8 hrs a day, can carry 490 tons of goods from a godown, how many trucks should be engaged with 12 buffalo carts to carry 3136 tons of goods in 7 days?

16. A contractor undertakes the work of covering 1200 m of road by pitch in 15 days. 700 m of road is covered by pitch by 42 workers, working 8 hrs a day in 9 days. After this 2 workers fell sick. How many additional workers, should be employed to finish the work in time working 8 hrs a day ?

17. 60 persons can build a house in 250 days. 200 days after beginning,, the work had to be stopped for 10 days because of rain. How many more persons have to be employed to finish the work in time? [S.F. 1958]

18. 12 persons, working 9 hrs a day, can finish a work in 30 days. How many persons, working 5 hrs a day, will finish 10 times the work in 24 days?

19. A contractor employed 160 labourers to make 1920 m of an embankment in 120 days. After 24 days it was found that only 240 m had been constructed. How many more labourers should be employed to finish the work in time?

20. 8 men or 16 boys can finish a work in 53 days. In how many days will 19 men and 15 boys finish a work 3 times the previous work?

21. If 3 men or 4 boys, working 8 hrs a day , can do a work in 65 days, in how many days will 7 men and 8 boys working 10 hrs a day, do $2\frac{1}{2}$ times the previous work ? is work?

22. To finish a development work of a village, 60 villagers started working 8 hrs a day. But after elapsing of $\frac{3}{4}$th time it was found that $\frac{2}{5}$ of the work had been finished. How may more villagers should be involved in the work so that all of them, working 10 hrs a day, can finish the work in time?

Harder problems on time and work.

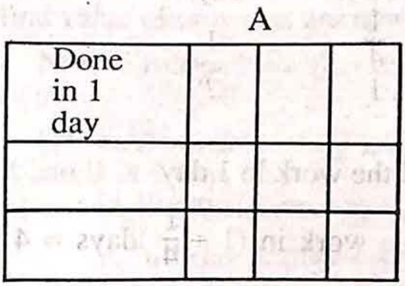
Ex. 1. A alone can do a work in 12 days; B alone can do the same work in 6 days. In how many days will A and B together do the work?

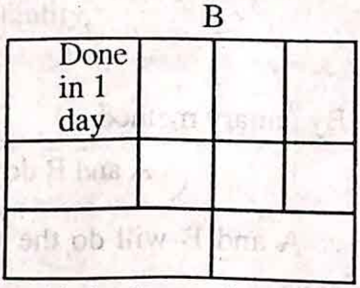
Let the work be ploughing a peice of land of definite area and let us take the whole work as 1.

Let us now observe what happens when A and B separately plough land of same area:

A alone does the work in 12 days. B alone does the work in 6 days.

$\therefore$ In 1 day A does $\frac{1}{12}$ part. In 1 day B does $\frac{1}{6}$ part





Now let us observe how the work is done when they work together :

A does in 1 day	A	A	A
B does in 1 day	B	B	B

In the figure we find that in 1 day A+B do $1\frac{1}{12}$ part + $\frac{1}{6}$ part = $\frac{1}{4}$ part, Again we find that in this manner both of them have to work for 4 days. Hence they will finish the work in 4 days.

Let us now work out the sum following the steps of the above analysis :

In 12 days A does the whole work

$\therefore$ In 1 day A does $\frac{1}{12}$ of the work

In 6 days B does the whole work

$\therefore$ In 1 day B does $\frac{1}{6}$th of the work

$\therefore$ In 1 day A and B do $(\frac{1}{12} + \frac{1}{6}) = \frac{1}{4}$ of the work

Now the problem is:

Work	days
$\frac{1}{4}$	1
1	?

By unitary method :

A and B do $\frac{1}{4}$ of the work in 1 day

$\therefore$ A and B will do the whole work in $(1 \div \frac{1}{4})$days = 4 days.

N.B. We know that the relation between amount of work and time is directly related; that is, time will increase with the increase of the amount of work. Thus when the amount of work was $\frac{1}{4}$ of the whole work the time was 1 day; when the amount of the work increased to whole work the time increased to 4 days. Again, it should be remembered that if any number is divided by a fraction less than 1 its value is increased.

Ex.2. A and B can do a work individually in 12 and 8 days respectively. They started doing the work together. But after 3 days A fell sick. B alone did the rest of the work. In how many days was the work finished?

A and B in 3 days ? part	B in? days ? part

We find that the work was done in two parts—

(1) First part was finished by A and B together in 3 days.

(2) Second part was finished by B alone in some days.
Further we find that in the first part there is one unknown quantity; whereas in the second part there are two unknown quantities. Hence the working of the sum should be started with the first part,for by unitary method we can find value of only one unknown quantity.

Now observe how the sum should be worked out in steps:

(1) First we are to find what part of the work is done by A and B in 1 day.

In 12 days A does the whole work

$\therefore$ In 1 day A does $\frac{1}{12}$ of the work

In 8 days B does the whole work

$\therefore$ In 1 day B does $\frac{1}{8}$ of the work

$\therefore$ In 1 day A and B do $(\frac{1}{12} + \frac{1}{8}) = \frac{5}{24}$ th of the work.

(2) Now we are to find what part of the work is done by A and B in 3 days.

In 1 day A and B do $\frac{5}{24}$ th of the work

$\therefore$ In 3 days A and B do$(\frac{5}{24} \times 3)$th $= \frac{5}{8}$ th of the work.

(3) Now we are to find what part of the work is left which has to be done by B alone.

Part of work left $= 1 - \frac{5}{8} = \frac{3}{8}$

(4) Now we are to find in how many days B alone will do $\frac{3}{8}$ of the work.

B alone does the whole work in 8 days

∴ B alone will do $\frac{3}{8}$th of the work in $(8 \times \frac{3}{8})$days
$= 3$ days

(5) Now we are to find in how many days the whole work has been finished.

A and B do the first part in 3 days
B alone does the second part in 3 days
Time required to finish the whole work is 6 days.
∴ The work is finished in 6 days.

Ex. 3. A man does double the work done by a boy at the same time. In how many days will 3 men and 4 boys finish a work which can be done by 10 man in 8 days ?

It is given that work of 2 boys = work of 1 man.

∴ work of 1 boy = work of $\frac{1}{2}$ man

∴ work of 4 boys = work of$(\frac{1}{2} \times 4)$ men
= work of 2 men

∴ work of 3 men and 4 boys
= work of (3+2) men = work of 5 men.

Thus the problem is :

no. of men	days
10	8
5	?

10 men can do the work in 8 days
∴ 1 " " " " " " (8×10) days
∴ 5 " " " " " " $\frac{8 \times 10}{5}$ days = 16 days

Hence 3 men and 4 boys will do the work in 16 days.

Exercise 11

1. A and B individually can do a work in 10 hrs and 12 hrs respectively. How long will they take to finish the work if they work together?

2. A, B and C individually can do a work in 6 days, 8 days and 12 days respectively. How long will they take to finish the work if they work together?

3. Rahim alone can clean the weeds of a jute field in 24 hrs. But his younger brother takes 30 hrs to do the same work. Working together how long will they take to finish the work?

4. Ram, Shyam and Jadu individually can plough a garden in 18, 21 and 24 hours respectively. How long will they take to finish the work if they work together?

5. Working individually Mahim and Naresh can plant paddy in a field in 8 and 12 hours respectively. In the morning Mahim worked for 2 hrs and left. Naresh came afterwards and finished the work. In what time was the whole field planted?

6. A, B and C individually can do a work in 10, 12 and 15 days respectively. If each of them work separately for 2 days one after another, what protion of the work will be left?

7. Working individually Nabin, Suresh and Abhay can do a work in 16, 20, 24 days respectively. How long will each of the pairs Nabin and Suresh, Suresh and Abhay, Nabin and Abhay take to finish the work?

8. A certain work can be done by A and B in 12 days, by B and C in 15 days and by A and C in 20 days. How long will they take to finish the work if three of them work together? How long will each one of them take to finish the work?

9. A B and C individually, can do a peice of work in 10, 12 and 15 days respectively. They start working together. After 3 days B leaves because of illness. How long will A and C take to finish the rest of the work?

10. Three masons Shripati, Selim and Haran, working with their own groups of workers, can separately build a house in 30, 36 and 45 days respectively. All the three groups started working together. Shripati left with his group after 3 days and after 5 more days Haran also left with his group. Rest of the work was finished by Selim with his group. What time was taken to build the house?

11. A and B individually can do a work in 20 and 30 days respectively. After working together for 7 days they left. C came and finished rest of the work in 10 days. How long will C take to to finish the whole work?

12. 2 men do as much work as 3 women. How long will 5 men and 15 women take to finish a work which can be done by 4 men and 9 women in 30 days?

13. A man does double the work of a boy. 6 men and 12 boys can do a piece of work in 18 days. How long will 24 men take to do that work?

14. 2 men and 6 boys can do a work in 6 days.That same work can be done by 5 men and 1 boy in 3 days. How long will 1 man and 1 boy, working together, take to finish the same work?

15. A work can be done by 1 man and 1 woman in $7\frac{1}{5}$ days, by 1 woman and 1 boy in 12 days and by 1 man and 1 boy in 9 days. How long will 1 man, 1 woman and 1 boy, working individually, take to do the work?

16. Manju and Sandhya can paint a design in 12 hours. After they worked for 4 hours their friend Manika joined them to do the work and the rest of the work was finished in 2 hrs 40 mins. How long would Manika have taken to finish the whole work had she worked alone?

17. 1man, 1 woman and 1 boy together can finish a work in 5 days. But 5 women and 5 boys can do $\frac{7}{12}$ th of the work in 1 day. How long will 1 man take to finish the work alone?

18. A and B individually can do a work in 10 days and 14 days respectively. A alone worked for 4 days and then B alone worked for 5 days. The rest of the work was finished by C in 8 days. How long would A, B and C have taken to do the work had they worked together?

19. 3 men and 5 women can do a work in $2\frac{2}{5}$ days. 6 men and 4 women can do that work in $1\frac{7}{8}$ days. How long will 9 men and 10 women take to do the work?

20. A and B together can do a work in 6 days and B and C together can do it in 12 days. If C alone can do it in 36 days, how long will A, B and C, working together, take to finish the work?

21. A and B together can do a work in 15 days. They worked together for 9 days and then A alone finished rest of the work in 12 days. How long would each one of them have taken to do it had they worked separately. [S.F. 1963]

22. A work can be done by A and B in 8 days and by B and C in 12 days. But if three of them work together they can finish the work in 6 days. In what time will A and C, working together, finish the work?

23. A work can be done by A and B in 3 hrs 44 mins and by A alone in 7 hrs. A starts the work and they work alternately after every 1 hr. In what time will the work be finished?

24. A B and C individually can do a work in 10, 12 and 15 days respectively. They start working together. But 5 days before the work is finished A leaves and B leaves 3 days before it is finished. What time is taken to finish the work ? [S.F. 1954]

Problems on container of liquid:

It is generally found that tap is used to fill cistern or other container of the same kind with water or any other liquid or to take out liquid from the container. The time required to do the same varies with the size of the tap. Again if there be a hole in the container liquid will go out of it. Problems connected with the above are as follows:

(1) To fill whole or part of an empty container by a tap through which liquid enters (inlet tap)

(2) To empty whole or part of a full container by a tap through which liquid goes out (outlet tap).

(3) To find the result when more than one tap of the same type or of different types work together.

Problems in nos. 1 and 2 can be solved by general rules of time and work. You have already learnt how to work out such problems.

Problems in no. 3 may be of two types. Let us consider the following examples.

(1) When more than one tap of same type work together the result is the sum of their individual work.

Ex. 1. A cistern is connected with two inlet taps for filling water. First and second taps can fill the empty cistern in 6 and 12 hours respectively. If both the taps are opened together how long will it take to fill the cistern?

Let us suppose that the capacity of the whole cistern be 1.

Now in 6 hrs the first tap fills the whole cistern

$\therefore$ in 1 hr it fills $\frac{1}{6}$th of the cistern

in 12 hrs the second tap fills the whole cistern

$\therefore$ in 1 hr it fills $\frac{1}{12}$th of the cistern

---

$\therefore$ in 1 hr the two taps fill $(\frac{1}{6} + \frac{1}{12})$th or $\frac{1}{4}$ th of the cistern.

Now the problem is:

work	time (in hr)
$\frac{1}{4}$	1
1	?

Two taps fill $\frac{1}{4}$th of the cistern in 1 hr

$\therefore$ two taps fill the whole cistern in $(1 \div \frac{1}{4})$hrs or 4 hrs.

$\therefore$ It will take 4 hrs to fill the cistern if both the taps are kept open.

Ex. 2. Petrol can be taken out of a petrol tanker by two taps. The two taps, separately, can empty the full tanker in 4 and 12 hours respectively. If both the taps are opened together how long will it take to emtpy the full tanker?

In 4 hrs first tap empties the full tanker

$\therefore$ In 1 hr first tap empties $\frac{1}{4}$th of the tanker.

In 12 hrs second tap empties the full tanker

$\therefore$ In 1 hr second tap empties $\frac{1}{12}$th of the tanker

$\therefore$ in 1 hr two taps empty $(\frac{1}{4}+\frac{1}{12})$ or $\frac{1}{3}$ of the tanker.

Now the problem is :	work	time (in hr)
	$\frac{1}{3}$	1
	1	?

The two taps empty $\frac{1}{3}$ of the tanker in 1 hr

$\therefore$ the two taps empty the full tanker in $(1 \div \frac{1}{3})$ hrs or 3 hrs

$\therefore$ If the two taps are opened it will take 3 hrs to empty the full tanker.

N.B. Compare the above two examples with Ex. 1 at page 63.

(2) When more than one tap of two types work together the container is filled up by the inlet tap and simultaneously it is emptied by the outlet tap. Because of this two things may happen.

(i) When the inlet tap fills more than what is emptied by the outlet tap at the same time, an empty or partially full container gradually gets more and more filled up. The rate of this filling = rate of filling by the inlet tap—rate of emptying by the outlet tap.

Ex. 3. A cistern is fitted with two taps. By the first the whole cistern can be filled in 10 minutes and by the second the full cistern can be emptied in 15 minutes. When the cistern is half full both the taps are opened. In how much time will the whole cistern be full or empty ?

In 10 min the first tap fiills the whole cistern

$\therefore$ in 1 min the first tap fills $\frac{1}{10}$ of the cistern.

In 15 min the second tap empties the whole cistern
$\therefore$ in 1 min the second tap empties $\frac{1}{15}$ of the cistern.

Here we find that the rate of filling is more than that of emptying, because $\frac{1}{10}$ part is more than $\frac{1}{15}$ part. Hence in this case the half full cistern will be more and more full.

In 1 min the first tap fills $\frac{1}{10}$ part
In 1 min the second tap empties $\frac{1}{15}$ part

$\therefore$ in 1 min the two taps will fill $(\frac{1}{10}-\frac{1}{15})$ part
$=\frac{1}{30}$ part.

Application of algebraic principle : We have discussed positive and negative numbers in class VII. Similarly in this case if 'filling' is taken as positive, 'emptying' will be negative. That is, if in 1 min the first tap "fills $\frac{1}{10}$ part" be taken as positive in 1 min the second tap "empties $\frac{1}{15}$ part" will be negative and when expressed by negative number it will be in 1 min the second tap "fills$-\frac{1}{15}$ part". Then the result of their joint work will be :

In 1 min the first tap fills $\frac{1}{10}$ part
In 1 min the second tap fills $-\frac{1}{15}$ part

$\therefore$ in 1 min the two taps fill $\{\frac{1}{10}+(-\frac{1}{15})\}$ part
$=(\frac{1}{10}-\frac{1}{15})$ part $=\frac{1}{30}$ part.

Now the problem is :

part	time (in min)
$\frac{1}{30}$	1
$\frac{1}{2}$	?

The two taps fill $\frac{1}{30}$ part in 1 min.
The two taps fill the whole cistern in $(1\div\frac{1}{30})$ min$=30$ min

The two taps fill $\frac{1}{2}$ of the cistern in $(30 \times \frac{1}{2})$ min $= 15$ min.

$\therefore$ When both taps are opened together the half full cistern will be full in 15 min.

(ii) When the outlet tap empties more than what is filled by the inlet tap at the same time full or partially full container gradually gets more and more empty. The rate of this emptying is

rate of emptying by outlet tap—rate of filling by inlet tap.

Ex. 4. A cistern has two taps. First tap can fill the whole cistern in 18 minutes and the second tap can empty the full cistern in 12 minutes. Both the taps are opened simultaneously when the cistern is half full with water. In how much time will the whole cistern be full or empty ?

In 18 min the first tap fills the whole cistern

$\therefore$ in 1 min the first tap fills $\frac{1}{18}$ of the cistern.

In 12 min the second tap empties the whole cistern

$\therefore$ in 1 min the second tap empties $\frac{1}{12}$ of the cistern.

Here we find that the rate of emptying is more than that of filling, because $\frac{1}{12}$ part is greater than $\frac{1}{18}$ part.

Hence in this case the half-full cistern will become more and more empty.

In 1 min the second tap empties $\frac{1}{12}$ part

In 1 min the first tap fills $\frac{1}{18}$ part

$\therefore$ in 1 min the two taps empty $(\frac{1}{12} - \frac{1}{18})$ part

$= \frac{1}{36}$ part.

Application of algebraic principle :

In 1 min the first tap fills $\frac{1}{18}$ part means in 1 min the first tap empties $-\frac{1}{18}$ part.

In 1 min the first tap empties $-\frac{1}{18}$ part

In 1 min the second tap empties $\frac{1}{12}$ part

$\therefore$ in 1 min the two taps empties $\{(-\frac{1}{18})+(\frac{1}{12})\}$ part $= (\frac{1}{12}-\frac{1}{18})$ part $= \frac{1}{36}$ part.

Now the problem is :

part	time (in min)
$\frac{1}{36}$	1
$\frac{1}{2}$	?

$\therefore$ the two taps empty $\frac{1}{36}$ part in 1 min

$\therefore$ the two taps will empty the whole cistern in

$$(1 \div \tfrac{1}{36}) \text{ min} = 36 \text{ min}$$

$\therefore$ the two taps will empty $\frac{1}{2}$ of the cistern in

$$(36 \times \tfrac{1}{2}) \text{ min} = 18 \text{ min.}$$

$\therefore$ When the two taps are opened simultaneously the half-full cistern will be empty in 18 min.

Exercise 12

1. There are two taps fitted to a cistern. The cistern may be filled in 12 and 15 minutes respectively by the two taps. How long will it take to fill the empty cistern if the two taps are opened simultaneously ?

2. Petrol can be taken out of a tanker by two taps. The two taps, individually, can empty the full tanker in 20 and 30 minutes respectively. If the two taps are opened simultaneously, how long will it take to empty half-full tanker ?

3. An empty cistern can be filled in 12 minutes by a tap. There is a hole in the cistern which can empty the full cistern in 18 minutes. If the inlet tap is opened without closing the hole, how long will it take to fill the empty cistern ?

4. Total amount of water that the bottom of a boat holds can be taken by a pipe in 20 min. But there is a hole at the bottom of the boat through which water enters and fills the empty boat in 40 min. If the outlet pipe is opened without closing the hole how long will it take to empty the boat fully ?

5. There are two taps through which oil can be taken out from a barrel. The two taps can empty the full barrel in 25 and 35 minutes respectivley. 5 minutes after opening of the first tap the second tap is also opened. How long will it take to take all oil out of the full barrel ?

6. An empty cistern can be filled by an inlet tap in 20 min and the full cistern can be emptied by an outlet tap in 70 min. How long will it take to fill the empty cistern if the two taps are opened simultaneously ?

7. An inlet tap can fill an empty barrel of kerosene oil in 40 min. An outlet tap can take out all oil from the full barrel in 60 min. When the barrel was $\frac{2}{3}$ full, oil was taken out of it by the outlet tap for 15 min. Then this tap was closed and the inlet tap was opened. In what time was the barrel full ?

8. The water tank on the roof of a house can be filled by a tap in 30 min. If the tap for supplying water to the house be opened the full tank is emptied in 45 min. If the two taps work simultaneously when the tank is half-full, how long will it take to fill it ,

9. Because of sudden development of a hole oil falls from a full tanker for 10 min. The hole is then closed and the tanker was filled again by the inlet tap in 4 min. If the empty tanker can be filled by the inlet tap in 28 min, in what time all oil would have fallen through the hole had it not been detected ?

10. An empty cistern is filled by the inlet tap in 45 min and the full cistern is emptied by the outlet tap in 20 min. When $\frac{2}{3}$ of the cistern is full the outlet tap is opened in order to clean the cistern. After the cistern is emptied it is cleaned for 10 min. Then the outlet tap is closed and the inlet tap is opened. How long will it take to fill $\frac{1}{2}$ of the cistern right from the start ?

11. The cistern of a house can be filled by the municipal water supply pipe in 30 min. By opening the tap for household work the water of full cistern can be used for 4 hrs. One day the water supply pipe was open only for 25 min. How long could the household work be done with the water supplied on that day ?

12. The water tank on the roof of a house can be filled by the water supply pipe in 35 min. There are two more pipes to distribute water to different parts of the house. Through these two pipes, individually, all water of a full tank goes out in 120 and 140 min respectively. The tank was half-full when all the three pipes were opened at 6 a.m. If the supply pipe is closed after 28 min, when will all water of the tank be exhausted ?

Miscellaneous Exercise 1

[Give the answer correct upto two decimal places, if it is not otherwise stated in the question.]

Averages

1. Answer the following questions orally :

(i) What is the average of 5, 6, 8, 10 and 11 ?

(ii) The averagc of 5 numbers is 13, what is the sum of the numbers ?

(iii) The average of some numbers is 9 and their sum is 99. What is number of those numbers ?

(iv) The average age of three students is 10 years, the average age of those three students and another new student is 11 years. What is the age of the new student ?

(v) Uttam, Sudip and Anup have 17 marbles to them. How many more marbels should they have so that average of their marbles be 7 ?

(vi) Price of three pencils of first kind ts 60 paise each and price of two pencils of second kind is 70 paise each. What is the average price of each of those pencils ?

(vii) Average rainfall on Monday and Tuesday is 30 cm and the rainfall on Wednesday is 18 cm. What is the average rainfall on these three days ?

(viii) The average of 10, 11, 12, 13, and another number is 10. What is the other number ?

2. Weights of 5 men are 55·2 kg, 57·5 kg, 60 kg, 45·4 kg and 50 kg. respectively. What is their average weight ?

3. Among 6 boys the heights of first 5 boys are 131·2 cm, 127·8 cm, 126·9 cm, 132·4 cm and 130 8 cm respectively. If the average height of the 6 boys be 129 cm, what is the height of the sixth boy ?

4. In 4 months the price of paddy per quintal was Rs. 125·50, Rs. 129.50, Rs. 132 and Rs. 135 respectively. What was the average price of paddy per quintal and what was the average rise in price of paddy per quintal during those 4 months ?

5. The average monthly salary of 8 men is Rs. 295, monthly salaries of 2 more men are Rs. 2400 and Rs. 2350 respectively. What is the average monthly salary of these 10 men ?

6. For the first 10 minutes a train travelled at a speed of 35 km/hr and for the next 5 minutes it travelled at a speed of 20 km/hr. What was the average speed of the train ?

7. 5 kg of tea at the rate of Rs. 22 per kg, 3 kg at the rate of Rs. 28 per kg and $2^1/_2$ kg at the rate of Rs. 35 per kg are mixed together. What is the average price of tea per kg ?

8. Average mark of 6 boys is 53. The first boy has got 2 marks more than the second boy. The average mark of the rest 4 boys is 49. What are the marks of the first and the second boys, individualy ?

9. A fruit vender sold 32 baskets of mango at Rs. 55 per basket, 41 baskets at Rs. 35 per basket and 15 basket at Rs. 46 per basket. What was the average price of mango per basket ?

10. Average daily income of 8 men is Rs. 13·20 and that of another group of 14 men is Rs. 12·10. What is the average daily income of those 22 men ?

11. A book seller sells 15, 24 and 23 books at the rates of

Rs. 4·50, Rs. 9·40 and Rs. 11·50 per book respectively. What is the average price of each book ?

12. Runs of a cricketer in 10 innings are 41, 47, 50, 39, 48, 12, 42, 32, 20 and 117. What is his average run per innings ?

13. What is the average of 75, 85 and 70. What is the average in nearest whole number ?

14 Runs of Kapil Dev, Gavaskar and Vengsarkar in three innings are as follows :

Kapil Dev : 41, 89, 65 ; Gavaskar : 48, 24, 2 ; Vengsakar : 2, 159, 12

What is the average run per innings of each in nearest whole numbers ? Whose average run is highest ?

15. A train travels 500 m in the first minute. In the next 4 minutes it travels in each minnte 125 m more than that in the previous minute. What is the average speed per hour of the train during those 5 minutes ?

16. The sale proceeds of a shopkeeper during the first five months are Rs. 6435, Rs. 6927, Rs. 6855, Rs. 7230 and Rs. 6562 respectively. What should be the sale proceed of the sixth month so that the average sale proceed during these 6 months is Rs. 6500 ?

17. Average age of 28 boys of class X of a school was 16 yrs 3 mths. 8 boys left the class and the average became 2 months less. What was the average age of those 8 boys ?

18. 11 kg of tea of Rs. 28·50 per kg is mixed with 9 kg of tea at Rs. 22 per kg. What will be the average price per kg of the mixed tea ?

19. For Saraswati Puja of the school Sujit bought $3^1/_2$ kg of apple at Rs. 8·50 per kg. What quantity of apple should be bought at Rs. 12 per kg, so that the average price of all apples becomes Rs. 10 per kg ?

20. A cultivator has 5 acres of paddy land and a vegetable garden. Yearly production of paddy is 8 quintal per acre. Average price of paddy per quintal is Rs. 128. His yearly income from the vegetable garden is Rs. 900. What is the average monthly income of that cultivator ?

21. The length of a sappling increases by 12 cm in the first three months. Average monthly increase of its length during the next 3 months is 3·3 cm. What is the average increase in length of the sappling during those 6 months ?

22. Mahim went to gopalpur at a speed of 4·8 km/hr and came back from there at a speed of 6 miles/hr. What was his average speed per hour ? [1 mile = 1·6 km]

23. Rahman went to a place by taxi at a speed of 1 km/min and came back at a speed of $1^1/_2$ km/min. What was the average speed of the taxi per hour ?

24. Runs of some Indian players in some cricket matches between India and New Zealand are given below. Find average run of each per innings.

Pankaj Roy—4 innings : 0, 28, 100, 173
Vinoo Mankad—5 innings : 30, 223, 25, 17, 231
Polly Umrigar—6 innings : 223, 15, 18, 1, 5, 79
Manjreker—5 innings : 118, 0, 177, 1, 90

25. 30 boys of a class are divided into six equal groups. Average ages of the boys of those six groups are 10 yrs 6 mths, 10 yrs 8 mths, 10 yrs 9 mths, 10 yrs 11 mths, 11 yrs and 11 yrs 1 mths respectively. What is the average age of all the boys of the class ?

26. The population of a town in 1931 and 1951 were 698078 and 854898 respectively. What was the average increase in population of the town during those 20 years ?

27. A man travelled $9\frac{1}{2}$ km and $8\frac{1}{2}$ km in the first 2 hrs and then took rest for half an hour. He travelled the rest of the distance in next 3 hrs walking uniformly at a speed of 7 km/hr. What is the total distance travelled ? What is the average speed of the man per hour ?

28. Mr. Sarkar bought a car for Rs. 85000, His yearly expenditure on driver, petrol and maintenance was Rs. 6300. After 5 years he sold the car for Rs. 55000. What was his average yearly expenditure during those five years for the car ?

29. Income and expenditure of Nirmalbabu for one year

are given in the following table. Find his average monthly income, expenditure and savings for that year.

Month	Total income (Rupees)	Total expenditure (Rupees)
From January to April	1575·50	1186·25
From May to September	2050·00	1680·50
From October to December	1476·75	1314·00

30. Five candidates of a political party polled 28330, 15154. 28170, 46810 and 35034 votes in five constituencies. What was the average vote per constituency ? How many votes should another candidate of that political party poll at the sixth constituency, so that the average vote per constituency may increase by 480 ? (give your answers in nearest whole numbers.)

31. Daily wage of a labourer is Rs. 8·50. In a year he got the opportunity for working 185 days. What was his average daily income during that year ?

32. (i) Marks obtained by some boys and girls in English. Mathematics and History are given in the following table.

Subject	Names of Boys						Name of Girls			
	Debu	Satya	Subha	Latif	John	Paltan	Ruma	Jhuma	Rinku	Chandra
Marks in English	46	31	20	31	41	54	35	45	56	40
Marks in Mathematics	56	47	66	61	58	51	50	60	73	74
Marks in History	41	47	31	42	36	40	51	61	47	90

Find the averages from the marks of the boys and girls given in the above table and fill up the gaps in the following table.

	English	Mathematics	History	Total marks
Average marks of boys				
Average marks of girls				
Average marks of boys & girls				

32. (ii) Let us suppose that a student will be marked 'good' if his mark is more than the average and he will be marked 'weak' if his mark is less than the average—following this definition compare the two tables of 32 (i) and write 'good' or 'weak' in the blank spaces of the following table.

	Debu	Subha	John	Ruma	Chandra
English					
Mathematics					
History					
All Subjects					

33. Average weight of 12 parcels is 1·85 kg. Addition of another new parcel makes the average weight 100 gm less. What is the weight of the new parcel ?

34. Number of seats and the price of 1 ticket per seat of a theatre hall are given in the following table, Find the average price per ticket for all seats of the hall.

Price per ticket	Rs. 2	Rs. 5	Rs. 10
No. of seats	385	235	80

35. Average height of 28 boys is 1·4 m. When 8 boys leave the group the average height increases by ·25 m, what will be the average height of the boys who leave ?

36. Numbers of unemployed in the first 4 years are 4069 thousand, 5100 thousand, 6895 thousand and 8218 thousand. Numbers of unemployed in second 4 years are 8433 thousand, 9320 thousand, 9784 thousand and 10924 thousand. What is the increase in average yearly unemployed of the seconnd 4 years over the first 4 years ?

37. Numbers of vehicles travelling on a road on a particular day in every half an hour from 10 a.m to 1 p.m. are given in the following table :

Time	10 to 10-30	10-30 to 11	11 to 11-30	11-30 to 12	12 to 12-30	12-30 to 1
No, of vehicles	93	125	97	90	80	45

Answer the following questions :

(1) How many vehicles pass in average per hour ?

(2) How many vehicles pass in average per hour from 10 to 11-30 ?

(3) How many vehicles pass in average per hour from 11-30 to 1 ?

38. Find average weight and height of boys from the following two tables.

Weight (kg)	No. of boys
42	7
45	13
46	10

Height (metre)	No. of boys
1·41	9
1·39	13
1·46	8

39. (i) There are 50 tomato plants in the garden of a cultivator. He collected 11 tomatoes from each of 10 plants. 20 tomatoes from each of 18 plants and 15 tomatoes from each of the rest 22 plants. Total weight of all tomatoes was 40 kg. (*a*) What was the average number of tomatoes collected from each plant ? (*b*) What average weight of tomato did he get from each plant ?

(ii) Out of 40 kg he sold 12 kg at Rs. 1·20 per kg, 20 kg at Rs. 1·25 per kg and 8 kg at Rs. 1·50 per kg. (*a*) What was the average price of tomatoes collected from each plant ? (*b*) What was the average price and weight of each tomato ?

40. Height, weight and examination marks of two groups of students are given below :

First Group

	Zafar	Subir	Anil	Tapasi	Suman	Nandini	Utpal
Hight (metre)	1·6	1·43	1·60	1·53	1·59	1·53	1·4
Weight (kg)	45·5	48·4	47·4	27·4	25·34	27·3	24·1
Exam. marks	445	536	323	245	378	426	495

Second Group

	Jeeban	Jahnabi	Baby	Ikbal	Swapan	Mustak
Height (metre)	1·42	1·5	1·54	1·42	1·39	1·3
Weight (kg)	41	44·2	46·4	47·1	43·6	31·2
Exam. marks	378	521	278	476	473	379

(i) Find average height, weight and examination marks of each group. (ii) Which group has more average height ? (iii) Which group has less average weight ? (iv) If a new group is formed taking all the students of the two groups, what will be the average height, weight and examination marks of the new group ?

Miscellaneous Exercise 2

Square root

(Try to answer questions 1 to 3 orally. If necessary do rough work. If not asked otherwise, give answer correct upto two decimal places.)

1. Find the square root :

(i) $\frac{4}{49}$ (ii) $\frac{1}{121}$ (iii) $\frac{49}{64}$ (iv) $\frac{1}{225}$ (v) $6\frac{1}{4}$
(vi) $7\frac{9}{16}$ (vii) ·81 (viii) ·09 (ix) ·01
(x) ·0001 (xi) ·0049 (xii) ·000001

2. Find the greatest and the smallest numbers :

(i) $\sqrt{\frac{1}{4}}$, $\sqrt{\frac{1}{9}}$, $\sqrt{\frac{1}{16}}$, $\sqrt{\frac{1}{25}}$ (ii) $\sqrt{\frac{26}{25}}$, $\sqrt{\frac{36}{25}}$, 1, $\sqrt{\frac{16}{25}}$
(iii) $\sqrt{\frac{4}{25}}$, $\sqrt{\frac{6}{25}}$, $\frac{\sqrt{5}}{5}$ (iv) $\sqrt{\frac{16}{25}}$, $\sqrt{\frac{25}{36}}$, $\sqrt{\frac{36}{49}}$
(v) $\sqrt{\frac{81}{100}}$ $\frac{\sqrt{89}}{10}$, $\frac{9}{11}$ (vi) $\sqrt{\frac{64}{49}}$, $\sqrt{2\frac{7}{25}}$, $\sqrt{\frac{121}{169}}$
(vii) $\sqrt{\cdot 01}$, $\sqrt{\cdot 0001}$, $\sqrt{\cdot 000001}$ (viii) $\sqrt{\cdot 04}$, $\sqrt{\cdot 09}$, $\sqrt{\cdot 0004}$, $\sqrt{\cdot 0009}$ (ix) $\sqrt{\cdot 64}$, $\sqrt{\cdot 0121}$, $\sqrt{\cdot 7}$
(x) $\sqrt{3}$, $\sqrt{2\cdot 25}$, $\sqrt{\cdot 81}$ (xi) ·09, $(0\cdot 9)^2$, $\sqrt{\cdot 09}$
(xii) 2, $\sqrt{5}$, $\frac{1}{\sqrt{5}}$

3. (i) Some boys decided to help a poor boy and to contribute as many paise as the number of boys. They raised altogether Rs. 6·25. What is the number of helping boys ?

(ii) Each child contributed as many 20 paise as the number of children taking part in a picnic. Total contribution amounted to Rs. 3·20. What was the number of children ?

(iii) What smallest number should be added to 1·56 to make it a perfect square ?

(vi) Length of a rectangular field is twice its breadth. If the area of the field be 200 sq.m. find its length and breadth.

(v) Product of two numbers is ·02 and their quotient is 2. What are the numbers ?

4. Find the square root :

(*a*) $3\frac{61}{100}$, $4\frac{21}{25}$, $272\frac{1}{4}$, $\frac{1521}{4096}$, $\frac{9409}{15129}$

(*b*) 70·56, 151 29, 15227·56, $\frac{16 \cdot 9}{22 \cdot 5}$, ·051076

(*c*) $1\frac{783}{4901}$; ·000324, $15\frac{21}{100}$, 1082·41, ·00053361

(*d*) 2, 3, 5, 7, 11 ; (Correct upto three decimal places)

(*e*) ·9, ·0013, ·00064, 23·1, 15·611

(*f*) $\frac{2}{5}$, $\frac{3}{7}$, $\frac{6}{11}$, $1\frac{3}{14}$, $131\frac{12}{75}$

(*g*) $\frac{\frac{1}{5}}{\frac{1}{3}}$, $\frac{2}{2\frac{1}{2}}$, $\frac{3}{4}+\frac{1}{5}$, 1·3+2·1, ·5

5 Which smallest number should be added to 119·678 to make the sum a perfect square ?

6. By which smallest number should $31\frac{1}{5}$ be multiplied to make the product a perfect square ?

7. Each student of a school contributed as many paise as the total number of students of the school and the total contribution was Rs. 640·09. What is the number of students in the school ? How much did each student contribute ?

8. It was decided that subscriptions would/be collected from the villagers to repair the village school. It was observed that if each villager pay as many two paise as the total number of villagers, the total subscription will be Rs. 915·92. What is the total number of villagers ?

9. The area of a square is 1·5 hectares What is the length of the square in metres ? (1 hectare = 10000 sq. m).

10. The length of a rectangular field is one and a half times its breadth and its area is ·4704 hectare. What are its length and breadth in metres ?

11. The length of floor of a room is 3 times its breadth. The cost of cementing the floor at Rs. 10·20 per sq.m. is Rs. 191·25. What are the length and breadth of the room ?

12. Area of a rectangular field is 160 hectares. If the length is $2\frac{1}{2}$ times the breadth, find the length and breadth of the rectangle.

13. Find the square root :

$$\sqrt{\frac{5\times22+7\times28+10\times33+3\times38}{30}}$$

14. 50 square stones of equal length were needed to cover a floor of area 72 Sq.m. What are the length and area of each stone ?

15. The area of circular park is 15400 Sq m. What is the radius of the park ? Suppose that

radius of a circle $= \sqrt{\frac{7}{22}\times \text{area of the circle}}$.

16. The square root of the product of two numbers is called the geometric mean of those two numbers.

Find the geometric mean of 1·21 and 6·25.

18. If a = 1·9, b = 2·1; find the value of $a^2+2ab+b^2$ and its square root.

19. Find the values of the following expressions correct upto nearest whole numbers by puting given values :

(a) l^2+b^2 ; when 1 = 2·5 cm, b=1·5 cm.

(b) $2\pi\sqrt{\frac{l}{g}}$ when $\Pi = \frac{22}{7}$, 1 = 98, g = 980.

(c) $\sqrt{\frac{3v}{\pi h}}$ -when V = 462, $\Pi = \frac{22}{7}$ h = 9.

(d) $\sqrt{\frac{A}{4\pi}}$ when A = 5544, $\Pi = \frac{22}{7}$

(e) $\sqrt{a^2+b^2+c^2}$, when $a = 3$, $b = 4$, $c = 12$.

(f) $\sqrt{s(s-a)(s-b)(s-c)}$; when $a = 3$ cm, $b = 4$ cm, $c = 5$ cm and $s = \frac{(a+b+c)}{2}$

(g) $\frac{1}{4}\sqrt{2a^2c^2+2c^2a^2+2a^2b^2-a^4-b^4-c^4}$ when a =15 cm, b = 14 cm, c =13 cm.

(h) $\sqrt{ab+bc+ca}$; when $a = b = c = 5$

Miscellaneous Exercise 3
Time and Distance

(Answer question no.1 orally. Give your answers correct upto two decimal places if not otherwise stated).

1. Indicate true or false :

(a) $\text{speed} = \frac{\text{distance traversed}}{\text{time}}$

(b) $\text{Distance} = \frac{\text{speed}}{\text{time}}$

(c) Speed = time × distance

(d) Time = speed × distance

(e) Speed remaining constant, more distance will be covered in less time.

(f) Speed remaining constant, more distance will be covered in more time.

(g) If the speed be more, a given distance will be covered in less time.

(h) If the speed be less, a given distance will be covered in less time.

(i) Time remaining constant, more distance will be covered if the speed be more.

(j) Time remaining constant, less distance will be covered if the speed be less.

(k) Two persons start walking from the same place and in the same direction at speeds of 30m./min. After 1 minute the distance between them will be 50 m.

(l) In the above question had the two persons walked in opposite directions, the distance between them would have been 10 m.

2. (a) Kamal covers $\frac{1}{2}$ km in 10 minutes. What is his speed per hour ?

(b) Suren travelled 5 km in an hour. How far would he travel in 5 minutes ?

(c) Rama runs 100 m in 2 seconds. How far will she run in 1 second ?

(d) Surajit and Kamaljit start at the same time from the same place in the same direction at speeds 4 km/hr and 6 km/hr. respectively. What will be the distance between them after 2 hrs ?

(e) Pareshbabu and Rameshbabu start at the same time from the same place in opposite directions at speeds 8 km/hr and 3 km/hr respectively. What will be the distance between them after 3 hrs ?

(f) A train, 100 m long, travels at a speed of 30 km/hr. What time will it take to cross a post and to cross a platform, 200m long ?

(g) Two trains, 100 m and 200 m long, start at speed 60 km/hr and 30 km/hr respectively from the same place and in the same direction. After what time will they cross each other ? In what time would they have crossed each other had they travelled in opposite directions ?

3. Fill up the blanks of the following table giving appropriate units :

speed	Distance covered	Time
5km/hr	500m	
150 m in 3 min	3 km	
100 m/sec	3 km	
5 km/hr		12 min
150 m in 3 min.		20 sec
100 m/sec		2 hr
per hr	300 m	5 min
per min.	15 m	5 sec
per sec.	1·8 km	2 hr

4. A man heard the sound of a lightning $3\frac{1}{2}$ seconds after observing its light. How far is the origin of lightning from the man ? (suppose that the man saw the light as soon as the lightning occured and the speed of sound is 330m/sec.)

5. A man travels 516 m in 6 min. What is his speed per hour ?

6. A cyclist crosses a bridge in $7\frac{1}{2}$ seconds at a speed of $13\frac{1}{2}$ km/hr. What is the length of the bridge ?

7. A train, 50 m long, travels at a speed of 36 km/hr. How long will it take to cross a post by the side of the track ?

8. A train, 56 m long, crosses a tree by the side of the track in 4 seconds. What is the speed of the train per hour ?

9. A bus travels at a speed of 54km/hr. It crosses two posts by the side of the road in 5 seconds. What is the distance between the two posts ?

10. A train, 70 m long, travels at a speed of 75 km/hr. How long will it take to cross a platform, 105 m long ?

11. The speed of a boat in still water is 10km/hr. The speed of the current of a river is 2 km/hr. How long will he take to cover 600 m along the river by the boating if the boat travels (a) along the current, (b) against the current ?

12. Distance between two hills is 1.15 km. A man standing on the first hill saw a man firing a gun from the second hill and heard the sound of firing5 seconds afterhesaw flash of light from the gun. What is the speed of sound per second ? A third man heard the sound 13 seconds after he saw flash of light of the firing. Whatn is the distance between the man with gun and the third man ?

13. A train leaves Burdwan at 8·30 a.m. and reaches Howrah at 11a.m On he same day another train leaves Howrah at 7·30 a.m. and reaches Burdwan at 12·30 p.m. When will they meet ?

14. A bus starts at 11 a.m. at a speed of 20 km/hr. At 12·24 p.m. a scooter starts from the same place in the same direction and meets the bus at 6 p.m. What is the speed of the scooter per hour ?

15. Two men 35 km apart started towards each other at speeds 6 km/hr and 8 km/hr respectively. When did they meet each other ?

16. A train, 100 m long, running at 48 km/hr crosses a tunnel through a hill in 21 sec. What is the length of the tunnel ?

17. The distance between two cars travelling in the same direction is 40 km. The speed of the car at the back is 25 km/hr and that of the one in front is 20 km/hr. When will the two cars meet each other and what distance will be covered by the car at the back ?

18. The distance between Amal and Asit is 700 m. When they start walking in the same direction along the same road at speeds $6\frac{1}{2}$ km/hr and 4 km/hr respectively, Asit was lagging behind. When will they meet eachother and what distances will be covered by each of them ?

19. A man leaves his house at 8 a.m. at 8 km/hr to go to a town for shopping and came back to the house at 2 p.m. at a speed of 6 km/hr. He spent $1\frac{1}{2}$ hrs. at the town for shopping. What is the distance of the town from his house ?

20. A boat travelled a distance of 28 km along the current in 4 hrs. To come back the same distance against the current it took 7 hrs. Find the speed of the boat and the current of the river ?

21. Two trains, 120 m and 90 m long, take 30 sec. and 5 sec. to cross each other moving in the same and opposite directions respectively. Find the speed of each per hour.

22. A man climbs on top of a hill at 2 km/hr and comes back to the starting point at $3\frac{1}{2}$ km/hr. He takes $4\frac{1}{2}$ hrs to go and come back. What distance does he travel in the above process.

23. Boatman Abdul went 20 km along the current in 4 hrs and came back the same distance against the current in 12 hrs. Find the speeds of the current and of the boat.

24. The paddy field of a cultivator is at a distance of 1·2 km from his house. He goes from home to the field at a speed of 60 m/min and goes back from there with head load of paddy at a speed of 80 m/min. In the field he takes 10 min everytime for trying the load and for resting. His working time is 6 a.m. to 12 noon and 2 p.m. to 5.30 p.m. What is the maximum number of times that he will be able to bring head loads from the field to home in a day ? In the process what distance will he travel in a day ?

25.(i) Two trains, 100 m and 120 m long, start at 10 a.m. and 10.30 a.m. respectively, from two stations 25 km apart, along two parallel tracks towards each other. The speeds of the two trains are 30 km/hr and 42 km/hr respectively.

(a) When will the engine-drivers of the two trains meet each other ?

(b) When will the engine-driver of the first train meet the ssenger of the last compartment of the second train ?

(c) When will the passenger of the last compartment of the first train meet the engine-driver of the second train ?

(d) When will passenger of the last compartment of the first train meet the passenger of the last compartment of the second train ?

(e) When will the distance between the two trains be 1 km ?

(i) (Suppose that the engine driver is at the front most part of the engine and the passenger of the last compartment is at the rearmost part of the train).

(ii) Had the two trains started together at 10 a.m. from the same place in the same direction what would have been the answers of questions (b), (c), (d) and (e) above ?

26. A steamer, 115 m long, travels at a speed of 45 km/hr and crosses a boat 65 m long, moving at a speed of 18 km/hr in the same direction. What time will the steamer take to cross the boat ?

Miscellaneous Exercise 4

Time and Work

1. Answer orally :
   1 man can do a work in 5 days.

(a) In how many days will 1 man do the work ?

(b) In how many days will 5 men do the work ?

(c) In how many days will 5 men do a work three times the work ?

(d) How many men will do the work in 5 days ?

(e) How many men will do a work twice the work in 5 days ?

(f) How many men will be needed to finish the work in 1 day ?

(g) How many men will be needed to finish three times a work in 15 days ?

(h) If more men are employed will more or less number of days be required to finish the work ?

2. Madhu can finish a work in 12 days and Bidhu does it in 10 days.

(a) In how many days will the work befinished if they work together ?

(b) Madhu alone works for 8 days and then Bidhu joins him to finish the work together. In how many days will the work be finished ?

(c) Madhu works alone for 8 days and leaves, in how many days will Bidhu finish the rest of the work ?

(d) Sidhu alone can do the work in 8 days. In how many days will Madhu Bidhu and Sidhu, working together, finish the work ?

(c) Madhu works for 4 days and then Kamal joins him to finish the work together in 6 days. What part of the work will Kamal do in 1 day ? How many days will he take to do the whole work ?

(f) In how many days will Madhu and Bidhu, working together, finish a work 5 times the work ?

(g) If each of Bidhu, Sidhu and Kamal had worked at a rate equal to that of Madhu, how many days would they working together, have taken to finish the work ?

If all of them start working together and if Madhu leaves at the end of the first day, Bidhu and Sidhu leave at the end of second and third days respectively, howmanydays will Kamal take to finish the work ?

3. 15 men earn Rs. 630 in 12 days. In how many days will 4 men earn Rs. 112 ?

4. 6 cows can plough12 hectares of land in 25 days. How many hectares of land will be ploughed by 10 cows in 20 days ?

5. 10 pumps can raise 75 tons of water in 10 hrs. How many hours will 15 pumps take to raise 100 tons of water ?

6. 25 men can dig a pond, 12 m long, in 20 days. How many men will dig a 21 m long pond in 25 days ? (The bredth and depth of the pond will remain same.)

7. 3 quintals of coal is necessary for keeping 25 ovens on for 5 hours. How many quintals of coal will be necessary for keeping 18 ovens on for 8 hours ?

8. 15 men can make a 3 m high wall in 5 days. How many men will make a 4 m high wall in 2 days ? (Length and breadth of the wall will remain same)

9. 6 men can plough 5 acres of land in 8 days working 6 hrs a day. How much land will be ploughed by 10 men in 12 days working 8 hours a day ?

10. If 125 men can do a piece of work in 120 days, how many men will be able to do the work in 100 days ?

11. 6 men can unload coal from one truck in 5 hrs. In how many hours will 15 men unload coal from 3 trucks ?

12.(i) Latif and Mahim can plant saplings in 1 acre of land in 10 and 12 days respectively. If they work together, how many days will they take to plant saplings in 3 acres of land ? Out of the 3 acres what portion is covered by each of them ?

(ii) If each of them could plant saplings in 1 acre of land in 10 days, how many days would they, working together have taken to cover 3 acres of land ?

13. 15 coal labourers can cut 2 metric tons of coal in 7 days working 8 hours a day. How many labourers will be needed to cut 500 metric tons of coal in 2 months working 7 hours a day ?

14. Subal and Amar together can do a work in $11\frac{1}{2}$ days. Subal alone can do that work in 20 days. Subal alone worked for 8 days and then Subal and Jaygopal worked together for 6 days. The rest of the work was done by Amar in 3 days.

In how many days will Amar and Jaygopal together do the work ?

15. 5 typists can type 25 lines in 5 minutes. How many lines will 16 typists type in 16 minutes ?

16.(a) A can do a work in 10 days and B can do it in 15 days. In how many days will A and B together do the work ?

(b) Suppose that A does the work in x days and B does it in y days. In how many days will they togetherdo the work ?

(Express this number of days in terms x and y). Verify the truth of the answer of (a) putting 10 as the value of x and 15 as that of y in the above formula. Solve the following two problems with the help of that formula :

(c) Kashinath can do a work in 8 days and Biswanath can do it in 10 days. In how many days will they, working together, do the work ?

(d) Sunila can do a work in 5 hrs and Subhra can do it in 9 hrs. In how many hours will they together do the work ?

17. Karim and Sudhir together can make a wall in 20 days. After they worked together for 12 days Karim alone finishes the rest of the work in 10 days. In how many days will each one of them make the wall ?

18. Labani, Srabani and Nandini together can make 75 dolls in 5 hours. They started working together with a contract of making 75 dolls. Labani left after working 2 hours. Srabani and Nandini worked together and made 15 dolls in $1\frac{1}{2}$ hrs. Then Srabani left Nandini alone made rest 30 dolls in 5 hours. How many dolls does each of Labani, Srabani and Nandini make in 1 hr ?

19. A, B and C together can do a work in 10 days. A alone can do the work in 30 days and B alone can do it in 40 days. In how many days can C alone do the work ?

20. There is food to feed 10 persons for 25 days. After 8 days 4 persons joined them. How many days will the rest of food last them ?

21. 4 kg rice is necessary to feed 4 men for 4 days.

(i) How many days will 20kg rice last for 20 men ?

(ii) How many kilogram of rice will be necessary to feed 20 men for 20 days ?

22. 12 men can make 144 chairs in 8 days.

(i) How many men will make 150 chairs in 10 days ?

(ii) How many chairs will 30 persons make in 12 days ?

(iii) In how many days will 200 men make 72 chairs ?

23. Ashoke does $1\frac{4}{5}$ of a work in 36 days and then he along with Arabinda finished the rest of the work in 33 days. In how many days can each of Ashoke and Arabinda alone do the work ?

24. (a) A can do a work in x days, B can do it in y days and C can do it in z days. Prove that

(i) A and B together can do the work in $\frac{xy}{x+y}$ days.

(ii) A and C together can do the work in $\frac{xz}{x+z}$ days.

(iii) B and C together can do the work in $\frac{yz}{y+z}$ days.

(iv) A, B and C together can do the work in $\frac{xyz}{xy+yz+zx}$ days.

(b) In each of the following rows some necessary conditions are given. Fill up the blanks by putting the values of x, y and z in appropriate formula :

Work	A alone does in (days)	B alone does in (days)	C alone does in (days)	A&B together do in (days)	A&C together do in (days)	B&C together do in (days)	A,B&C together do in (days)
1st work	5	7	8				
2nd work	10	12	6				
3rd work	4		8			4	
4th work	3	5				4	

In every case work out the sum by arithmetical method and verify the answers in the table.

25. 12 men can do a work in 4 days. How many men will do 8 times a work in half the time ?

26. 50 men can do a work in 50 days. If 50 men start doing the work and if 10 men leave after every 10th day, what part of the work will be left after all men had left ?

27. A cistern is filled by 1st and 2nd taps in 30 and 40 minutes respectively. In what time will the cistern be filled if both the taps are opened simultaneously ?

28. There are two taps fitted to a bathrooom cistern. The first tap fills the empty cistern in 2 hrs and the second tap empties the full cistern in 3 hrs.

(i) Both the taps are opened simultaneously when the cistern is empty. How long will it take to fill the cistern ?

(ii) When the cistern is empty the second tap is opened 15 minutes after opening the first tap. How long will it take to fill the cistern ?

29. An empty tank is filled by the first tap in 40 minutes and the full tank is emptied by the second tap in 30 minutes.

(i) If both the taps are opened simultaneously when the tank is full, how long will it take to empty it ?

(ii) When the tank is full the first tap is opened 5 minutes after opening the second tap. How long will it take to empty the tank ?

30. Three taps are fitted to a cistern. The empty cistern is filled by the first and the second taps in 3 and 4 respectively. The full cistern is emptied by the third tap in 5 hrs. How long will it take to fill the empty cistern if all the three taps are opened simultaneously ?

Arithmetic

Answers

Exercise 1 (a)

1. $\frac{1}{8}$, $1\frac{1}{40}$, $\frac{16}{3125}$, $10-\frac{31}{250}$, $\frac{1}{160}$, $\frac{5}{16}$, 7, $5\frac{77}{125}$

2. (a) ·0101, ·0011, ·001, ·00098, ·0009;
(b) ·51, $\frac{1}{2}$, ·499, $\frac{1}{3}$, ·299
(c) $\frac{\cdot 1}{\cdot 0001}$, $\frac{1}{\cdot 01}$, $\frac{1}{\cdot 1}$, $\frac{\cdot 01}{2}$, $\frac{\cdot 001}{5}$,

3. (a) $\frac{9}{110}$, $\frac{9}{11}$, $\frac{9}{1100}$, $2\frac{3}{370}$, $2\frac{12}{55}$,
(b) $\frac{27}{3700}$, $3\frac{533}{4950}$, $\frac{1}{7}$, $\frac{2}{35}$, $14\frac{131}{9990}$,

4. (a) $4{\cdot}66\dot{6}32\dot{1}$ (b) ${\cdot}00\dot{3}5\dot{2}$

5. (a) 1·64 m (b) 24·2 km (c) 2·31 gallon, 2 gallon, ·31gallon less.

Exercise 1 (b)

1. (a) multiple (b) factor (c) L.C.M. (d) multiple (e) L.C.M., H. C. F. (f) product.

2. (a) $1\frac{4}{5}$, $3\frac{3}{5}$, (b) 140, 280, 3. (a) 4 (b) 144 (c) 60 (d) 64·5 (e) 37·5 4. (a) $1\frac{3}{4}$, (b) $\frac{1}{4}$, (c) $\frac{9}{160}$, (d) ·05, (e) ·12.

Exercise 1 (c)

1. (a) 5, (b) 1,3, 7, 57 (c) 4 (d) 2 (e) 3 2. (a) 96, 315, 101, 807, 232 (b) 1001, 1092, 613, 325, 7564 3. no. of digits – 3, 5, 4 ; digit

in unit's place 3 or 7, 3 or 7, 2 or 8 4. 28 5. 99856, 10000 6. 13
7. 29 rows, 35 8. 23 9. 60 rows, 102 10. 25, 100.

Exercise 1 (d)

1. $21\frac{1}{19}$ sec, $21\frac{17}{23}$ sec, 18 km/hr, 17·1 km/hr, 16·56 km/hr.

2. 18 min 45 sec 3. 1 hr 52 min 4. 1.44 km 5. $74\frac{2}{7}$ m/hr 6. $5\frac{19}{25}$ sec, $20\frac{4}{25}$ sec 7. 1231 km. 8. They will reach together.

Exercise 1 (e)

1. Rs. 185·90 2. Rs. 1937·50 3. 6 4. $16\frac{2}{3}$ 5. 25 yrs 6. $2\frac{1}{2}$ yrs
7. 8 years 5 months 8. Rs. 810 9. Rs. 1090 10. Rs. 700 11. 8.

Exercise 1 (f)

1. 2m, 4 sq. m 2. 74·75 sq. m, Rs. 168·19 (approx.) 3. Rs. 27
4. 16m, 8 m 5. 10m, 40m 6. 516 sq. m, Rs. 774 7. 76.25 sq.m,
323·75 sq. m 8. 13 sq.m, Rs. 58·50.

Exercise 2

1. 63 2. 16 runs 3. 4·75 quintal 4. 145 5. 90 6. 87 7. 196
8. 11 yrs 9. 24 runs 10. 14·3 yrs 11. 49 12. ·02 cm 13. 146·8 cm
14. 149 15. 75·40 16. 132,66,33 17. 48 18. 11 yrs.
19. 1·1 quintal more per bigha.
20. 125 machine parts, 1500 machine parts.
21. A—Rs. 490, B—Rs. 520, C—Rs. 550.
22. ·54 cm 23. 3 yrs 24. 50 runs
25. Both have same average 26. 40 km/hr.

Exercise 3

1. 515 2. Rs. 1·35 3. 48 4. Rs. 7·25 5. 1·22 cm 6. 825 m/min 7. 9·40 quintal 8. 45·25 9. 29°C 10. 105 11. B team—19 runs 12. Rs. 43 13. Average mark in math is more by 7 marks. 14. 157·2 cm. 15. 14·48 yrs. 16. Rs. 1·50, Rs. 2·50, Rs. 8·25; Rs. 122·50.

Exercise 4

1. (i) $\frac{15}{16}$ (ii) $\frac{13}{23}$ (iii) $\frac{29}{30}$ (iv) $1\frac{8}{27}$ (v) $1\frac{1}{5}$ (vi) $2\frac{15}{26}$ (vii) $\frac{89}{96}$ (viii) $\frac{116}{225}$ (ix) $\frac{64}{73}$ (x) $1\frac{15}{71}$ (xi) $3\frac{23}{24}$ (xii) $9\frac{1}{13}$ (xiii) $10\frac{13}{27}$ (xiv) $\frac{65}{357}$ (xv) $3\frac{60}{79}$ (xvi) $\frac{83}{105}$ (xvii) $\frac{1705}{1871}$ (xviii) $12\frac{136}{175}$ (xix) $\frac{1487}{5460}$ (xx) $160\frac{3}{4}$

2. $2\frac{3}{23}$ 3. $\frac{11}{16}$ part 4. $1\frac{1}{5}$ 5. 3 6. $1\frac{1}{11}$ 7. 16 times.

Exercise 5

1. (i) ·7 (ii) ·12 (iii) ·09 (iv) ·56 (v) ·87 (vi) ·94 (vii) 3·5 (viii) 5·7 (ix) 7·42 (x) 1·86 (xi) ·013 (xii) ·225 (xiii) 11·02 (xiv) 13·66 (xv) 8·729 (xvi) 16·54 (xvii) 86·42 (xviii) 27·95
2. ·08 3. 1·05 4. 2·03 m 5. 2·4 m 6. ·25 7. 15·54 8. 18 cm.

Exercise 6

1. (i) 2·236 (ii) 2·646 (iii) 3·464 (iv) 6·083 (v) 9·274 (vi) 10·724 (vii) 16·031 (viii) 26·627 (ix) 33·650 (x) 71·099 (xi) 79·354 (xii) 95·021 (xiii) 107·345.

2. (i) ·692 (ii) ·378 (iii) ·816 (iv) 2·299 (v) 2·987 (vi) ·861 (vii) ·914 (viii) 8·415 (ix) ·775 3. (i) ·63 (ii) 1·58 (iii) ·47 (iv) ·48 (v) ·45 (vi) 3·77 (vii) ·04 (viii) 3·24 (ix) 10·05 (x) 10·73 (xi) 16·70.

Exercise 7

1. 40 km, 15 km/hr 2. 53 min 20 sec 3. 5 km/hr 4. 12 km/hr 5. 72 km/hr 6. 3 min 24 sec. 7. 33 sec 8. 250 m 9. 125 m, 45 km/hr 10. 250 m, 150 m; 40 sec 11. 2 min 15 sec 12. 21 sec 13. 9 km/hr 14. 176 m, 79.2 km/hr 15. 65 m, 39.6 km/hr 16. $7\frac{1}{2}$ km. 4 hr 12 min.

Exercise 8

1. 4 hr, 42 km 2. (i) second, 500 m (ii) 4.5 km (iii) 6 min 3. After 5 hr 12 min, 156 km 4. 12 - 24 p.m., 11.520 km 5. 90 km 6. After 20 sec 7. In 1 min. 8. After 20 min 9. At B 10. 4.9 m.sec 11. 1 km, 12 km/hr 12. 6 m./sec 13. 28 min 14. 18.04 km/hr approx 15. After 2 min 30 sec, 1.50 m. 16. 6 sec after A started his return journey on his way back, 450 m from the starting point. 17. 4km/hr 18.48 km/hr 19.14.4 sec.

Exercise 9

1. (i) $2\frac{1}{2}$ hr (ii) 5hrs 2.(i) 40 km (ii) 51 km 3.2 km/hr 4.7 km/hr 5. 11 hrs 6. 24 km 7. 8km/hr 8. 7km/h4, 2 km/hr 9. 5 hrs. 10. 4 hrs.11. 5 km/hr 12. 5 km/hr, 5.8 km/hr 13. 1. hr 15 min 14. 2.50 km/hr 15. At 3.30 p.m. 16. 120 km 17.2 hrs more, same time 18. Boat B, 2 hrs more.

Exercise 10

1. 253 m 2. 25 days 3. 3 days 4. 28 workers 5. 210 6. 6 days 7. 7 persons 8. 8 hours 9. 290 m 10. 80 workers 11. 4 workers 12. 12 days 13. 24 days 14. 16 days 15. 2 16. 5 .17. 15 18. 270 19. 120 20. In 48 days 21. In 30 days 22. 156

Exercise 11

1. $5\frac{4}{11}$ hr 2. $2\frac{2}{3}$ days 3. $13\frac{1}{3}$ hr 4. $6\frac{66}{73}$ hr 5. 11 hr 6. $\frac{1}{2}$ work 7. $8\frac{8}{9}$ days, $10\frac{10}{11}$ days, $9\frac{3}{5}$ days 8. 10 days, A—30 days, B—20 days, C—60 days 9. $1\frac{1}{2}$ days 10. 26 days 11. 24 days

12. 20 days 13. 9 men 14. 14 days 15. Man —12 days, Woman—18 days, boy —36 days 16. 6 hrs. 17. 12 days 18. 5 days 19. 1 day 20. $5\frac{1}{7}$ days 21. A—30 days, B—30 days 22. 8 days 23. $7\frac{3}{7}$ hr 24. 7 days

Exercise 12

1. $6\frac{2}{3}$ min 2. 6 min 3. 36 min 4. 40 min 5. $11\frac{2}{3}$ min 6. 28min 7. $23\frac{1}{3}$ min 8. 45 min 9. 70 min 10 $45\frac{5}{6}$ min 11. 3 hrs 20 min 12. At 7-24 a.m.

MiscellaneousExercise 1

1. (i) 8 (ii) 65 (iii) 11 (iv) 14 (v) 4 (vi) 64 paise (vii) 26 cm (viii) 4 2, 53.62 kg 3. 124.9 cm 4. Rs. 130.50, Rs. 3.17 5. Rs.7116.30 km/hr 7. Rs. 26.81 8. 62.60 9. Rs. 44.15 10. Rs. 12.50 11. Rs. 8.99 12. 44.8 13. 76.67, 77 14. 65, 25,58; Kapil Dev's

15. 45 km 16. Rs. 4991 17. 16 yr 8 months

18. Rs. 25·58 19. 2·625 kg 20. Rs. 501·67 21. 3·65 cm

22. 6·4 km/hr 23. 72 km/hr 24. 75·25, 105·2, 56·83, 77·2

25. 10 yr $9\frac{5}{16}$ months 26. 7841 27. 39 km, $7\frac{1}{11}$ km/hr

28. Rs. 1025 29. Av. income — Rs. 425·19, Av. Expenditure Rs. 348.40. Av. Saving — Rs. 76·79 30. 30700 33580

31. Rs. 4·31 32. (i)

Average marks	English	Mathematics	History	Total
Average marks of boys	37.17	56.5	39.5	133.17
Average marks of girls	44	64.25	62.25	170.5
Average marks of boys & girls	39.9	59.6	48.6	148.1

(ii)

	Debu	Subha	John	Ruma	Rinku	Chandra
English	good	weak	good	weak	good	weak
Mathematics	weak	good	good	weak	good	good
History	good	weak	weak	weak	weak	good
All subjects	good	weak	good	weak	good	good

33. 550 gm 34. Rs. 3·92 35. 77·5 cm 36. 3544·75 37. (1) 176·67 (2) 210 (3) 143·33 38. (i) 44·63 kg (ii) 1·41 m 39. (i) (*a*) 16 (*b*) 800 gm (ii) Rs. 1·03 ; 6·43 paise, 50 gm 40. First group : height—1·53 m, weight—35·06 kg, exam. marks—406·86 ; Second group. : height—1·43 m, weight—42·25 kg, exam. marks—417 5 ; New group : height—1·48 m, weight—38·38 kg exam. marks—411·77.

Miscellaneous Exercise 2

1. (i) $\frac{2}{7}$ (ii) $\frac{1}{11}$ (iii) $\frac{7}{8}$ (iv) $\frac{1}{15}$ (v) $2\frac{1}{2}$ (vi) $2\frac{3}{4}$ (vii) ·9 (viii) ·3 (ix) ·1 (x) ·01 (xi) ·07 (xii) ·001 2. (i) $\sqrt{\frac{1}{4}}$, $\sqrt{\frac{1}{25}}$ (ii) $\sqrt{\frac{36}{25}}$, $\sqrt{\frac{16}{25}}$ (iii) $\sqrt{\frac{6}{25}}$, $\sqrt{\frac{4}{25}}$ (iv) $\sqrt{\frac{36}{49}}$, $\sqrt{\frac{16}{25}}$ (v) $\sqrt{\frac{89}{10}}$, $\frac{9}{11}$ (vi) $\sqrt{2\frac{7}{25}}$, $\sqrt{\frac{121}{169}}$ (vii) $\sqrt{\cdot01}$, $\sqrt{\cdot000001}$ (viii) $\sqrt{\cdot09}$, $\sqrt{\cdot0004}$ (ix) $\sqrt{\cdot7}$, $\sqrt{\cdot0121}$ (x) $\sqrt{3}$, $\sqrt{\cdot81}$ (xi) $\sqrt{\cdot09}$, $(\cdot09)^3$ (xii) $\sqrt{5}$, $\sqrt{\frac{1}{5}}$ 3. (i) 25 (ii) 4 (iii) 1 (iv) 20 m, 10 m (v) ·2, ·1 4. (a) $1\frac{9}{10}$, $2\frac{1}{5}$, $16\frac{1}{2}$, $\frac{39}{64}$, $\frac{97}{123}$ (b) 8·4, 12·3, 123·4, $\frac{13}{15}$, ·226 (c) $1\frac{1}{13}$, 018, 3·9, 32·9, ·0231 (d) 1·414, 1·732, 2·236, 2·646, 3 317 (e) ·95, ·04, ·03, 4·81, 3·95 (f) ·63, ·65, ·74, 1·10, 11·45 (g) ·77, ·89, ·97, 1·84, ·71 5. 1·322 6. $\frac{5}{156}$ 7. 253, Rs. 2·53 8. 214 9. 122·47 m 10. 84 m, 56 m 11. 7·5 m, 2·5 m 12. 2 km., 800 m 13. 5 14. 12 dcm, 144 sq dcm 15. 70 m 16. 2·75 17. 7·07 cm 18. 16,4 19. (a) 3 cm (b) 2 (c) 7 (d) 21 (e) 13 (f) 6 sq. cm . (g) 84 sq. cm (h) 9.

Miscellaneous Exercise 3

1. (a) True (b) False (c) False (d) False (e) False (f) True (g) True (h) False (i) True (j) True (k) False (l) False 2. (a) 3 km (b) 416·67 m (c) 5 m (d) 4 km (e) 39 km (f) 12 sec, 36 sec (g) 36 sec, 12 sec 3. 6 min, 1 hr, 30 sec, 1 km, $16\frac{2}{3}$ m, 720 km, 3·6 km, 180 m, 25 cm 4. 1155 m 5. 5·16 km 6. $28\frac{1}{8}$ m 7. 5 sec 8. 50·4 km 9. 75 m 10. 8·4 sec 11. (a) 3 min (b) $4\frac{1}{2}$ min 12. 230 m, 2·99 km 13. 9-50 a.m. 14. 25 km/hr 15. After $2\frac{1}{2}$ hrs. 16. 180 m 17. 8 hrs, 200 km 18. 16·8 min, Amal—1820 m, Asit—1120 m. 19. $15\frac{3}{7}$ km 20. 5·5 km/hr, 1·5 km/hr, 21. 88·2 km/hr, 63 km/hr 22. $11\frac{5}{11}$ km 23. $3\frac{1}{3}$ km/hr, $1\frac{2}{3}$ km/hr 24. 12 times, 28·8 km 25. (i) (a) 10 hr 38 min 20 sec (b) 10 hr 38 min 26 sec (c) 10 hr 38 min 25 sec (d) 10 hr 38 min 31 sec (e) 10 hr 39 min 21 sec (ii) (b) 10 hr 30 sec (c) will not meet (d) at the star (e) 10 hr 5 min 30 sec. 26. 24 sec.

Miscellaneous Exercise 4

1. (a) 5 days (b) 1 day (c) 3 days (d) 1 (e) 2 (f) 5 (g) 1 (h) less 2. (a) $5\frac{5}{11}$ days (b) $1\frac{9}{11}$ days (c) $3\frac{1}{3}$ days (d) $3\frac{9}{37}$ days (e) $\frac{1}{36}$, 36 days (f) $27\frac{3}{11}$ days (g) 3 days (h) 3 days 3. 8 days 4. 16 hectares 5. $8\frac{8}{9}$ hrs 6. 35 7. $3\frac{57}{125}$ quintals 8. 50 9. $16\frac{2}{3}$ acres 10. 150 11. 6 hrs 12. (i) $16\frac{4}{11}$ days, Latif $1\frac{7}{11}$ acres, Mahim $1\frac{4}{11}$ acres (ii) 15 days 13. 500 14. $14\frac{2}{7}$ days 15. 256 lines 16. (a) 6 days (b) $\dfrac{xy}{x+y}$ days (c) $4\frac{4}{9}$ days (d) $3\frac{3}{14}$ hrs 17. Karim – 25 days, Sudhir – 100 days 18. Labani – 5, Srabani – 4, Nandini – 6 19. 24 days 20. $12\frac{1}{7}$ days 21. (i) 4 days (ii) 100 kg

(iii) 4 22. (i) 10 (ii) 540 (iii) $\frac{6}{25}$ day 23. Ashoke – 135 days, Arabinda – $7\frac{1}{2}$ days 24. (b) 1st work – $2\frac{11}{12}$ days, $3\frac{1}{13}$ days, $3\frac{11}{15}$ days, $2\frac{9}{131}$ days, 2nd work – $5\frac{5}{11}$ days, $3\frac{3}{4}$ days, 4 days, $2\frac{6}{7}$ days, 3rd work – 8 days, $2\frac{2}{3}$ days, $2\frac{2}{3}$ days, 2 days, 4th work – 20 days, $1\frac{7}{8}$ days, $2\frac{14}{23}$ days, $1\frac{5}{7}$ days. 25. 192 26. $\frac{2}{5}$ part 27. $17\frac{1}{7}$ min 28. (i) 6 hrs (ii) After 5 hrs 15 min 29. (i) After 2 hrs (ii) 1 hr 40 min 30. $2\frac{14}{23}$ hrs.

ALGEBRA

Chapter 1

Revision Exercises on previous lessons

Exercise 1

1. Write using sings :

(i) sum of p and q (ii) 5 less than a (iii) 10 times b (iv) 5 more than 3 times x (v) 7 more than the sum of a and b (vi) sum of a and b divided by c (vii) q less than p.

2. Write in words :

(i) $x-y$ (ii) $p+q$ (iii) $4m$ (iv) $\frac{n}{5}$ (v) $p+3q$

(vi) $\frac{a}{2}+b$

3. You have x colour pencils. If you give y pencils to your brother, how many pencils will you have ?

4. Madhabbabu is now 65 years old. (i) What was his age x years ago ? (ii) What will be his age y years hence ?

5. How many milimetres are there in a metres b centimetres.

6. The sum of two numbers is x. If one is y what is the other ?

7. During Durgapuja the students of a school raised a subscription of Rs. 250 to help the poor. From that they bought clothing worth x rupees and the rest money was equally distributed among y poor persons. Find the share of each.

8. The distance of B from A is x km. If the speed of a train is y km per hour, what time will it take to travel from A to B ?

9. The digits in the unit's and ten's places of a number of two digits are x and 3 respectively, what is the number ? What will be the number, if the digits in the unit's and ten's places are x and y respectively ?

10. You are elder than your brother by x years and your sister is elder than you by y years. If your present age is 12 years, what are present ages of your brother and your sister ? What will be the ages of your brother and your sister when your age will be the same as the present age of your sister ?

Exercise 2

If $a=3,\ b=4,\ c=2,\ d=1,\ x=5,\ y=0$, find the values of the following expressions :

1. $a+2b+3c+4d$
2. $7a-4x-4y+2c-3b$
3. $a\times b-c$
4. $b\div c-d$
5. $7a\times b\div 6c\times d$
6. $5a-\frac{cd}{2}+\frac{abc}{3}$
7. $\frac{2a+5b}{5c}+\frac{3d-4a}{a+b-c}$
8. $3bcx-10acx+3abx$
9. $ax+by+cd\div ab$
10. $4\times\frac{a-b}{c}+3\times\frac{b-c}{d}-6\times\frac{c-d}{a}$

Exercise 3

1. What do you mean by the following statements ?

(i) profit of − Rs. 15 (ii) −20 km below

(iii) −10 metres to the north (iv) −10°C increase in temperature

2. Fill up the blanks :

(i) Loss of Rs. 10 = profit of··· = ···of − Rs. 10

(ii) −5 km. to the south = ···to the north = 5 km. to the

(iii) To go up by 15 km = To go down by··· = ···by − 15 km

(iv) −5° decrease in temperature = ···increrse in temperature = 5°···in temperature.

3. Arrange the following numbers in ascending order ;

(i) $0, 1, -3, 4, -8$ (ii) $-5, 0, -1, -9, -7$

(iii) $3, -2, 6, -4, 8$ (iv) $1, -1, -3, -5$

(v) $\frac{1}{2}, -\frac{1}{4}, \frac{3}{4}, -\frac{5}{2}, \frac{9}{4}$ (vi) $-1, \frac{3}{2}, 3, -\frac{2}{3}, \frac{5}{2}, -4$

4. If $x=2, y=-1, z=3$, find the values of (i) $x^2+y^2+z^2$ and (ii) $x^3+y^3+z^3$.

5. Is the expression $a \times b-c$ same as the expression $a \times (b-c)$? Verify thc result taking $a=3, b=4, c=2$.

6. Simplify :

(i) $3a-4b-(3b+a)-(5a-8b)$

(ii) $-2-[-2-\{-2-(-2-2)\}]$

(iii) $a-[-a-\{-a-(-a)\}]$

(iv) $1-[1-\{1-(1-\overline{1-x})\}]$

7. If $a=3$ and $b=2$, show that

(i) $\dfrac{a^3+b^3}{a+b}=a^2-ab+b^2$ (ii) $\dfrac{a^3-b^3}{a-b}=a^2+ab+b^2$

8. If $a=1, b=2, c=3$ show that

$$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

9 Room charges of a hotel are a rupees per day for the first two days and b rupees per day after that. What will the total charges be for c days (assuming $c>2$) ?

10. Fill up the blanks :

(i) $a+3b-3c=a+3(\cdots\cdots)$

(ii) $x-2y+2z=x-2(\cdots\cdots)$

(iii) $a-b+c-d=a-(\cdots\cdots)$

Exercise 4

1. Add :

(i) $a^2, 2b^2, 3c^2$ (ii) $x^2y, 3x^2y, -4x^2y$

(iii) $5a^2-2a+4, -3a^2+3a+1, 7a^2-a, -2a+3$

2. Simplify :

(i) $5x+(-3x+8y+(-6y)+7z+(-3y)$

(ii) $(a-b)+(b-c)+(c-a)+(a+b+c)$

(iii) $(p^2+3pq+2q^2)+(-2pq+q^2-2p^2)+(4q^2+p^2-pq)$

3. Subtract :

(i) $a+b+c$ from $2a-3b+c$

(ii) $4x^2-2y^2-z^2$ from $8x^2-3y^2+4z^2$

(iii) m^2+n^3 from $4m^2-3mn+n^2$

4. Multiply :

(i) $8x^2, -2y^2, z^2$ (ii) $a^2b, -2ab^2, 4ab$

(iii) $-\frac{1}{2}ab^2, -\frac{2}{3}a^3c^3, -3ca^2$

5. Multiply :

(i) $3x^2-5x$ by $x+1$

(ii) b^2-c^2 by b^2+c^2

(iii) $yz+zx$ by $x-y$

6. Simplify :

(i) $a(b-c)+b(c-a)+c(a-b)$

(ii) $(y-z)(y+z)+(z-x)(z+x)+(x-y)(x+y)$

7. If $x=al-m$, $y=bm-n$, $z=cn-l$,

simplify : $x(l+m)+y(m+n)+z(n+l)$

Find the value of the expression when $a=b=c=1$

8. Divide :

(i) $12a^3b^2-15a^2b^3+9ab$ by $3ab$

(ii) $36x^3y^2-24x^2y^5-20x^4y^3$ by $4x^2y$

(iii) $3pqr+12p^2qr^2-9p^3q^2r^2$ by $-3pqr$

(iv) $30m^2n^3+40m^3n^4-50m^4n^5$ by $10m^2n^2$

Exercise 5

1. Find squares of the following expressions

(i) $4x+3y$ (ii) $8a-4b$ (iii) $xy+c$

(iii) $4x-abc$ (v) $x+y+2$ (vi) $2a+3b-5c$

(vii) $x+y+zp$ (viii) $a+b-c-1$

(ix) $1+2x+2y+3z$ (x) $m+2n-3p+4q$.

2. Simplify :

(i) $(3a+5b)^2+2(3a+5b)(2a-5b)+(2a-5b)^2$

(ii) $(3x-2y+z)^2-2(3x-2y+z)(x-2y+z)+(x-2y+z)^2$

3. Express as perfect square :

(i) $16a^2+8a+1$ (ii) $36x^2-84xy+49y^2$

(iii) $1+\frac{4}{x}+\frac{4}{x^2}$ (iv) $4x^2+\frac{9}{16y^2}-\frac{3x}{y}$

4. Find the values after expressing as perfect squares :

(i) $5{\cdot}71 \times 5{\cdot}71 + 2 \times 5{\cdot}71 \times 4{\cdot}29 + 4{\cdot}29 \times 4{\cdot}29$

(ii) $6{\cdot}47 \times 6{\cdot}47 - 2 \times 6{\cdot}47 \times 3{\cdot}47 + 3{\cdot}47 \times 3{\cdot}47$

(iii) $36x^2 + 132xy + 121y^2$ when $x = 8$, $y = -5$

(iv) $x^2y^2 - 10xyz + 25z^5$ when $x = 1$, $y = -2$, $z = 3$

5. If $x - y = 3$, $xy = 28$, what is the value of $x^2 + y^2$?

6. If $x + \frac{1}{x} = 4$. show that (i) $x^2 + \frac{1}{x^2} = 14$, (iii) $x^4 + \frac{1}{x^4} = 194$

7. If $m + \frac{1}{m} = -5$, show that $m^2 + \frac{1}{m^2} = 23$

8. If $m - \frac{1}{m} = p$, prove that (ii) $m^2 + \frac{1}{m^2} = p^2 + 2$

(ii) $\left(m + \frac{1}{m}\right)^2 = p^2 + 4$

9. If $a + b = 5$, $a - b = 1$, find the value of $8ab(a^2 + b^2)$

10. (i) What expression involving x should be added to $4x^2 + 16$ to make the sum a perfect square ?

(ii) What expresion involving ab should be subtracted from $16a^2 + 25b^2$ to make the difference a perfect square ?

Exercise 6

1. Find the product using formula :

(i) $(a+6)(a-6)$ (ii) $(6a+5b)(6a-5b)$

(iii) $(ac+bd)(ac-bd)$ (iv) $(4xy+5pq)(4xy-5pq)$

(v) $(2m+n+p)(2m+n-p)$ (vi) $(a+2b+3c)(a+2b-3c)$

(vii) $(2x-y+3z)(2x+y+3z)$

2. Multiply with the help of formula :

(i) 1002×998 (ii) 1511×1489 (iii) 556×544

3. Simplify with the help of formula :

(i) $(a+2b-3c)^2 - (a+2b+3c)^2$

(ii) $(4x+5y+3z)^2 - (3x-2y+3z)^2$

(iii) $4533 \times 4533 - 4531 \times 4531$

(iv) $(757\;71)^2 - (242{\cdot}29)^2$

4. Express as product of two expressions :

(i) $81a^2 - 49$ (ii) $121m^2 - 169n^2$

(iii) $(3x+4y)^2 - (2x-y)^2$ (iv) $a^2 + 2ab + b^2 - 1$

(v) $x^2 - 9 - 4xy + 4y^2$

5. Express as difference of two squares :

(i) 983×17 (ii) 541×409

(iii) $(2a+3b)(3a+b)$ (iv) $(4x+3y)(2x-3y)$

6. Faetorise with the help of formula :

(i) $4-25a^2$ (ii) x^2y^2-1 (iii) $a^2b^2-c^2d^2$

(iv) $25m^2-4n^2$ (v) $(a+b)^2-16$

(vi) $(4x+3y)^2-(3x-2y)^2$ (vii) $64ax^2-49a(x-2y)^2$

(viii) $(x^2+xy+y^2)^2-(x^2-xy+y^2)^2$

(ix) $4b^2c^2-(b^2+c^2-a^2)^2$ (x) $x^2+y^2+2xy-z^2$

(xi) $16x^2-4a^2+12ab-9b^2$ (xii) $a^2-b^2-2ac+2bc$

(xiii) x^2-2x-y^2+2y (xiv) $2mn-m^2-n^2+p^2$

(xv) $(a^2-b^2-c^2+d^2)^2-4(ad-bc)^2$

(xvi) $c+bc^2-ba^2-a$ (xvii) $2+a-a^2$

Exercise 7

Solve the following equations (1—17) :

1. $3x=0$ 2. $52x=39$ 3. $11x=-121$

4. $8x+5x-2x=17-9+33+3$

5. $6+7x-4x=5x+10$ 6. $\frac{1}{4}x-\frac{1}{2}x=3\frac{1}{4}-\frac{1}{3}x$

7. $3x+4+10x-17=14-23x+16-7x$

8. $\frac{x}{3}-\frac{1}{2}=\frac{x}{5}+1\frac{1}{2}$ 9. $\frac{x}{3}+1\frac{1}{2}=\frac{2x}{9}-\frac{x}{6}+4$

10. $5x-3(2x-1)=3(6-x)-1$

11. $4(3+x)-3(2x-5)=6-x-2(3-x)$

12. $2x-5\{7-(x-6)+3x\}-28=39$

13. $20(7x+4)-18(3x+4)-5=25(x+5)$

14. $\frac{2x-1}{3}-\frac{3x-2}{4}=\frac{5x-4}{6}-\frac{7x+6}{12}$

15. $13-\frac{9-x}{1}=\frac{3x}{22}+12\frac{1}{2}$

16. $\frac{1}{3}(x+1)+\frac{1}{4}(x+3)=\frac{1}{5}(x+4)+16$

17. $x-\frac{x-1}{2}-\frac{x}{3}=\frac{x-1}{3}-\frac{x-2}{4}+\frac{x-3}{5}$

18. Your present age is 8 years more than that of your younger brother. 4 years hence your age will be double that of your younger brother. Find out your age by forming an equation.

19. You went to the market and bought 2 kg potato at the rate of Rs. 2 per kg., 2 kg atta at Rs. 2·50 per kg and some rice at Rs. 3 per kg. Your total expenditure was Rs. 24. Form an equation to find out the quantity of rice and solve it.

20. Total marks in arithmetic scored by Nasifa, Sulema and Mandira was 162. Nasifa scored 8 marks more than Sulema and Sulema scored 10 marks less than Mandira. Form an equation to find out marks scored by each and solve it.

Chapter 2

Multiplication and division of polynomials

In class VII you have learnt how to multiply a binomial by another binomial. We shall discuss in this chapter how to multiply a polynomial by another polynomial. In this case also the basic rule for multiplication is same. When we multiply a polynomial by another polynomial, we generally arrange the terms of the multiplicand (the expression to be multiplied) and of the multiplier (the expression by which to multiply) conveniently.

First, we write the expressions, so that the powers of one or more alphabets are either in ascending or in descending order. Then we multiply by writing the multiplicand first and the multiplier below it (as in arithmetic). We consider the following example :

Ex. 1. Multiply $3a^3-4a+2a^2+1$ by $2a+a^2-3$

Here both the multiplicand and the multiplier are expressions with one alphabet ; but the terms are not arranged according to the ascending or descendig order of powers. Arranging the multipicand. $3a^3-4a+2a^2+1$, according to the descending order of powers of a we get $3a^3+2a^2-4a+1$. Similarly, arranging the multiplier $2a+a^2-3$, we get a^2+2a-3. After arranging the multiplicand and the multiplier in the above manner we shall multiply in the following way :

Multiplicand $= 3a^3 + 2a^2 - 4a + 1$

Multiplier $= \qquad a^2 + 2a - 3$

$3a^5 + 2a^4 - 4a^3 + a^2$ (multiplying by a^2)

$6a^4 + 4a^3 - 8a^2 + 2a$ (multiplying by $2a$)

$-9a^3 - 6a^2 + 12a - 3$ (multiplying by -3)

$3a^5 + 8a^4 - 9a^3 - 13a^2 + 14a - 3$

$\therefore$ required product $= 3a^5 + 8a^4 - 9a^3 - 13a^2 + 14a - 3$

Now let us consider how to multiply an expression with two alphabets by another expression with two alphabets.

Ex. 2. Multiply $x^2 + xy - y^2$ by $y^2 + x^2 - xy$

Arranging the multiplicand and the multiplier in the descending order of powers of x, we get

Multiplicand $= x^2 + xy - y^2$

Multilier $= x^2 - xy + y^2$

$x^4 + x^3y - x^2y^2$ (Multiplication by x^2)

$-x^3y - x^2y^2 + xy^3$ (multiplication by $-xy$)

$x^2y^2 + xy^3 - y^4$ (multiplication by y^2)

$x^4 \qquad - x^2y^2 + 2xy^3 - y^4$

$\therefore$ Required product $= x^4 - x^2y^2 + 2xy^3 - y^4$

Note that multiplicand and multiplier have been arranged in the descending order of powers of x. Note further that the partial products have been so arranged that the like terms of different rows are placed one below the other in the same column. A line has been drawn below the lowest row and the algebraic sum of each column has been written in a row. Then result,so obtained is the required product.

Ex. 3. Multiply $3a^2-4ax+x^2$ by $5x^2+2ax-2a^2$

Arranging both multiplicand and multiplier in the ascending order of powers of x, we get

$$\begin{array}{lrrrrr}
\text{Multiplicand} = & 3a^2 & -4ax & +x^2 & & \\
\text{Multiplier} = & -2a^2 & +2ax & +5x^2 & & \\
\hline
& -6a^4 & +8a^3x & -2a^2x^2 & & \\
& & 6a^3x & -8a^2x^2 & +2ax^3 & \\
& & & 15a^2x^2 & -20ax^3 & +5x^4 \\
\hline
& -6a^4 & +14a^3x & +5a^2x^2 & -18ax^3 & +5x^4
\end{array}$$

Required product $=-6a^4+14a^3x+5a^2x^2-18ax^3+5x^4$

Ex. 4. Multiply $a^3+3a^2b+3ab^2+b^3$ by $a^3-3a^2b+3ab^2-b^3$

$$\begin{array}{lllllll}
a^3 & +3a^2b & +3ab^2 & +b^3 & & & \\
a^3 & -3a^2b & +3ab^2 & -b^3 & & & \\
a^6 & +3a^5b & +3a^4b^2 & +a^3b^3 & & & \\
& -3a^5b & -9a^4b^2 & -9a^3b^3 & -3a^2b^4 & & \\
& & +3a^4b^2 & +9a^3b^3 & +9a^2b^4 & +3ab^5 & \\
& & & -a^3b^3 & -3a^2b^4 & -3ab^5 & -b^6 \\
\hline
a^6 & & -3a^4b^2 & & +3a^2b^4 & & b^6
\end{array}$$

$\therefore$ required product $=a^5-3a^4b^2+3a^2b^4-b^6$

Note : In the above example both mulitiplicand and multiplier are expressions with terms of same order and the order or maximum power of each of them is three. Note that in both multiplicand and multiplier the power of a in the first term is 3, in the second term in the power of a is 2 and that of b is 1. Thus the total power is $2+1=3$. Similarly, the total power of each of the 3rd and 4th terms is also 3. Thus both multiplicand and multiplier are with terms of the same order. It may also be noted that the product is also an expression with terms of the same order and the order of each term is 6.

Multiplication of more than two polynomials :

First we multiply any two expressions in the above manner. Then taking this product as the multiplicand and taking the 3rd expression as the multiplier we find the product in the above manner. In this way taking the successive products as multiplicands and multiplying it by successive expressios we complete the process of multiplication.

Ex. 5. Find the successive products of x^2+x+1, x^2-x+1 and x^4-x^2+1.

$$\begin{array}{lllll}
x^2 & +x & +1 & & \\
x^2 & -x & +1 & & \\ \hline
x^4 & +x^3 & +x^2 & & \\
 & -x^3 & -x^2 & -x & \\
 & & x^2 & +x & +1 \\ \hline
x^4 & & +x^2 & & +1
\end{array}$$

Thus the product of the first two expressions $=x^4+x^2+1$. Taking this product as multiplicand and the third expression as the multiplier we shall have to multiply as before.

$$\begin{array}{lllll}
x^4 & +x^2 & +1 & & \\
x^4 & -x^2 & +1 & & \\ \hline
x^8 & +x^6 & +x^4 & & \\
 & -x^6 & -x^4 & -x^2 & \\
 & & x^4 & +x^2 & +1 \\ \hline
x^8 & & +x^4 & & +1
\end{array}$$

$\therefore$ required product $=x^8+x^4+1$

In case of more than three expressions we shall have to multiply in the same manner.

Multiplication by the method of 'detached co-efficients' :

If both multiplicand and multiplier are two expressions containing terms with different powers of the same alphabet then it will be less laborious if we find out the product by the method of detached co-efficients. In this method we only write successive co-efficients of the multiplicand and multiplier and then multiply as in the previous method. After obtaining the product of the co-efficients the alphabet is written with successive powers. In this method also the expressions are first arranged in ascending or descending order of powers. If any term containing a particular power is absent either in the multiplicand or in the multiplier, then the corresponding coefficient is to be taken as 0(zero). Observe the following examble :

Ex. 6. Multiply $5x^2-6x+3$ by $x-4$.

First we detach the co-efficients of multiplicand and multiplier.

$$\begin{array}{rrrr}
5 & -6 & +3 & \\
1 & -4 & & \\
\hline
5 & -6 & +3 & \\
 & -20 & +24 & -12 \\
\hline
5 & -26 & +27 & -12
\end{array}$$

Here highest powers of the alphabet in the multiplicand and multiplier are 2 and 1 respectively. Hence the highest power of the alphabet in the product will be $(2+1)=3$.

$\therefore$ required product $5x^3-26x^2+27x-12$

Ex. 7. Multiply $3a^3-2a^2+5$ by $2a^2+3a-3$.

$$\begin{array}{rrrrrr}
3 & -2 & +0 & +5 & & \\
2 & +3 & -3 & & & \\
\cline{1-4}
6 & -4 & +0 & +10 & & \\
 & +9 & -6 & +0 & +15 & \\
 & & -9 & +6 & -0 & -15 \\
\hline
6 & +5 & -15 & +16 & +15 & -15
\end{array}$$

[Here the term containing a is absent in the multiplicand. Hence the co-efficient of a has been taken as 0.]

Here the highest power of the alphabet a of the product will naturally be 5 and the other terms will be in the descending order of the power of a.

$\therefore$ required product $=6a^5+5a^4-15a^3+16a^2$
$+15a-15$

Remark : Two homogenous expressions with two alphabets can also be multiplied by the method of detached co-efficients.

Exercise 8

Multiply :

1. $2a+3b+5$ cy $3a+4b+2$
2. $3x-4y+5z$ by $x+y-z$
3. $5a^2-6a+8$ by $3a^2+2a-3$
4. $5x^3+3x-2$ by $2x^2-x+3$
5. x^4+x^2+1 by x^4+x^2+1
6. $2k^2-3k-1$ by $3k^2-k-1$
7. $8a^3-3a-2a^2$ by $3a^2-5a+1$
8. $5m^2+3-4m$ by $5-4m+3m^2$
9. $4x+3x^3-2x^2+1$ by $x-2x^2+2$
10. $x^2-5+x^4-3x+2x^3$ by $5+x^2-3x$
11. $2a^2+3ab+b^2$ by a^2-ab+b^2
12. $x^3+3x^2y+3xy^2+y^3$ by x^2-xy+y^2

13. $p^3-p^2q^2+q^2$ by p^2+q^2+1
14. $a^3-4a^2b+ab^2-b^3$ by a^2+ab+b^2
15. $2ax^2+3bx+4c$ by ax^2-bx-c
16. $4x+3x^3-2x^2+1$ by $x-2x^2+2$
17. $6y^2+y^4+1-4y^3-4y$ by $1+y^2-2y$
18. $x^2+y^2+z^2-xy-yz-zx$ by $x+y+z$
19. $2a^3b-3ab^3-a^4+5a^2b^2$ by $2a^4-3a^3b+2ab^3-4a^2b^2$
20. $a^2-ab+b^2+a+b+1$ by $a-b+1$
21. px^3+qx^2+rx+s by px^2-qx-r
22. $a^3b^3-c^3+abc^3-a^3b^2c$ by $a^3b^2-a^2bc-ac^2$

Find the product of :

23. $(a+b), (a-b), (a^2+b^2)$
24. $(x-y), (x^2+xy+y^2), (x^3+y^3)$
25. $(3x+1), (2x-5), (x+2)$
26. $(2x-3y), (4x+5y), (2x^2-3xy+y^2)$
27. $(a-b), (a+b), (a^4+a^2b^2+b^4)$
28. $(ax+by), (ax-by), (a^2x^2-abxy+b^2y^2)$
29. $(x^2+2xy+3y^2,)(x^2-2xy+3y^2), (x^4-2x^2y^2+3y^4\}$
30. $(a+b+c), a-b+c), (b+c-a), b+a-c)$
31. $(a^2-ab+b^2), (a^2+ab+b^2), (a^4-a^2b^2+b^4)$
32. $(xy+yz+zx), (yz-zx+xy), (xy+yz)$
33. $(2x-3y), (4x^2+6xy+9y^2), (x+y), (x^2-xy+y^2)$

Find the product by the method of detached co-efficients :

34. $3x^2-5x+6$ by $2x-4$
25. $5a^2-6a+8$ by $3a^2+5a-6$
36. $2x^3-4x^2-5$ by $3x^2+4x\quad 2$
37. px^2+qx+r by $ax+b$
38. **Simplify :** $(x+y)(x^2-xy+y^2)+(x-y)(x^2+xy+y^2)$
39. **If** $a=lx+my+n,\ b=mx+ny+l,\ c=nx+ly+m$;
find the value of $a(m+n)+b(n+l)+c(l+m)$
40. **If** $p=a^2+ab+b^2,\ q=b^2+bc+c^2,\ r=c^2+ca+a^2$
find the value of $(a+b)p+(b-c)q+(c-a)r.$

Division of a polynomial by a polynomial :

You have learnt in class VII how to divide a polynomial by a monomial. Now we shall discuss how to divide a polynomial by another polynomial.

You know that the expression which is divided is called dividend and the expression which divides is called divisor. In this case also, like multiplication, the dividend and the divisor are arranged in the ascending or descending order of powers of a common alphabet in those expressions and then the process of division is completed.

First of all, in the following example, we discuss how to divide a polynomial containing one alphabet by another polynomial containing the same alphabet.

Ex. 1. Divide $a^4 - 7a^2 - 2a^3 + 8a + 12$
by $-a + a^2 - 6$

First we arrange both the dividend and the divisor in descending order of powers of a.

Dividend $= a^4 - 2a^3 - 7a^2 + 8a + 12$
Divisor $= a^2 - a - 6$

$$
\begin{array}{rl}
a^2 - a - 6) & a^4 - 2a^3 - 7a^2 + 8a + 12\,(a^2 - a - 2 \\
 & \underline{a^4 - a^3 - 6a^2\phantom{{}+8a+12}} \\
 & - a^3 - a^2 + 8a + 12 \\
 & \underline{ - a^3 + a^2 + 6a\phantom{{}+12}} \\
 & - 2a^2 + 2a + 12 \\
 & \underline{ - 2a^2 + 2a + 12}
\end{array}
$$

$\therefore$ required quotient $= a^2 - a - 2$.

Let us now discuss how we have carried out the division. First dividing the first term of the dividend by the first term of the divisor, we get $a^4 \div a^2 = a^2$. We take this a^2 as the first term of the quotient. All the terms of the divisor are now multiplied by this term and each term of the product is placed under corresponding term of the dividend. Subtracting this product from the dividend we get the second step of the dividend. Now dividing the first term of the second step by the first term of the divisor, we get $-a^3 \div a^2 = -a$. This $(-a)$ is taken as the second term of the quotient. As before, whole of the divisor is multiplied by this term and the product is subtracted from the second step of the divisor. In this way we proceed as far as possible.

Remark : We can also divide by arranging the dividend and the divisor in ascending instead of descending order of powers of the common alphabet.

Now let ns see how to divide a polynomial with two alphabets by another polynomial of two alphabets.

Ex. 2. Divide $3a^3+8a^2b+9ab^2+10b^3$ by $a+2b$

Here the expressions are already arranged in descending order of powers of a.

$$
\begin{array}{l}
a+2b)\ 3a^3+8a^2b+9ab^2+10b^3\ (3a^2+2ab+5b^2 \\
\qquad\quad \underline{3a^3+6a^2b} \\
\qquad\qquad\quad 2a^2b+9ab^2+10b^3 \\
\qquad\qquad\quad \underline{2a^2b+4ab^2} \\
\qquad\qquad\qquad\qquad 5ab^2+10b^3 \\
\qquad\qquad\qquad\qquad \underline{5ab^2+10b^3}
\end{array}
$$

$\therefore$ required quotient $=3a^2+2ab+5b^2$

Now we discuss the above method of division. As before, $3a^3 \div a = 3a^2$ is taken as the first term of the quotient. Then the product of the whole of the divisor and $3a^2$ is arranged under the corresponding terms of the dividend and the second step of the dividend is obtained by subtraction. Then $2a^2b \div a = 2ab$ taken as the second term of the quotient. In this way we proceed till we come to the end of the process of division.

Ex. 3. Divide $16x^4+36x^2+81$ by $4x^2+6x+9$

Here both the dividend and the divisor are already arranged in the descending order of powers of x. Note that in the dividend there is no term containing x^3 and x, that is. the co-efficients of x^3 and x are zero. For this reason in writing the dividend some space is left in the places of x^3 and x so that we can write like terms just one below the other. Now observe the process of division.

$$\begin{array}{r|rrrrr|l}
4x^2+6x+9 & 16x^4 & & +36x^2 & & +81 & 4x^2-6x+9 \\
& 16x^4 & +24x^3 & +36x^2 & & & \\ \cline{2-6}
& & -24x^3 & & & +81 & \\
& & -24x^3 & -36x^2 & -54x & & \\ \cline{3-6}
& & & 36x^2 & +54x & +81 & \\
& & & 36x^2 & +54x & +81 & \\ \cline{4-6}
\end{array}$$

$\therefore$ required quotient $= 4x^2-6x+9$.

Ex. 4. Divide $2+x-8x^2+4x^3+x^4-3x^5+x^6$ by $1+2x-x^2$

Here both the dividend and the divisor are already arranged in the ascending order of powers of x.

$$
\begin{array}{l}
1+2x-x^2)\,2+x-8x^2+4x^3+x^4-3x^5+x^6\,(2-3x+x^3-x^4\\
\qquad\quad 2+4x-2x^2\\
\hline
\qquad\quad -3x-6x^2+4x^3+x^4-3x^5+x^6\\
\qquad\quad -3x-6x^2+3x^3\\
\hline
\qquad\qquad\qquad\quad x^3+x^4-3x^5+x^6\\
\qquad\qquad\qquad\quad x^3+2x^4-x^5\\
\hline
\qquad\qquad\qquad\qquad -x^4-2x^5+x^6\\
\qquad\qquad\qquad\qquad -x^4-2x^5+x^6\\
\hline
\end{array}
$$

$\therefore$ required quotient $=2-3x+x^3-x^4$

Ex. 5. Divide $a^3+b^3+c^3-3abc$ by $a+b+c$

$$
\begin{array}{l}
a+b+c)\,a^3 \quad -3abc \quad +b^3 \quad +c^3\,(a^2-ab-ca+b^2-bc+c^2\\
\qquad a^3+a^2b+a^2c\\
\hline
\qquad -a^2b-a^2c-3abc \quad +b^3 \quad +c^3\\
\qquad -a^2b-ab^2-abc\\
\hline
\qquad -a^2c+ab^2-2abc \quad +b^3 \quad +c^3\\
\qquad -a^2c \quad -abc-ac^2\\
\hline
\qquad\qquad ab^2-abc+ac^2+b^3 \quad +c^3\\
\qquad\qquad ab^2 \quad +b^3+b^2c\\
\hline
\qquad\qquad\quad -abc+ac^2 \quad -b^2c+c^3\\
\qquad\qquad\quad -abc \quad -bc^2-b^2c\\
\hline
\qquad\qquad\qquad ac^2+bc^2 \quad +c^3\\
\qquad\qquad\qquad ac^2+bc^2 \quad +c^3\\
\hline
\end{array}
$$

$\therefore$ required quotient $=a^2+b^2+c^2-ab-bc-ca.$

Ex. 6. Divide $2x^6-2$ by x^3+2x^2+2x+1

$$
\begin{array}{l}
x^3+2x^2\\
\quad +2x+1)\,2x^6 \qquad\qquad\qquad -2\,(2x^3-4x^2+4x-2\\
\qquad\quad 2x^6+4x^5+4x^4+2x^3\\
\hline
\qquad\qquad -4x^5+4x^4-2x^3 \qquad -2\\
\qquad\qquad -4x^5-8x^4-8x^3-4x^2\\
\hline
\qquad\qquad\qquad 4x^4+6x^3+4x^2 \qquad -2\\
\qquad\qquad\qquad 4x^4+8x^3+8x^2+4x\\
\hline
\qquad\qquad\qquad\quad -2x^3-4x^2-4x-2\\
\qquad\qquad\qquad\quad -2x^3-4x^2-4x-2\\
\hline
\end{array}
$$

$\therefore$ required quotient $=2x^3+4x^2+4x-2$

Exercise 9

Divide :

1. a^2+3a+2 by $a+1$
2. x^2+4x+4 by $x+2$
3. $m^2+7m+12$ by $m+4$
4. a^2+5a+6 by $a+3$
5. $x^2-9x+20$ by $x-5$
6. $m^2-4m-21$ by $m+3$
7. $x^2-13x+22$ by $x-11$
8. $a^2-3a-40$ by $a+5$
9. $p^2-8p-65$ by $p-13$
10. $3x^2+8x+4$ by $3x+2$
11. $3y^2+y-2$ by $y+1$
12. $6c^2-7c+2$ by $3c-2$
13. $-15x^2+17x+4$ by $5x+1$
14. $-21m^2+58m-21$ by $3m-7$
15. $4a^2-9b^2$ by $2a+3b$
16. $36c^2-6cd-2d^2$ by $-6c+2d$
17. $12a^2-31ab+20b^2$ by $4a-5b$
18. $6x^3y-x^2y^2-7xy^3+12y^4$ by $2x+3y$
19. $24x^2-65xy+21y^2$ by $8x-3y$
20. $2a^4c-7a^3bc^2+9ab^3c^4$ by $2a^2-3abc$
21. $2a^3-7a^2-a+2$ by a^2-3a-2
22. x^3-4x^2+x+6 by x^2-x-2
23. $6a^3-17a^2-16a+7$ by $3a^2+2a-1$
24. $1-a^2+3a^4-5a^3+3a$ by $1-3a^2+2a$
25. $m^4-2m^3-7m^2+8m+12$ by m^2-m-6
26. $81x^4-1$ by $3x-1$
27. $5y^3+1+10y^4-4y^2$ by $5y^2-2y+1$
28. $33p^2-13p^3+15p^4-9p+10$ by $5p^2-p+2$
29. $15a^4-32a^3+50a^2-32a+15$ by $5a^2-4a+3$
30. $4c^2-12c-d^2+9$ by $2c+d-3$
31. $24x^2-65xy+21y^2$ by $8x-3y$
32. $a^2-4ab+4b^2-9c^2$ by $a-2b+3c$

33. $6m^4+22mn^3-8m^2n^2-11m^3n-8n^4$ by $6m^2+4n^2-11mn$

34. $a^2+4b^2+9c^2+4ab+12bc+6ac$ by $a+2b+3c$

35. $x^3y^3-xy-9-y^4+3y^3+3y-x^4-3x^3-3x$ by $3-y+x^3$

36. $2a^4x-7a^3bx^2+9ab^3x^4$ by $2a^3-3abx$

Inexact Division :

When a dividend is not totally divisible by the divisor i.e., at the end of the process of division there is such a non zero remainder that cannot be divided by the divisor, the division is called inexact division. Let us take the following example.

Ex. 1. Divide $6x^5-x^4+4x^3-5x^2-3x-18$ by $2x^2-x+3$ and find out the quotient and the remainder

$$
\begin{array}{r}
2x^2-x+3\,\big)\,6x^5-x^4+4x^3-5x^2-3x-18\,\big(\,3x^3+x^2-2x-5 \\
\underline{6x^5-3x^4+9x^3} \\
2x^4-5x^3-5x^2-3x-18 \\
\underline{2x^4-x^3+3x^2} \\
-4x^3-8x^2-3x-18 \\
\underline{-4x^3+2x^2-6x} \\
-10x^2+3x-18 \\
\underline{-10x^2+5x-15} \\
-2x-3
\end{array}
$$

$\therefore$ quotient $=3x^3+x^2-2x-5$, remainder $=-2x-3$.

Note that $-2x-3$ can no longer be divided by $2x^2-x+3$, because $-2x-3$ is of first order and this order is less than the order of $2x^2-x+3$ which is 2.

Remark : The process of division will have to be continued until the order of the remainder is less than that of the divisor and in doing so sometimes we may even get fractional co-efficient in the quotient.

Division by the method of 'detached co-efficients':

You have already learnt how to multiply by the method of 'detached co-efficients'. This method is also applicable in division

If both the dividend and the divisor are expressions with same alphabet, then we can carry on the process of division by 'arranging the co-efficients in the descending order of powers of the alphabet. Observe the following example.

Ex. 2. Divide $6x^4+19x^3+25x^2+21x+9$ by $2x^2+5x+3$

Here both the dividend and the divisor are arranged in the desconding order of powers of x.

First we divide by the general method.

$$\begin{array}{l}
2x^2+5x+3)6x^4+19x^3+25x^2+21x+9(3x^2+2x+3 \\
\qquad\qquad\quad \underline{6x^4+15x^3+9x^2} \\
\qquad\qquad\qquad\qquad 4x^3+16x^2+21x+9 \\
\qquad\qquad\qquad\qquad \underline{4x^3+10x^2+6x} \\
\qquad\qquad\qquad\qquad\qquad\quad 6x^2+15x+9 \\
\qquad\qquad\qquad\qquad\qquad\quad \underline{6x^2+15x+9}
\end{array}$$

$\therefore$ required quotient $=3x^2+2x+3$

Now we divide by the method of detached co efficients.

$$\begin{array}{l}
2+5+3)6+19+25+21+9(3+2+3 \\
\qquad\qquad \underline{6+15+9} \\
\qquad\qquad\quad 4+16+21+9 \\
\qquad\qquad\quad \underline{4+10+6} \\
\qquad\qquad\qquad\quad 6+15+9 \\
\qquad\qquad\qquad\quad \underline{6+15+9}
\end{array}$$

Highest power of x in the quotient will definitely be 2. Hence the required quotient is $3x^2+2x+3$.

Remark : If the term containing any particular power of the common alphabet in dividend or divisor is absent, the above method of division may be applied by taking the co-efficient of the missing term as 0 (zero). We consider the following example.

Ex. 3. Divide $2x^5 - 5x^2 + 3$ by $x^2 - 2x + 1$

Note that the terms containing x^4, x^3, x are missing in the dividend. Hence the the co-efficients of those terms are to be taken as 0 (zero).

$$\begin{array}{l}
1-2+1)2+0+0-5+0+3(2+4+6+3 \\
\qquad\quad \underline{2-4+2} \\
\qquad\qquad\quad 4-2-5+0+3 \\
\qquad\qquad\quad \underline{4-8+4} \\
\qquad\qquad\qquad\quad 6-9+0+3 \\
\qquad\qquad\qquad\quad \underline{6-12+6} \\
\qquad\qquad\qquad\qquad\quad 3\quad 6+3 \\
\qquad\qquad\qquad\qquad\quad \underline{3-6+3}
\end{array}$$

$\therefore$ required quotient $= 2x^3 + 4x^2 + 6x + 3$.

In case of inexact division also we may apply the method of detached co-efficients.

Ex. 4 Divide $a^4 - 2a^3 - 7a^2 + 10a + 13$ by $a^2 - a - 6$

$$\begin{array}{l}
1-1-6)1-2-7+10+13(1-1-2 \\
\qquad\quad \underline{1-1-6} \\
\qquad\qquad -1-1+10+13 \\
\qquad\qquad \underline{-1+1+\ 6} \\
\qquad\qquad\qquad -2+\ 4+13 \\
\qquad\qquad\qquad \underline{-2+\ 2+12} \\
\qquad\qquad\qquad\qquad\quad 2+\ 1
\end{array}$$

$\therefore$ required quotient $= a^2 - a - 2$ and
remainder $= 2a + 1$

Remark : The process of division will have to be continued until the number of terms in the remainder falls short of the number of terms in the divisor.

Exercise 10

Find out the quotient and remainder in the division of:

1. $12a^2+14a+8$ by $3a+2$
2. $10x^2-8x+5$ by $5x-1$
3. $5a^3-4a^2+2a-3$ by a^2+2a-1
4. $x^3+x^2-8x-13$ by x^2+3x+3
5. $12x^4+5x^3-33x^2-3x+16$ by $4x^2-x-5$
6. $p^4-6p^3+13p^2-10p+7$ by p^2-3p+2
7. $15x^4+22-32x^3-30x+50x^2$ by $3-4x+5x^2$
8. $x^5-2x^4-4x^3+23x^2-31x+12$ by x^2-2x+3
9. $18a^3+24b^3-40ab^2-9a^2b$ by $9a^2+7b^2+18ab$
10. $x^5+x^4y-x^3y^2+x^3-2xy^2$ by x^2+xy-y^2

Find out, by the method of detached co-efficients, the quotient and the remainder, if any, in the division of :

11. $2a^3-9a^2b+13ab^2-9b^3$ by $2a-3b$
12. $63x^2-19x-20$ by $9x+5$
13. $21a^3-26a^2-15a+20$ by $7a^2+3a-4$
14. $2a^4+11a^3-13a^2-99a-45$ by $a^2+2a-15$
15. $m^4-4m^3-18m^2-11m+2$ by m^2-7m+1
16. $x^5-2x^4-4x^3-19x^2-31x+15$ by x^3-7x+5
17. $2a^5-5a^2+3$ by a^2-2a+1
18. $3x^5-5x^3+2$ by x^2-2x+1
19. a^3+64 by $a^2-4a+16$
20. $81x^4+2$ by $3x-1$
21. $a^3-a^2-8a-13$ by a^2+3a+3
22. $12x^4+5x^3-33x^2-3x+16$ by $4x^2-x-5$
23. $a^5-2a^4-4a^3+20a^2-31a+10$ by a^2-2a+3
24. $2a^6-2$ by a^3+2a^2+2a+1
25. Find the remainder in the division of x^3+px^2+qx+r by x^2+px+q.
26. In a division the divisor is x^2-x+1, the quotient is $2x^2+x-3$ and the remainder is $x+2$, what is the dividend ?

Chapter 3

Formulae and their applications

Formula for the cube of the sum of two terms :

In class VII you have learnt the following formula : $(a+b)^2=a^2+2ab+b^2$. This result was obtained by multiplying $a+b)$ by itself two times. Similarly, we get the formula for $(a+b)^3$ by multiplying $(a+b)$ by itself three times.

Proof of the formula

$$\begin{aligned}(a+b)^3 &= (a+b)(a+b)(a+b)\\ &= (a+b)(a+b)^2\\ &= (a+b\ (a^2+2ab+b^2)\\ &= a(a^2+2ab+b^2)+b(a^2+2ab+b^2)\\ &= a^3+2a^2b+ab^2+a^2b+2ab^2+b^3\\ &= a^3+3a^2b+3ab^2+b^3\end{aligned}$$

Thus we get the following formula :

$$\begin{aligned}(a+b)^3 &= a^3+3a^2b+3ab^2+b^3\\ &= a^3+b^3+3ab(a+b)\end{aligned}$$

Geometric representation of the formula with the help of blocks :

Prepare two cubes of sides 7 cm and 3 cm Prepare three parallelopipeds of length and breadth 7 cm each and height 3 cm Prepare three more parallelopipeds of length and breadth 3 cm each and height 7 cm. Thus we get altogether eight solids of the following volumes.

7^3 c.c. = one
3^3 c.c. = one
$7^2 \times 3$ c c. = three
7×3^2 c c. = three

Total volume of these solids $=(7^3+3\times7^2\times3+3\times3^2\times7+3^3)$ c.c.

By arranging these eight solids in proper manner we get a cube, the length of each side of whose is $(7+3)$ cm and hence its volume is $(7+3)^3$ c.c.

$$(7+3)^3=7^3+3\times7^2\times3+3\times7\times3^2+3^3$$

Now taking 7 cm $=a$ and 3 cm $=b$, we get

$$(a+b)^3=a^3+3a^2b+3ab^2+b^3$$

Ex. 1. Find the cube of $(x+5)$.

$$(x+5)^3=(x)^3+3(x)^25+3x(5)^2+(5)^3$$
$$=x^3+15x^2+75x+125$$

Ex. 2. Find the cube of $(3a+4)$

$$(3a+4)^3=(3a)^3+3(3a)^2.4+3(3a)(4)^2+(4)^3$$
$$=27a^3+108a^2+144a+64$$

Ex. 3. Find the cube of $(4x+7y)$

$$(4x+7y)^3=(4x)^3+3(4x)^2.7y+3\ 4x(7y)^2+(7y)^3$$
$$=64x^3+336x^2y+588xy^2+343y^3$$

Ex. 4. Find the cube of 75 with the help of formula $(75)^3=(70+5)^3$

$$=(70)^3+3.(70)^2.5+3\ 70\ (5)^2+(5)^3$$
$$=343000+73500+5250+125$$
$$=421875$$

Ex. 5. Find the value with the help of formula.

$$6{\cdot}33\times6{\cdot}33\times6{\cdot}33+3\times6{\cdot}33\times6{\cdot}33\times3{\cdot}67+3\times6{\cdot}33\times3{\cdot}67\times3{\cdot}67+3{\cdot}67\times3{\cdot}67\times3{\cdot}67$$

Given expression $=(6{\cdot}33)^3+3\times(6{\cdot}33)^2\times3{\cdot}67+3\times6{\cdot}33\times(3{\cdot}67)^2+(3{\cdot}67)^3$

$$=(6{\cdot}33+3{\cdot}67)^3$$
$$=(10{\cdot}00)^3$$
$$=(10)^3=1000$$

Ex. 6. Simplify :

$$(3a+2b)^3+3(3a+2b)^2(a-2b)+3(3a+2b)(a-2b)^2+(a-2b)^3$$

Taking $3a+2b=x$ and $a-2b=y$, we get

Given expression $=x^3+3x^2y+3xy^2+y^3$

$$=(x+y)^3$$
$$=(3a+2b+a-2b)^3$$
$$=(4a)^3=64a^3$$

Cube of a polynomial

Ex. 7. Find the cube of $x+y+z$.

$(x+y+z)^3=\{(x+y)+z\}^3$

If we use the formula taking $(x+y)$ as the first term and z as the second term, we get

$$\therefore\ (x+y+z)^3=(x+y)^3+3(x+y)^2z+3(x+y)z^2+z^3$$
$$=x^3+3x^2y+3xy^2+y^3+3(x^2+2xy+y^2)z+3xz^2+3yz^2+z^3$$
$$=x^3+3x^2y+3xy^2+y^3+3x^2z+6xyz+3y^2z+3xz^2+3yz^2+z^3$$
$$=x^3+y^3+z^3+3y^2z+3yz^2+3z^2x+3zx^2+3x^2y+3xy^2+6xyz$$

Ex. 8. Use formula to find the value of $x^3+9x^2+27x+27$, when $x=-3$.

$$x^3+9x^2+27x+27$$
$$=x^3+3\,x^2.3+3.x.(3)^2+(3)^3$$
$$=(x+3)^3=(-3+3)^3=0.$$

Fx. 9. Use formula to find the value of $8a^3+36a^2b+54ab^2+27b^3$, when $a=2$ and $b=1$

$$8a^3+36a^2b+54ab^2+27b^3$$
$$=(2a)^3+3.(2a)^2\,b+3.2a(3b)^2+(3b)^3$$
$$=(2a+3b)^3=(2.2+3.1)=(7)^3=343$$

Ex. 10. If $x + y = 6$, find the value of $x^3 + y^3 + 18xy$

$$\begin{aligned} x^3 + y^3 + 18xy &= x^3 + y^3 + 3.x.y.6 \\ &= x^3 + y^3 + 3xy\,(x + y) \text{ [putting } 6 = x + y] \\ &= (x+y)^3 \\ &= (6)^3 = 216 \end{aligned}$$

Ex. 11. If $m + n = 4$, prove that $m^3 + n^3 + 12mn = 64$.

$$\begin{aligned} \text{Given expression} &= m^3 + n^3 + 12mn \\ &= m^3 + n^3 + 3mn.4 \\ &= m^3 + n^3 + 3mn\,(m + n) \\ &= (m + n)^3 \text{ [putting } 4 = m + n] \\ &= (4)^2 = 64 \text{ (proved)} \end{aligned}$$

Exercise 11

Find the cubes of the following expressions with the help of formula :

1. $x + 2$
2. $3a + 4$
3. $4c + d$
4. $3m + 2n$
5. $a^2 + 2b$
6. $ab + cd$
7. $3a + 2bc$
8. $x^2 + yz$
9. $x + yz^2$
10. $p2 + q2$
11. $a + b + 2c$
12. $a + 2b + 3c$
13. 95
14. 110
15. 125

Find the values of the following with the help of formula :

16. $(29)^3 + 3 \times (29)^2 \times 21 + 3 \times 29 \times (21)^2 + (21)^3$
17. $31{\cdot}63 \times 31{\cdot}63 \times 31{\cdot}63 + 3 \times 31{\cdot}63 \times 31{\cdot}63 \times 28{\cdot}37 + 3 \times 31{\cdot}63 \times 28{\cdot}37 \times 28{\cdot}37 \times 28{\cdot}37 \times 28{\cdot}37 \times 28{\cdot}37$
18. $2{\cdot}89 \times 2{\cdot}89 \times 2{\cdot}89 \times 2{\cdot}11 \times 2{\cdot}11 \times 2{\cdot}11 + 15 \times 2{\cdot}89 \times 2{\cdot}11$

Simplify after expressing as cubes :

19. $(5a+2b)^3+3(5a+2b)^2(a-2b)+3(5a+2b)(a-2b)^2+(a-2b)^3$
20. $(3x-2y+4z)^3+3(3x-2y+4z)^2(2y-4z)+ 3(3x-2y+4z)(2y-4z)^2+(2y-4z)^3$
21. $(2m+3n)^3+(2m-3n)^3+12m(4m^2-9n^2)$
22. $(3x-7y)^3+(7y+x)^3+12x(3x-7y)(7y+x)$
23. Find the value of $8x^3+36x^2+54x+27$, when $x = -1$

24. Find the value of $27m^3+54m^2n+36mn^2+8n^3$, when $m=1, n=3$.
25. If $a+b=8$, find the value of a^3+b^3+24ab
26. If $m+n=-3$, find the value of m^3+n^3-9mn
27. If $x+y=5$, prove that $x^3+y^3+15xy=125$
28. If $a+b=2\frac{1}{3}$, prove that $a^3+b^3+7ab=12\frac{19}{27}$
29. If $a+b=-\frac{1}{3}$, prove that $a^3+b^3-ab=-\frac{1}{27}$
30. If $xy(x+y)=m$, prove that $x^3+y^3+3m=\frac{m^3}{x^3y^3}$

Formula for the cube of difference of two terms :

Cube of the difference of two terms may be found in the same way as the cube of the sum of two terms.

$$\begin{aligned}(a-b)^3 &= (a-b)(a-b)(a-b)\\ &= (a-b)(a-b)^2\\ &= (a-b)(a^2-2ab+b^2)\\ &= a(a^2-2ab+b^2)-b(a^2-2ab+b^2)\\ &= a^2-2a^2b+ab^2-a^2b+2ab^2-b^3\\ &= a^3-3a^2b+3ab^2-b^3\end{aligned}$$

Thus the formula is

$$\begin{aligned}(a-b)^3 &= a^3-3a^2b+3ab^2-b^3\\ &= a^3-b^3-3ab(a-b)\end{aligned}$$

Remark : This formula can also be obtained from the formula of $(a+b)^3$

$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$

Now replacing b by $-b$, we get

$$\{a+(-b)\}^3=a^3+3a^2(-b)+3a(-b)^2+(-b)^3$$

or, $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

Geometric representation of the formula with the help of blocks :

Prepare two cubes of sides 7 cm and 3 cm and a set of three parallelopipeds of length 10 cm, breadth 7 cm and height 3 cm. Arranging these five solids properly you will get a cube whose side is 10 cm. If you now remove the cube of side 3 cm and the three parallelopipeds from the new cube you will be left with the cube of side 7 cm.

$\therefore\ 7^3=10^3-3.10.7.3-3^3$

or, $(10-3)^3=10^3-3.10.3.(10-3)-3^3$ [replacing 7 by $(10-3)$]

$$=10^3-3.10^2.3+3.10.3^2-3^3$$

$\therefore$ in general, $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

Ex.1. Find the cube of $(x-5)$

$$\begin{aligned}(x-5)^3 &= x^3-3(x)^2.5+3.x.(5)^2-(5)^3\\ &= x^3-15x^2+75x-125\end{aligned}$$

Ex.2. Find the cube of $(3x-2y)$

$$\begin{aligned}(3x-2y)^3&=(3x)^3-3(3x)^2.2y+3(3x)(2y)^2-(2y)^3\\ &=27x^3-54x^2y+36xy^2-8y^3\end{aligned}$$

Ex.3. Find the cube of $(a-2b-3c)$

$$(a-2b-3c)^3=\{(a-2b)-3c\}^3$$

$$=(a-2b)^3-3(a-2b)^2.3c+3(a-2b)(3c)^2-(3c)^3$$

$$=a^3-3a^2.2b+3.a(2b)^2-(2b)^3-9c(a^2-4ab+4b^2)+27c^2(a-2b)-27c^3$$

$$=a^3-6a^2b+12ab^2-8b^3-9a^2c+36abc-36b^2c+27ac^2-54bc^2-27c^3$$

Ex.4. Find the cube of 199

$$\begin{aligned}(199)^3 &= (200-1)^3\\ &= (200)^3-3(200)^2.1+3.200.(1)^2-(1)^3\\ &= 8000000-120000+600-1\\ &= 8000600-120001\\ &= 7880599\end{aligned}$$

Ex.5. Find the value of the following using formula :

$$16{\cdot}45\times16{\cdot}45\times16{\cdot}45-3\times16{\cdot}45\times16{\cdot}45\times14{\cdot}45+3\times16{\cdot}45\times14{\cdot}45\times14{\cdot}45-14{\cdot}45\times14{\cdot}45\times14{\cdot}45$$

Given expression $= (16{\cdot}45)^3 - 3(16{\cdot}45)^2.14{\cdot}45$

$\qquad 3.16{\cdot}45(14{\cdot}45)^2 - (14{\cdot}45)^3$

Taking $16{\cdot}45 = a$ and $14{\cdot}45 = b$, we get

Given expression $= a^3 - 3a^2b + 3ab^2 - b^3$

$= (a-b)^3$

$= (16{\cdot}45 - 14{\cdot}45)^3$

$= (2)^3 = 8.$

Ex.6. Find the value of $a^3 - 12a^2b + 48ab^2 - 64b^3$ when $a = 5$, $b = 3$.

Given expression $= a^3 - 12a^2b + 48ab^2 - 64b^3$

$= a^3 - 3.a^2.4b + 3.a.(4b)^2 - (4b)^3$

$= (a - 4b)^3$

$= (5 - 4.3)^3$ [putting values of a, b]

$= (5-12)^3 = (-7)^3 = -343.$

Ex.7. If $x - 2y = 5$, prove that $x^3 - 8y^3 - 30xy = 125$.

$x^3 - 8y^3 - 30xy = x^3 - (2y)^3 - 3.x.\,2y.5$

$= x^3 - (2y)^3 - 3.\,x.\,2y.(x - 2y)$

$= (x - 2y)^3 = (5)^3 = 125.$

Exercise 12

Find the cubes of the following expressions :

1. $a-3$ 2. $2x-y$ 3. $3m-n$
4. $2p-3q$ 5. a^2-2b 6. $ab-cd$
7. $2x-3bc$ 8. a^2-bc 9. $x-yz^2$
10. m^2-n 11. $1+2b-2c$ 12. $p-2q-3r$
13. $a-1$ 14. $2-b^3$ 15. 97
16. 298 17. 999 18. 249

Find the value of the following using formula :

19. $31\times31\times31 - 3\times31\times31\times26 + 3\times31\times26\times26 - 26\times26\times26$

20. $41{\cdot}5\times41{\cdot}5\times41{\cdot}5 - 3\times41{\cdot}5\times41{\cdot}5\times38{\cdot}5 + 3\times41{\cdot}5\times38{\cdot}5$
$\times38{\cdot}5 - 38{\cdot}5\times38{\cdot}5\times38{\cdot}5$

21. $5{\cdot}73\times5{\cdot}73\times5{\cdot}73 - 3{\cdot}73\times3{\cdot}73\times3{\cdot}73 - 6\times5{\cdot}73\times3{\cdot}73$

Simplify expressing as cubes :

22. $(4a-3b)^3-3(4a-3b)^2(4a-2b)+3(4a-3b)(4a-2b)^2-(4a-2b)^3$

23. $(4x-3y-2z)^3-3(4x-3y-2z)^2(4x-2z)+3(4x-3y-2z)$
$(4x-2z)^2-(4x-2z)^3$

24. $(2p+3q)^3-(2p-3q)^3-18q(4p^2-9q^2)$

25. $(3m-2n)^3-(2m-n)^3-3(3m-2n)(2m-n)(m-n)$

26. Find the value of $8x^3-36x^2+54x-27$, when $x = 2$

27. Find the value of $27a^3-108a^2b+144ab^2-64b^3$ when $a = 4$, $b = 3$.

28. Find the value of $27m^3-54m^2n+36mn^2-8n^3$. when $m=1\frac{2}{3}$, $n=2\frac{1}{2}$

29. If $a-b=6$, find the value of a^3-b^3-18ab.

30. If $2x-3y = 4$, prove that $8x^3-27y^3-72xy = 64$

Corollary :

We have got the formula $(a+b)^3=a^3+b^3+3ab(a+b)$
Transposing, we get $a^3+b^3=(a+b)^3-3ab(a+b)$
Similarly, $(a-b)^3=a^3-b^3-3ab(a-b)$. Transposing we get $a^3-b^3=(a-b)^3+3ab(a-b)$

Applications of corollary :

Ex. 1. If $a+b=5$, $ab=6$, find the value of a^3+b^3.
Now, $a^3+b^3=(a+b)^3-3ab(a+b)$
$= (5)^3-3\times6\times5$
$= 125-90 = 35.$

Ex. 2. If $a-b=3$, $ab=40$, find the value of a^3-b^3.
Now, $a^3-b^3=(a-b)^3+3ab(a-b)$
$= (3)^3+3.40.3=27+360=387$.

Ex. 3. If $x+\frac{1}{x}=5$, find the value of $x^3+\frac{1}{x^3}$

$$x^3+\frac{1}{x^3}=(x)^3+\left(\frac{1}{x}\right)^3$$
$$=\left(x+\frac{1}{x}\right)^3-3x.\frac{1}{x}\left(x+\frac{1}{x}\right)$$
$$=(5)^3-3.5=125-15=110.$$

Ex. 4. If $m-\frac{1}{m}=a$, **find** the value of $m^3-\frac{1}{m^3}$

$$m^3-\frac{1}{m^3}=(m)^3-\left(\frac{1}{m}\right)^3$$

$$=\left(m-\frac{1}{m}\right)^3+3.m.\frac{1}{m}\left(m-\frac{1}{m}\right)$$

$$=a^3+3a.$$

Ex. 5. If $\left(a+\frac{1}{a}\right)^3=3$, prove that $a^3+\frac{1}{a^3}=0.$

$$a^3+\frac{1}{a^3}=\left(a+\frac{1}{a}\right)^3-3.a.\frac{1}{a}\left(a+\frac{1}{a}\right)$$

$$=\left(a+\frac{1}{a}\right)^2\left(a+\frac{1}{a}\right)-3\left(a+\frac{1}{a}\right)$$

$$=3\left(a+\frac{1}{a}\right)-3\left(a+\frac{1}{a}\right)$$ [putting the value of $\left(a+\frac{1}{a}\right)^2$]

$$=0.$$

Exercise 13

1. Find the value of a^3+b^3 when $a+b=6$, $ab=8$
2. Find the value of m^3+n^3 when $m+n=8$, $mn=15$.
3. Find the value of x^3-y^3 when $x-y=2$, $xy=15$.
4. Find the value of p^3-q^3 when $p-q=5$, $pq=14$.
5. If $a+\frac{1}{a}=4$, find the value of $a^3+\frac{1}{a^3}$
6. If $\left(p+\frac{1}{p}\right)^2=3$ find the value of $p^3+\frac{1}{p^3}$
7. If $2a+\frac{1}{2a}=5$, prove that $8a^3+\frac{1}{8a^3}=110.$
8. If $m-\frac{1}{m}=6$, find the value of $m^3-\frac{1}{m^3}$
9. If $c+\frac{1}{c}=k$, prove that $c^3+\frac{1}{c^3}=k^3-3k$
10. If $x-\frac{1}{x}=p$, prove that $x^3-\frac{1}{x^3}=p^3+3p$

11. If $a+b=5$, $ab=6$, find the value of $(a^2+b^2)(a^3+b^3)$.

12. If $m-n=2$, $mn=15$, find the value of $(m^2-n^2)(m^3-n^3)$

Formula : $a^3+b^3=(a+b)(a^2-ab+b^2)$

Verfication : R.H.S.

$$
\begin{aligned}
&=(a+b)(a^2-ab+b^2)=a(a^2-ab+b^2)+b(a^2-ab+b^2)\\
&\qquad=a^3-a^2b+ab^2-a^2b+ab^2-b^3\\
&\qquad=a^3+b^3 \qquad\qquad =\text{L.H.S.}
\end{aligned}
$$

$\therefore a^3+b^3=(a+b)(a^2-ab+b^2)$

Proof :
$$
\begin{aligned}
a^3+b^3&=(a+b)^3-3ab(a+b)\\
&=(a+b)\{(a+b)^2-3ab\}\\
&=(a+b)(a^2+2ab+b^2-3ab)\\
&=(a+b)(a^2-ab+b^2)
\end{aligned}
$$

Application of the formula :

Ex. 1. Multiply x^2-2x+4 by $x+2$

$$
\begin{aligned}
\text{Required product} &=(x+2)(x^2-2x+4)\\
&=(x+2)(x^2-x.2+2^2)\\
&=x^3+2^3=x^3+8
\end{aligned}
$$

Ex. 2. Multiply $9x^2-12xy+16y^2$ by $3x+4y$

$$
\begin{aligned}
\text{Required product} &=(3x+4y)(9x^2-12xy+16y^2)\\
&=(3x+4y)\{(3x)^2-3x.4y+(4y)^2\}\\
&=(a+b)(a^2-ab+b^2)\ [\text{taking } 3x=a,\ 4y=b]\\
&=a^3+b^3\\
&=(3x)^3+(4y)^3\ [\text{putting values of } a,b]\\
&=27x^3+64y^3
\end{aligned}
$$

Ex. 3. Multiply $a^2b^2-abc+c^2$ by $ab+c$

$$
\begin{aligned}
\text{Required product} &=(ab+c)(a^2b^2-abc+c^2)\\
&=(ab+c)\{(ab)^2-ab.c+c^2\}\\
&=(ab)^3+c^3=a^3b^3+c^3.
\end{aligned}
$$

Exercise 14

Find the following products by using formula :

1. $(x+3)(x^2-3x+9)$
2. $(1+3a)(1-3a+9a^2)$
3. $(2a+3b)(4a^2-6ab+9b^2)$
4. $(mn+p)(m^2n^2-mnp+p^2)$
5. $(a+bc)(a^2-abc+b^2c^2)$
6. $(x^2+y^2)(x^4-x^2y^2+y^4)$
7. $(4p+5q)(16p^2-20pq+25q^2)$
8. $(a+b+c)(\{(a+b)^2-(a+b)c+c^2\}$
9. $(x^2+1)(x^4-x^2+1)$
10. $(x+2y+c)\{(x+y)^2-(x+y)(y+z)+(y+z)^2\}$
11. $(2a+3)\{(a+2)^2-(a+2)(a+1)+(a+1)^2\}$
12. $3x\{(2x-1)^2-(2x-1)(x+1)+(x+1)^2\}$

Formula : $a^3-b^3=(a-b)(a+ab+b^2)$

Verification : R.H.S. $= (a-b)(a^2+ab+b^2)$

$$= a(a^2+ab+b^2)-b(a^2+ab+b^2)$$
$$= a^3+a^2b+ab^2-a^2b-ab^2-b^3$$
$$= a^3-b^3 = \text{L.H.S.}$$
$$\therefore a^3-b^3=(a-b)(a^2+ab+b^2)$$

Proof : $a^3-b^3=(a-b)^3+3ab(a-b)$

$$= (a-b)\{a-b)^2+3ab\}$$
$$= (a-b)(a^2-2ab+b^2+3ab)$$
$$= (a-b)(a^2+ab+b^2)$$

N.B. Expressions of the form a^3+b^3 or a^3-b^3 may be factorised by applying these two formulae.

Applications of the formula :

Ex. 1. Multiply $9x^2+6xy+4y^2$ by $3x-2y$.

Required product $= (3x-2y)(9x^2+6xy+4y^2)$

$$= (3x-2y)\{(3x)^2+3x.2y+(2y)^2\}$$
$$= (3x)^3-(2y)^3$$
$$= 27x^3-8y^3$$

Ex. 2. Multiply $(m+n)^2+(m+n)(n+p)+(n+p)^2$ by $m-p$.

Required product $= (m-p)\{(m+n)^2+(m+n)(n+p)+(n+p)^2\}$

$= \{(m+n)-(n+p)\}((m+n)^2+(m+n)(n+p)+(n+p)^2\}$

$= (a-b)(a^2+ab+b^2)$ [Taking $m+n = a$, $n+p = b$]

$= a^3-b^3$

$= (m+n)^3-(n+p)^3$ [Putting values of a,b]

$= m^3+3m^2n+3mn^2+n^3-n^3-3n^2p-3np^2+p^3$

$= m^3+3m^2n+3mn^2-3n^2p-3np^2-p^3$

Exercise 15

Find the products with the help of formula :

1. $(x-3)(x^2+3x+9)$
2. $(1-4x)(1+4x+16x^2)$
3. $(3a-5b)(9a^2+15ab+25b^2)$
4. $(ab-cd)(a^2b^2+abcd+c^2d^2)$
5. $(x^2-y^2)(x^4+x^2y^2+y^4)$
6. $(a^2-2bc)(a^4+2a^2bc+4b^2c^2)$
7. $(2x-3y)(2x+3y)(4x^2+6xy+9y^2)(4x^2-6xy+9y^2)$
8. $(a-2b)(a^2+2ab+4b^2)(a^3+8b^3)$

Simplify :

9. $(x-y)(x^2+xy+y^2)+(y-z)(y^2+yz+z^2)+(z-x)(z^2+zx+x^2)$
10. $(2a+3)(4a^2-6a+9)-(2a-3)(4a^2+6a+9)$
11. $(a+b)(a-b)(a^2+ab+b^2)(a^2-ab+b^2)$
12. $(x+1)(x^2-x+1)+(2x-1)(4x^2+2x+1)-(x-1)(x^2+x+1)$

Chapter 4

Factorisation with the help of formulae

Let us first consider the following two formulae that we have discussed in the last chapter :

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$
$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

Here we find that each of a^3+b^3 and a^3-b^3 is equal to the product of two algebraic expressions. Hence, we may say that two factors of a^3+b^3 are $a+b$ and a^2-ab+b^2 and similarly, those of a^3-b^3 are $a-b$ and a^2+ab+b^2.

We can factorise a number of algebraic expressions with the help of the above two formulae. Observe the following examples.

Ex. 1. Factorise x^3+8.

$$\begin{aligned} x^3+8 &= (x)^3+(2)^3 \\ &= (x+2)(x^2-x.2+2^2) \text{ [from formula]} \\ &= (x+2)(x^2-2x+4) \end{aligned}$$

Ex. 2. Factorise $27m^3-8n^3$

$$\begin{aligned} 27m^3-8n^3 &= (3m)^3-(2n)^3 \\ &= (3m-2n)\{(3m)^2+3m.2n+(2n)^2\} \\ &= (3m-2n)(9m^2+6mm+4n^2) \end{aligned}$$

Ex. 3. Factorise $x^3+3x^2y+3xy^2+2y^3$

$$\begin{aligned} x^3+3x^2y+3xy^2+2y^3 &= x^3 + 3x^2y + 3xy^2 + y^3 + y^3 \\ &= (x+y)^3+y^3 \\ &= (x+y+y)\{(x+y)^2-(x+y)y+y^2\} \\ &= (x+2y)(x^2+2xy+y^2-xy-y^2+y^2) \\ &= (x+2y)(x^2+xy+y^2) \end{aligned}$$

Ex. 4. Factorise $8(a+b)^3-27(b+c)^3$.

$8(a+b)^3-27(b+c)^3=\{2(a+b)\}^3-\{3(b+c)\}^3$

$=(2a+2b)^3-(3b+3c)^3$

$=\{(2a+2b)-(3b+3c)\}\ \{(2a+2b)^2$
$\quad+(2a+2b)(3b+3c)+(3b+3c)^2\}$

$=(2a+2b-3b-3c)\ (4a^2+8ab+4b^2+6ab+6b^2+6ca$
$\quad+6bc+9b^2+18bc+9c^2)$

$=(2a-b-3c)(4a^2+19b^2+9c^2+14ab+24bc+6ca)$

Ex. 5. Factorise $a^6+3a^4b^2+3a^2b^4+b^6+a^3b^3$.

$a^6+3a^4b^2+3a^2b^4+b^6+a^3b^3$.

$=(a^2)^3+3.(a^2)^2.b^2+3a^2.(b^2)^2+(b^2)^3+a^3b^3$

$=(a^2+b^2)^3+(ab)^3$

$=(a^2+b^2+ab)\{(a^2+b^2)^2-(a^2+b^2)ab+(ab)^2\}$

$=(a^2+ab+b^2)(a^4+2a^2b^2+b^4-a^3b-ab^3+a^2b^2\}$

$=(a^2+ab+b^2)(a^4-a^3b+3a^2b^2-ab^3+b^4)$

Ex. 6. Factorise $m^3-n^3-m(m^2-n^2)+n(m-n)^2$

$=m^3-n^3-m(m^2-n^2)+n(m-n)^2$

$=(m-n)(m^2+mn+n^2)-m(m+n)(m-n)+n(m-n)(m-n)$

$=(m-n)(m^2+mn+n^2-m(m+n)+n(m-n)\}$

$=(m-n)(m^2+mn+n^2-m^2-mn+mn-n^2)$

$=(m-n).mn$

$=mn(m-n)$.

Exercise 16

Resolve into factors :

1. a^3+8
2. $8x^3+1$
3. a^3+27b^3
4. $8m^3+27n^3$
5. x^3+64y^3
6. $8p^3+343q^3$
7. $a^3b^3+c^3$
8. $192a^3+3$
9. $8am^3+27am^3p^3$
10. $243x^3+512y^3$
11. $729a^3b^3c^3+1000m^3n^3p^3$
12. $16a^4x^3+54ay^3$
13. $1-8a^3$
14. $64a^3-b^3$
15. $125x^3-216y^3$
16. m^4-8m
17. a^6-b^6
18. $b^3c^3-8d^3$
19. $27m^3-64n^3p^3$
20. $216m^3-343n^3$
21. $729a^3b^3c^3-125$
22. $(a-2b)^3+c^3$
23. $m^3-(n-p)^3$
24. $(2a^3-b^3)^3-b^9$
25. $8m^3+12m^2n+6mn^2+2n^3$
26. $(p+q)^3-(p-q)^3$
27. x^3+3x^2+3x+2
28. $8(x+y)^3+27(y+z)^3$
29. $1+9x+27x^2+28x^3$
30. $8-a^3+3a^2b-3ab^2+b^3$
31. $8x^3+12x^2+6x-y^3+9y^3-27y+28$
32. $m^3+n^3-m(m^2-n^2)+n(m+n)^2$

Chapter 5

Factorisation of quadratic expression by the method of splitting middle co-efficient

If the highest power in an algebraic expression involving one alphabet (say x) is 2, then the expression is called a quadratic expression. For example, in the expression x^2+3x+2, highest power of x is 2; hence this is a quadratic expression. The co-efficients of the three terms of this expression are 1,3,2 respectively. Now, let us see how we can factorise this expression by splitting the middle co-efficient, 3. We have to split this co-efficient in two parts in such a way that their product is the same as the product of the first and the third co-efficients. This has to be done by inspection. In the present example middle co-efficient 3=1+2 and the product of these two splitted parts of 3 is 1.2=2= product of the first and third co-efficients or of the two end co-efficients. This is the main principle. Now, let us see the actual process of factorisation.

$$x^2+3x+2 = x^2+(1+2)x+2$$
$$= x^2+x+2x+2$$
$$= x(x+1)+2(x+1)$$
$$= (x+1)(x+2)$$

Remark : (i) Main point of the above method can easily be seen in the following algebraic product.

$$(x+a)(x+b)=x^2+(a+b)x+ab.$$

(ii) the co-efficient of the term, which comes in the middle after arranging the expression in ascending or descending order of powers is called the middle co-efficient. This middle co-efficient is to be splitted.

(iii) in fact 2 is the third term of the expression x^2+3x+2. This term is independent of x. But we can write 2 as 2.1. What you have learnt in class VII, you can say $1=x^0$. Hence $2=2.x^0$. Therefore, all terms of the expression when written in terms of x,2 is the co-efficient of the third term.

Ex. 1. Resolve into factors : $x^2+11x+24$

Here middle co-efficient $11 = 3+8=2+9=4+7$ etc. But here we will accept only the first set of splitted parts, as $3\times8=24$ and this is the product of the two end co-efficients.

Therefore,
$$\begin{aligned} x^2+11x+24 &= x^2+(3+8)x+24 \\ &= x^2+3x+8x+24 \\ &= x(x+3)+8(x+3) \\ &= (x+3)(x+8) \end{aligned}$$

N.B. : Instead of splitting the middle terms in the manner shown above, we can first find out the factors of the product of the two end co-efficients and then find out the factors whose algebraic sum is equal to the middle co-efficient. For example, in the previous example the product of the two end co-efficients is 24; now $24=2\times12=3\times8=4\times6$. Obviously, the sum of 3 and 8 = middle co-efficient 11. We can split the middle co-efficient in this manner also.

Ex. 2. Resolve into factors : $x^2-8x+15$.

Here middle co-efficient $= -8$ and correct splitted parts will be $(-3)+(-5)$, because $(-3)\times(-5) = 15$ = product of the two end co-efficients.

$$\begin{aligned} \therefore x^2-8x+15 &= x^2-(3+5)x+15 \\ &= x^2-3x-5x+15 \\ &= x(x-3)-5(x-3) \\ &= (x-3)(x-5) \end{aligned}$$

Ex. 3. Resolve into factors : $x^2+9x-36$.

Here the middle co-efficient 9 is to be splitted in two such parts whose products is -36 and it is evident that one of the two parts will be positive and the other will be negative. By inspection, required splitted parts of the middle co-efficient $9 = 12+(-3)$.

$$\begin{aligned}\therefore x^2+9x-36 &= x^2+(12-3)x-36\\ &= x^2+12x-3x-36\\ &= x(x+12)-3(x+12)\\ &= (x+12)(x-3)\end{aligned}$$

Ex. 4. Resolve into factors : $x^2-3x-28$.

Here the middle co-efficient -3 is to be splitted in two such parts that their sum is -3 and their product is -28. Here also one of the parts will be positive and the other will be negative. By inspection we find that the required parts of the middle co-efficient $-3 = 4+(-7)$

$$\begin{aligned}\therefore x^2-3x-28 &= x^2+(4-7)x-28\\ &= x^2+4x-7x-28\\ &= x(x+4)-7(x+4)\\ &= (x+4)(x-7)\end{aligned}$$

Ex. 5. Resolve into factors : $a^2+8ab+15b^2$.

(It will be convinient if this expression is considered as a quadratic expression in ***a***.)

Here middle mumerical co-efficient $=8=5+3$

Again, the product of end numerical co-efficients$=15=5\times3$.

$$\begin{aligned}\therefore a^2+8ab+15b^2 &= a^2+(5+3)ab+15b^2\\ &= a^2+5ab+3ab+15b2\\ &= \alpha(a+5b)+3b(a+5b)\\ &= (a+5b)(a+3b)\end{aligned}$$

Ex. 6. Resolve into factors : $p^2-9pq+18q^2$

Here middle numerical co-efficient $= -9= (-3)+(-6)$

Again, the product of end numerical co-efficients$=18$

$=(-3)\times(-6)$

$\therefore p^2-9pq+18q^2 = p^2-9pq+18q^2$

$= p^2-(3+6)pq+18q^2$

$= p^2-3pq-6pq+18q^2$

$= p(p-3q)-6q(p-3q)$

$= (p-3q)(p-6q)$

Ex. 7. Resolve into factors : $(x+y)^2-7(x+y)-18$,
Taking $x+y=a$,
Given expression $= a^2-7a-18$

$= a^2+(-9+2)a-18$ [by inspection]

$= a^2-9a+2a-18$

$= a(a-9)+2(a-9)$

$= (a-9)(a+2)$

$= (x+y-7)(x+y+2)$[putting value of a]

Ex. 8. Resolve into factors : a^4-3a^2-4
Given expression $= a^4-4a^2+a^2-4$

$= a^2(a^2-4)+1(a^2-4a)$

$= (a^2-4)(a^2+1)$

$= \{(a)^2-(2)^2\}(a^2+1)$

$= (a-2)(a+2)(a^2+1)$

Remark : Note that here in the given expression highest power of a is 4; but it can be considered as a quadratic expression in a^2.

Ex. 9. Resolve into factors :$(x+1)(x+3)(x+5)(x+7)+15$
Given expression

$= (x+1)(x+3)(x+5)(x+7)+15$

$= \{(x+1)(x+7)\}\{(x+3)(x+5)\}+15$

$= (x^2+8x+7)(x^2+8x+15)+15$

$= (a+7)(a+15)+15$ [Taking $x^2+8x=a$]

$= a^2+22a+105+15$

$= a^2+22a+120$

$= a^2+12a+10a+120$ [Breaking middle co-efficient by inspection]

$= a(a+12)+10(a+12)$

$= (a+12)(a+10)$

$= (x^2+8x+12)(x^2+8x+10)$ [Putting value of a]

$= (x^2+2x+6x+12)(x^2+8x+10)$

$= \{x(x+2)+6(x+2)(x+8x+10)$

$= (x+2)(x+6)(x^2+8x+10)$

Remark : Here first four factors have been arranged in two pairs in such a way that the two terms of the product of both the pairs, in which x is involved,become same.

Exercise 17

Resolve into factors :

1. x^2+4x+3
2. x^2+5x+6
3. $x^2+7x+10$
4. a^2+3a+2
5. $a^2+7a+12$
6. $a^2+10a+24$
7. $m^2+11m+24$
8. $p^2+14p+48$
9. $60+19x+x^2$
10. $x^2-7x+12$
11. $z^2-9z+14$
12. $x^2-16x+39$
13. $a^2-12a+35$
14. $m^2-18m+65$
15. $x^2-15x+54$
16. $a^2+2a-35$
17. $c^2+12c-45$
18. $x^2+5x-36$
19. $p^2+6p-16$
20. x^2+x-56
21. x^2-x-6
22. $a^2-2a-35$
23. $x^2-4x-21$
24. $x^2-3x-28$
25. $m^2-21m-72$
26. $a^2-a-110$
27. $b^2-14b-51$
28. $x^2-4x-117$
29. $a^2-2a-63$
30. $x^2-16x+63$
31. m^2+m-72
32. $156-25a+a^2$
33. $x^2+8x-105$
34. $p^2+2p-143$
35. $a^2-23a+60$
36. $1-5x-36x^2$
37. $x^2+24x-81$
38. $a^2-13a-90$
39. $x^2-49x-102$
40. $80+11x-x^2$
41. a^4+4a^2-5
42. x^4-9x^2+20
43. x^4+6x^2+9
44. x^6-10x^3+16
45. a^6+7a^3-8
46. m^4-2m^2-35
47. $a^2+ab-12b^2$
48. $x^2-xy-210y^2$

49. $m^2+2mn-35n^2$ 50. $p^2-16pq+39q^2$ 51. $x^2+20xy-96y^2$
52. $a^2+3ab-40b^2$ 53. $(x+y)^2-(x+y)-6$
54. $(a^2+2a)^2-(a^2+2a)-2$
55. $(x^2-2x)^2+5(x^2-2x)-36$
56. $(m^2+2m)^2-21(m^2+2m)-72$
57. $(p^2-3q)^2-16(p^2-3q)+63$
58. $(x^2-8x)^2-29(x^2-8x)+180$
59. $(x+1)(x+9)(x+5)^2+63$
60. $(x-1)(x+3)(x-2)(x-6)+96$.

Factorisation of expressions of the form $acx^2+(ad+bc)x+bd$:

We can apply the method of splitting middle co-efficient for factorising expressions of the above form.

$$\begin{aligned}&acx^2+(ad+bc)x+bd\\&=acx^2+adx+bcx+bd\\&=ax(cx+d)+b(cx+d)\\&=(cx+d)(ax+b)\\&=(ax+b)(cx+d)\end{aligned}$$

Remark : Quadratic expressions of this form are generally written in the form px^2+qx+r

Ex. 1. Resolve into factors : $3x^2+7x+4$.

Here the middle co-efficient = 7. This middle co-efficient 7 is to be splitted in two parts in such a way that their product is equal to the product of the two end co-efficients=3·4=12. By inspection we find 7=3+4 and 3·4=12.

$$\begin{aligned}\text{Given expression} &= 3x^2+(3+4)x+4\\&=3x^2+3x+4x+4\\&=3x(x+1)+4(x+1)\\&=(x+1)(3x+4)\end{aligned}$$

Ex. 2. Resolve into factors : $5x^2+6x-8$.

Here the middle co-efficient 6 is to be splitted in two such parts whose product is $5\times(-8)=-40$. It is clear that one part will be positive and the other will be negative. By inspection we find $6=10+(-4)$ and $10\times(-4)=-40=$ product of the two end co-efficients.

$$\begin{aligned}\text{Given expression} &= 5x^2+6x-8\\ &= 5x^2+(10-4)x-8\\ &= 5x^2+10x-4x-8\\ &= 5x(x+2)-4(x+2)\\ &= (x+2)(5x-4)\end{aligned}$$

Ex. 3. Resolve into factors : $6x^2-5x-6$.

Here also from the product of the two end co-efficients we can correctly given how to split middle co-efficient product of the two end co-efficient $=6\times(-6)=-36=(-9)\times(4)$ and $-9+4=-5$.

$$\begin{aligned}6x^2-5x-6 &= 6x^2+(-9+4)x-6\\ &= 6x^2-9x+4x-6\\ &= 3x(2x-3)+2(2x-3)\\ &= (2x-3)(3x+2)\end{aligned}$$

Ex. 4. Factorise $13a^2-20ab+7b^2$.

Here middle numerical co-efficient $=-20$. Now -20 has to splitted into two such parts whose product is $13\times7=91$. Clearly both the parts will be negative. By inspection we find $-20=(-13)+(-7)$ and $(-13)\times(-7)=91$.

$$\begin{aligned}13a^2-20ab+7b^2 &= 13a^2+(-13-7)ab+7b^2\\ &= 13a^2-13ab-7ab+7b^2\\ &= 13a(a-b)-7b(a-b)\\ &= (a-b)(13a-7b)\end{aligned}$$

Ex. 5. Resolve into factors :

$3(a^2+2a)^2-22\,(a^2+2a)+40.$

Given expression

$= 3(a^2+2a)^2-22(a^2+2a)+40$

$= 3x^2-22x+40$ [Taking $a^2+2a = x$]

$= 3x^2+(-12-10)x+40$ [By inspection]

$= 3x^2-12x-10x+40$

$= 3x\,(x-4)-10(x-4)$

$= (x-4)(3x-10)$

$= (a^2+2a-4)\{3(a^2+2a)-10\}$ [Putting value of x]

$= (a^2+2a-4)(3a^2+6a-10)$

Exercise 18

Resolve into factors :

1. $2a^2+5a+2$
2. $3x^2+8x+5$
3. $3a^2+14a+8$
4. $2a^2+9a+4$
5. $2m^2+7m+6$
6. $2c^2+7c+3$
7. $2p^2+p-15$
8. $6x^2-x-15$
9. $5a^2-4a-9$
10. $9x^2-x-8$
11. $3k^2-22k-25$
12. $2x^2-x-1$
13. $10a^2-27a-28$
14. $6m^2-11mn-10n^2$
15. $9x^2+30x-11$
16. $5a^2-8ab-13b^2$
17. $12p^2+23pq-24q^2$
18. $5a^2-8ab-13b^2$
19. $7a^2+48ab-7b^2$
20. $10x^2+17xy-20y^2$
21. $11a^2+2ab-13b^2$
22. $3x^2-6xy-9y^2$
23. $9p^2-24pq-9q^2$
24. $8a^2+63ab-8b^2$
25. $12x^2-19xy-10y^2$
26. $21a^2-58a+21$
27. $10x+25-3x^2$
28. $14a-3a^2+5$
29. $12+x-6x^2$
30. $6+5a-6a^2$
31. $a^2b^2-10ab-39$
32. $x^2y^2-xy-72$
33. $3a^4-2a^2-5$
34. $8x^6+x^3-9$

35. m^4+m^2-132 (36. a^6+13a^3-48

37. p^4-11p^2-152 38. $6a^6+17a^3-45$

39. $3(a+b)^2+5(a+b)+2$

40. $6(x-y)^2-(x-y)-15$

41. $3(x^2+x)^2-22(x^2+x)-25.$

42. $2(3x+2y)^2-5(3x+2y)+3$

43. $3(a+b)^2-10(a+b)(a-b)+3(a-b)^2$

44. $5(x^2-y^2)^2-8xy(x^2-y^2)-13x^2y^2$

45. $ax^2+(a^2+1)x+a$

46. $x^2-(a+\frac{1}{a})x+1$

Chapter 6

H.C.F. AND L.C.M.

H.C.F. (Highest Common Factor) :

In arithmetic you have found out H.C.F. There is basically no difference between the methods of finding H.C.F.of algebraic expressions and of arithmetical numbers. You know that if two numbers have the same factor, this factor is called common factor of the two numbers. The factor which cannot be further factorised is called a prime factor. Both the numbers are always divisible by the common factor.

Take two numbers 18 and 24. Now, the factors of 18 will be 1,2,3,6,9 and 18 and the factors of 24 are 1,2,3,4,6,8,24. Hence the common factors of two number will be 1,2,3,6. Greatest of all these common factors and hence the highest common factor of 18 and 24 is 6. Note that 6 is the product of two prime factors 2 and 3 of the given numbers.

In algebra also H.C.F. is found in an almost similar manner. If two or more algebraic expressions are divisible by an algebraic expression then the latter is called a **common factor** of the former two or more expressions. The product of all prime factors, which are common to two or more algebraic expressions, is called the **Highest Common Factor** or H.C.F. of the given two or more expressions.

Let us take two algebraic expressions ab^2 and a^2bc. Factors of ab^2 are a, b, b^2, ab, and factors of a^2bc and a, a^2, b, c, ab, ac, bc, a^2b, a^2c, a^2bc . [Factor 1 has not been considered.] Common factors of the two given expressions are a, b, ab. Of these factors the term having highest power is the highest common factor or H.C.F. Here ab has the highest power and so the H.C.F. of ab^2 and a^2bc is ab.

Consider the following example. Let us suppose that we have to find the H.C.F. of $2a^2b^3c^2, 4a^3b^2c^4, 6a^4b^4c^3$. It may be easily seen that the common factors of the given expression are $2, a, b, c, a^2, b^2, c^2$ $ab, bc, ca, a^2b, a^2c, b^2c, bc^2, abc, a^2bc, ab^2c, abc^2, a^2b^2c^2, 2a^2b^2c^2$. Of these $2a^2b^2c^2$ has the highest power and so the H.C.F. of the three given expressions is $2a^2b^2c^2$.

Remark : Note that each given expression is divisible by the H.C.F.

Ex. 1. Find the H.C.F. of $a^3b^3, a^2b^4c, a^4b^3c^2$.

Here maximum power of a which is common to all the expressions is 2 and maximum power of b which is common to all is 3. c is not common to all the expressions.

Hence the expression having highest power which is common to all expressions is a^2b^3.

H.C.F. of the three expressions is a^2b^3

Ex. 2. Find the H.C.F. of $12ab^2x^3$, $21a^2c^2x^2$, $9c^3x^3y^3$.

$$12ab^2x^3 = 2^2\times3\times a\times b^2\times x^3$$
$$21a^2c^2x^2 = 3\times7\times a^2\times c^2\times x^2$$
$$9c^3x^3y^3 = 3^2\times c^3\times x^3\times y^3$$

It is readily seen that only prime factors which are common to the three given expressions are 3 and x. Maximum power of 3 that is common to the three expressions is 1 and the maximum power of x that is common to them is 2.

Hence, required H.C.F. = $3x^2$

Remark : Note that we have not taken any power of 2, a, b, c, y as none of them is common to all the three expressions.

Exercise 19

Find the H.C.F. :

1. a^2b, b^2c
2. xy, x^3y^2
3. m^3n^2, m^2n^3
4. $4a^3b^2, 8a^2b^3$
5. $2pq^2, 8p^4q^3$
6. $7, 14a^4b$
7. $3a^3b^2c, 6ab2c^2$
8. $8xyz, 2x^2ya$
9. $6xy^2z^3, 9x^2y^4z^5$
10. $7m^3np^4, 21m2n^5p^3$
11. $5abxy, 25ab^2y^3$
12. $12a^6x^5y^6, 16a^3x^4y^5$
13. $2ab, 4bc, 6b^2$
14. m^3, m^2n, mp^2
15. a^2b, ab^2, a^2b^2
16. $3a^2x^2, 6axy, 9ay^2$
17. $p^2, 2p^2q^2, 3p^3q$
18. $15a^3, 45a^2b, 25b^4$
19. $3x^2y^2z^2, 6x^3y^2z^4, 18x^4y^3z^3$
20. $16a^2b^3x^4y^5, 40a^3b^2x^3y^4, 24a^4b^5x^6y^4$
21. $12m^3n^4l, 32m2l^3n, 28mn^3$
22. $24a^2bc^5, 56ab^2c, 72a^3c^2$
23. $25bc^4d^3, 35b^2c^5, 45c^2d$
24. $35p^3q^2r^4, 49p^2q^4r^3, 63p^4q^3r^5$
25. $70a^3b^2c, 42a^2bc^2, 105ab^3c^3$
26. $37a^7b^3c^2, 111a^5b^3c^4, 148a^8b^6c^3$
27. $27a^5b^4c^3x^2, 81a^3b^4c^2x^4, 108a^2c^4x^3y^2, 117a^4c^3y^3$
28. $75x^4y^3z^5d^0, 90x^3y^5z^6d^4, 105x^6y^4z^3d^5, 135x^5y^6d^3$

H.C.F. by factorisation :

H.C.F of the polynomials, which can be easily factorised, can also be found in the above statyed manner.

Ex. 1. Find the H.C.F. of a2b+ab2 and a^3b-ab^3

1st expression $=a^2b+ab^2=ab\,(a+b)$

2nd expression$=a^3b-ab^3$

$$=ab\,(a^2-b^2)$$

$$=ab\,(a+b)(a-b)$$

Since a,b, a+b are common prime factors of both the expressions, the H.C.F. of the two given expressions $=ab\,(a+b)$.

Ex. 2. Find the H.C.F. of $x^4y-2x^3y^2+x^2y^3$ and $x^5y-x^2y^4$.

1st expresssion $=x^4y-2x^3y^2+x^2y^3$

$$=x^2y\,(x^2-2xy+y^2)=x^2y\,(x-y)^2$$

2nd expression $=x^5y-x^2y^4$

$$=x^2y(x^3-y^3)=x^2y(x-y)(x^2+xy+y^2)$$

$\therefore$ required H.C.F. $=x^2y\,(x-y)$

Ex. 3. Find the H.C.F. of $2x^2+3x-2$ and $2x^2-9x+4$.

1st expression $=2x^2-9x+4$.

$$=2x^2+4x-x-2$$

$$=2x(x+2)-1(x+2)$$

$$=(x+2)(2x-1)$$

2nd expression $=2x^2-9x+4$

$$=2x^2-8x-x+4$$

$$=2x(x-4)-1(x-4)$$

$$=(x-4)(2x-1)$$

$\therefore$ required H.C.F. $=2x-1$.

Ex. 4. Find the H.C.F. of $48(a^2+ab-2b^2)$ and $60(2a^2+3ab-2b^2)$

1st expression $= 48(a^2+ab-ab^2)$

$= 2^4\times3(a^2+2ab-ab-2b^2)$

$= 2^4\times3\{a(a+2b)-b(a+2b)\}$

$= 2^4\times3(a+2b)(a-b)$

2nd expression $= 60(2a^2+3ab-2b^2)$

$= 2^2\times3\times5(2a^2+4ab-ab--2b^2)$

$= 2^2\times3\times5\{2a(a+2b)-b(a+2b)\}$

$= 2^2\times3\times5(1+2b)(2a-b)$

$\therefore$ required H.C.F. $= 2^2\times3(a+2b) = 12(a+2b)$

Remarks : Sometimes it may be convenient to obtain the H.C.F. of the co-efficients (48 and 60) of the given expressions by arithmetical method.

Ex. 5. Find the H.C.F. of x^2+3x+2, $3x^2+x-2$ and $2x^3-x^2-3x$.

1st expression $= x^2+3x+2$

$= x^2+2x+x+2$

$= x(x+2)+1(x+2)$

$= (x+2)(x+1)$

2nd expression $= 3x^2+x-2$

$= 3x^2+3x-2x-2$

$= 3x(x+1)-2(x+1)$

$= (x+1)(3x-2)$

3rd expression $= 2x^2-x^3-3x$

$= x(2x^2-x-3)$

$= x(2x^2-3x+2x-3)$

$= x\{x(2x-3)+1(2x-3)\}$

$= x(2x-3)(x+1)$

Since $(x+1)$ is the only prime factor common to all the three expressions, the required H.C.F. is $x+1$.

Ex. 6. Find the H.C.F. of $3x^2-2x-8$ and $10+x-3x^2$

1st expression $= 3x^2-2x-8$

$= 3x^2-6x+4x-8$

$= 3x^2-6x+4x-8$

$= 3x(x-2)+4(x-2)$

$= (x-2)(3x+4)$

2nd expression $= 10+x-3x^2$

$= 10+6x-5x-3x^2$

$= 2(5+3x)-x(5+3x)$

$= (5+3x)(2-x)$

It appears that there is no common factor of the two given expressions. But if we take out -1 as one of the factors from the factor $(2-x)$ of the 2nd expression, then

2nd expression $= -(5+3x)(x-2)$

In that case required H.C.F. will be $(x-2)$.

On the other hand if we take out -1 as one of the factors from the factor $(x-2)$ of the 1st expression then

1st expression $= -(2-x)(3x+4)$

and in that case required H.C.F. will be $(2-x)$.

Hence H.C.F. of the two given expressions may be either $(x-2)$ or $(2-x)$.

Remark : it $(x-a)$ is H.C.F. of two or more expressions, their H.C.F. may also be $a-x$.

Exercise 20

Find H.C.F. of following expressions :

1. $a(b+c)^2, a^2(b+c)$
2. $x^2+xy, xy+y^2$
3. $a^2-b^2, (a+b)^2$
4. $a^2+ab, ac+bc$
5. a^2-b^2, a^3-b^3
6. c^2-d^2, c^2-cd
7. $4x^2-1, 2x^2=x$
8. $3a^2(b+c)^2, 6a(b+c)$
9. $a^2-2ab, 2ab-4b^2$
10. a^3-3a^2b, a^2-9b^2

11. $(x-1)^3, x^3-1$ 12. $ax(a-x)^3, 2a^2x(a-x)^2$
13. $a^3-b^3, a^3-a^2b+ab^2-b^3$
14. $x^2-y^2, x^3-y^3, 3x^2-5xy+2y^2$
15. a^2-2a-3, a^2-5a+4
16. a^2-1, a^3-1, a^2+a-2
17. $8(x^2-4), 12(x^3+8), 36(x^2-3x-10)$
18. $x^3-3x^2-10x, x^4-8x^3+15x^2$
19. $x^2-x-2, x^2+1, (x+1)^2$
20. $c^2-c, (c-1)^2, c^3-1$
21. $p^2+7p-18, p^2+10p+9$
22. $m^2-3m-18, m^2+5m+6$
23. x^3-x^2-42x, x^4-49x^2
24. $3a^2+7a-6, 2a^2+7a+3$
25. $x^2+3xy+2y^2, x^2+5xy+6y^2$
26. $a^3-3a^2-10a, a^3+6a^2+8a, a^4-5a^3-14a^2$
27. $24x^3+2x^2a+2xa^2+a^2), 32(2x^3+5x^2a+5xa^2+2a^3)$
28. $27(x^4-2ax^3-8a^2x^2), 45(x^5-ax^4-6a^2x^3)$
29. $a^2+b^2-c^2+2ab, a^2-b^2-c^2+2bc$
30. $x^3-16x, 2x^3+9x^2+4x, 2x^3+x^2-28x.$

H.C.F. by the method of division :

The method of division for finding H.C.F. of two or more numbers as applied in arithmetic may also be applied in algebra.

One expression is divided by another and the remainder is taken as the second divisor and the first divisor is taken as the second dividend. Similarly, the second remainder is taken as the third divisor and the second divisor is taken as the third dividend. This process is continued until the remainder is zero. The last divisor which makes the remainder zero is the H.C.F. of the two given expressions. If there is a third expression, it is then taken as the dividend and the previous last divisor as the divisor. The process of division is then continued as discussed above and the H.C.F. is found. When dividing the general rule of division is to be applied. That is both divisor and dividend

are to be **arranged** in ascending or descending powers of an alphabet in the given expressions.

Remark : In finding H.C.F. generally we apply the mtehod of factors. But in certain case it is not easy to factorise the given expressions. In these cases only we are to use the method of division.

To important informations :
In finding H.C.F. by the method of division we can multiply or devide the divisor or the dividend by any number which is not a common factor of the divisor and the dividend. Since this multiplier is not a common factor of the given expressions, the H.C.F. will not be altered by the above multiplication.

Ex. 1. Find H.C.F. of x^2+5x+4 and x^2+6x+5
Here both the expressions are arranged in descending powers of x. If the order of both the expressions is same any one of them may be taken as the divisor ro dividend.

$$
\begin{array}{rl}
x^2+5x+4\,) & x^2+6x+5\,(1 \\
 & \underline{x^2+5x+4} \\
 & \qquad\quad x+1\,)\,x^2+5x+4\,(x+4 \\
 & \qquad\qquad\qquad\quad \underline{x^2+x} \\
 & \qquad\qquad\qquad\qquad\quad 4x+4 \\
 & \qquad\qquad\qquad\qquad\quad \underline{4x+4}
\end{array}
$$

$\therefore$ required H.C.F. $= x+1$

Ex. 2. Find H.C.F. of $2x^3-10x^2+20x-16$ and $3x^3-12x^2+21x-18$.
First Expression $= 2x^3-10x^2+20x-16$
Second expression $= 3x^3-12x^2+21x-18$

Here both the expression are arranged in descending powers of x. If we multiply the second expressions by 2, its first term will be divisible by the first term of the first expression.

$$
\begin{array}{l}
\qquad\qquad\qquad\quad 3x^3-12x^2+21x-18. \\
\qquad\qquad\qquad\quad \underline{2} \\
2x^3-10x^2+20x-16)\ 6x^3-24x^2+42x-36(3 \\
\qquad\qquad\qquad\quad \underline{6x^3-30x^2+60x-48} \\
\qquad\qquad\qquad\qquad 6\underline{|6x^2-18x+12} \\
\qquad\qquad\qquad\qquad\quad x^2-\ 3x+2
\end{array}
$$

6 is a factor of the remainder and so it will be easier to work out the sum if the remainder is divided by 6.

$$
\begin{array}{l}
x^2-3x+2)2x^3-10x^2+20x-16(2x-4 \\
\qquad\quad \underline{2x^2-\ 6x^2+4x\qquad\quad} \\
\qquad\qquad\quad -4x^2+16x-16 \\
\qquad\qquad\quad \underline{-4x^2+12x-8} \\
\qquad\qquad\qquad\quad 4\underline{|\,4x-8} \\
\qquad\qquad\qquad\qquad x-2
\end{array}
$$

$$
\begin{array}{l}
x-2)x^2-3x+2(x-1 \\
\quad\ \underline{x2-2x} \\
\qquad\ -x+2 \\
\qquad\ \underline{-x+2}
\end{array}
$$

$\therefore$ required H.C.F. $= x-2$

Ex. 3. Find H.C.F. of $2x^2+5x+2$, $2x^3+5x^2-4x-3$ and $6x^3+x^2+3x+2$.

Let us first find the H.C.F. of the first two given expression.

$$
\begin{array}{l}
2x^2+5x+2)2x^3+5x^2-4x-3(x \\
\qquad\quad \underline{2x^3+5x^2+2\quad} \\
\qquad\qquad -3\underline{|-6x-3} \\
\qquad\qquad\qquad 2x+1)2x^2+5x+2(x+2 \\
\qquad\qquad\qquad\qquad\ \underline{2x^2+x\qquad} \\
\qquad\qquad\qquad\qquad\qquad 4x+2 \\
\qquad\qquad\qquad\qquad\qquad \underline{4x+2}
\end{array}
$$

Thus the H.C.F. of the first two expressions is $(2x+1)$. Now, we have to find the H.C.F. of $2x+1$ and the thrid given expression.

$$\begin{array}{l}
2x+1)6x^3+x^2+3x+2(3x^2-x+2\\
\qquad\quad \underline{6x^3+3x^2\qquad\quad}\\
\qquad\qquad -2x^2+3x+2\\
\qquad\qquad \underline{-2x^2+x\qquad\;}\\
\qquad\qquad\qquad\quad 4x+2\\
\qquad\qquad\qquad\quad \underline{4x+2}
\end{array}$$

$\therefore$ required H.C.F. $= 2x+1$

Ex. 4. Find H.C.F. of $6a^3-17a^2-5a+6$,

$6a^3-5a^2-3a+2$ and $3a^3-7a^2+4$.

Let us first find the H.C.F. of the first two given expressions.

$$\begin{array}{l}
6a^3-17a^2-5a+6)6a^3-5a^2-3a-2(1\\
\qquad\qquad\qquad\qquad \underline{6a^3-17a^2-5a+6}\\
\qquad\qquad\qquad\qquad 2\,\underline{\lfloor 12a^2+2a-4}\\
\qquad\qquad\qquad 6a^2+a-2)6a^3-17a^2-5a+6(a-3\\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{6a^3+a^2-2a\qquad}\\
\qquad\qquad\qquad\qquad\qquad\qquad\quad -18a^2-3a+6\\
\qquad\qquad\qquad\qquad\qquad\qquad\quad \underline{-18a^2-3a+6}
\end{array}$$

Hence H.C.F. of the first two expression is $6a^2+a-2$ Now we find the H.C.F. of $6a^2+a-2$ and the third given expression.

Here if we multiply the third expression its first term will be divisible by the first term of $6a^2+a-2$.

$$\begin{array}{l}
\qquad\qquad\quad 3a^3-\;7a^2\quad +4\\
\qquad\qquad\quad \underline{2\qquad\qquad\qquad\quad}\\
6a^2+a-2)6a^3-14a^2\quad +8(a-5\\
\qquad\qquad\quad \underline{6a^3+\;\;a^2-2a\;\;}\\
\qquad\qquad\qquad -15a^2+2a+8\\
\qquad\qquad\qquad \underline{\qquad 2\qquad\qquad}\\
\qquad\qquad\qquad -30a^2+4a+16\\
\qquad\qquad\qquad \underline{-30a^2-5a+10}\\
\qquad\qquad\qquad\qquad\quad 3\;\underline{|9a+6}\\
\qquad\qquad\qquad\qquad\qquad 3a+2
\end{array}$$

$$\begin{array}{r}
3a+2)\,6a^2+a-2\,(2a-1\\
\underline{6a^2+4a}\\
-3a-2\\
\underline{-3a-2}
\end{array}$$

∴ required H.C.F.$=3a+2$

Remark : Note that H.C.F. of the three given expressions is different from H.C.F. of the first two expressions.

Exercise 21

Find H.C.F. of the following expressions (by method of division) :

1. a^2+5a+4, a^2+6a+5
2. $x^2+3x+2, 3x^2+x-2$
3. $2a^2-5a+3, 4a^2-4a-3$
4. x^2-2x-3, x^3-2x^2-2x-3
5. a^3-3a-2, a^3-4a^2+6a-4
6. $x^3-7x^2+14x-8, x^3-6x^2+11x-6$
7. $a^3-18a+35, a^3-21a+20$
8. $x^3-5x^2+7x-3, x^3-4x^2+5x-2$
9. $3a^3-13a^2+23a-21, 6a^3+a^2-44a+21$
10. $4a^3-11a^2+25a+7, 2a^3-5a^2+11a+7$
11. $2y^3-9y^2+9y-7, y^3-5y^2+5y-4$
12. $a^3+3a^2-16a+12, a^3+a^2-10a+8$
13. $x^3+4x^2+2x-28, x^3-21x+344$
14. $a^3+2a^2-19a-20, a^3+9a^2+23a+15$
15. $x^3+4x^2-2x-15, x^3-21x-36$
16. $6x^2-5x-4, 6x^3-5x^2+6x+5, 10x^3+11x^2-x-2$
17. $2a^3+7a^2-5a-4, a^3+8a^2+11a-20, 2a^3+19a^2+49a+20$
18. $x^3+4x^2+4x+1, x^3+5x^2+7x+2, x^4+3x^3+2x^2+3x+1$
19. $6x^3+13x^2+x-2, 3x^3+14x^2+13x-6, 15x^3-11x^2+8x-2$
20. $a^3+a^2-7a+2, 2a^3-a^2-7a+2, a^3-4a^2+4a$

L.C.M. (Lowest Common Multiple)

(By factorisation)

If an expression is divisible by another expression, then the first expression is called a **multiple** of the second expression. For example, in arithmetic the number 24 is a multiple of all the numbers 2,3,4,6,8,12,etc..Similarly, in algebra the expression x^3y is a multiple of the expressions x, x^2, xy, x^2y, etc.

If an expression is divisible by each of two or more expressions, then the first expression is called **common multiple** of those two or more expressions. For examle, x^2y^2 is a common multiple of the three expressions xy, x^2y, xy^2 as x^2y^2 is divisible by each of these three expressions.

Lowest Common Multiple (L.C.M.)

Of all the expressions the one with the lowest power (and with L.C.M. of co-efficients) is called the **Lowest Common Multiple** (or in short L.C.M.) of the given two or more expressions.

It has been stated above that x^2y^2 is a common multiple of xy, x^2y, xy^2. Other common multiples of these three expressions may easily be written as x^3y^2, x^2y^3, x^3y, $2x^2y^2$, ax^3y^3 etc. Of all these multiples the value of x^2y^2 is lowest and hence this is lowest common multiple or L.C.M. of the three given expressions.

Method of finding L.C.M. of single term expressions :

Each expression is first factorised. The successive product of all factors with highest power is the L.C.M. of the given expressions.

Ex. 1. Find L.C.M. of a^3bc, $a^2b^3c^2$, ab^2c^3.

Different powers of a, b and c are factors of these expression. Here

factors with highest power of a is a^3

factor with highest power of b is b^3

factor with highest power of c is c^3

Hence L.C.M. of the three given expressions $=a^3b^3c^3$

Ex. 2. Find L.C.M. of $4x^2y^4z$, $12x^3yz^5$, $18x^2y^3z^2$

Different powers of x, y, z are factors of these expressions. Here

highest power of x is 3
highest power of y is 4
highest power of z is 5.

Again L.C.M. of the numerical co-efficients (4, 12,8) is 36.

$\therefore$ L.C.M. of the three expressions $= 36x^3y^4z^5$

Exercise 22

Find L.C.M. of the following expressions :

1. $2a^2, a^3b$ 2. x^2y, y^2z 3. a^3b^2, a^2b^3
4. $8m^3n^2p, 12mn^2p^3$ 5. $3x^2yz^2, 5yz^3$
6. ab, bc, ca 7. x^2y, xy^2, yz^2
8. $a^2, 2b^2, 3c^2$ 9. a^2bc, ab^2c, abc^2
10. $3m^2n^2p, 9m^3n^2, 15np^3$ 11. $4x^2yz^3, 6x^3y^2z^4, 8xyz^2$
12. $13c^3d^2, 26cd^4, 39c^5$
13. $27a^2b^2c^2, 81b^2x^2, 9ca^3$
14. $5x^2y^2, 10xz^2, 15y^3z^4$
15. $32a^4b^3c, 48abc^5, 16a^4c^3$
16. $21a^4x^2y, 35a^2x^4y, 28a^3xy^4$
17. $4ab, 9a^2c, 3ac, 12a^2c^3$
18. $3a^2x^3, 9x^2b^3, 12c^3y^2, 24a^3z^4$
19. $9x^2y^4a^5, 12b^2z^5x, 18c^2y^3a, 4a^3b^3z^4$
20. $3a^5b^4, 5a^2b^2c, 7abc, 10a^2b^4c^2$

L.C.M. of expressions with more than one term by factiorisation :

L.C.M. is to be found by factorising each expression with more than one term.

Ex. 1. Find L.C.M. of $a^2-b^2, a^3-b^3, (a+b)^2$

first expression $=a^2-b^2=(a+b)(a-b)$
second expression $=a^3-b^3=(a-b)(a^2+ab+b^2)$
third expression $=(a+b)^2$

Thus the factors of the three expressions are $a+b$, $a-b$, a^2+ab+b^2 and their maximum powers are 2,1,1 respectively.

Hence required L.C.M. $=(a-b)(a^2+ab+b^2)(a+b)^2$

$$=(a^3-b^3)(a+b)^2$$

Ex. 2. Find L.C.M. of $x^5y^2-x^2y^5$ and $x^4y^2-2x^3y^3+x^2y^4$

1st expression $=x^5y^2-x^2y^5$

$=x^2y^2(x^3-y^3)$

$=x^2y^2(x-y)(x^2+xy+y^2)$

2nd expression $=x^4y^2-2x^3y^3+x^2y^4$

$=x^2y^2(x^2-2xy+y^2)$

$=x^2y^2(x-y)^2$

Here highest power of x $=2$

highest power of y $=2$

highest power of $x-y$ $=2$

highest power of x^2+xy+y^2 $=1$

$\therefore$ required L.C.M. $=x^2y^2(x-y)^2(x^2+xy+y^2)$

Ex. 3. Find L.C.M. of $4(x^2-4)$, $6(x^2-x-2)$ and $12(x^2+x-2)$

1st expression $=4(x^2-4)=4(x+2)(x-2)$

2nd expression $=6(x^2-x-2)=6(x^2-2x+x-2)$

$=6\{x(x-2)+1(x-2)\}$

$=6(x-2)(x+1)$

3rd expression $=12(x^2+x-2)=12(x^2+2x-x-2)$

$=12\{x(x+2)-1(x+2)\}$

$=12(x+2)(x-1)$

L.C.M. of numerical co-efficients 4,6,12=12

$\therefore$ required L.C.M. $=12(x+1)(x-1)(x+2)(x-2)$

Ex. 4. Find L.C.M. of $(x^2+2x^2)^2$, $2x^4+3x^3-2x^2$ and $2x^3-3x^2-14x$.

1st expression $=(x^2+2x)^2=\{x(x+2)\}^2=x^2(x+2)^2$

2nd expression $=2x^4+3x^3-2x^2$
$=x^2(2x^2+3x-2)$
$=x^2(2x^2+4x-x-2)$
$=x^2\{2x(x+2)-1(x+2)\}$
$=x^2(x+2)(2x-1)$
3rd expression $2x^3-3x^2-14x$
$=x(2x^2-3x-14)$
$=x(2x^2-7x+4x-14)$
$=x\{x(2x-7)+2(2x-7)\}$
$=x(2x-7)(x+2)$
$\therefore$ required L.C.M.$=x^2(x+2)^2(2x-1)(2x-7)$

Ex. 5. Find L.C.M. of $3a^3-18a^2x+27ax^2$, $4a^4+24a^3x+36a^2x^3$ and $6a^4-54a^2x^2$

1st expression $=3a^3-18a^2x+27ax^2$
$=3a(a^2-6ax+9x^2)$
$=3a\{a^2-2.a.3x+(3x)^2\}=3a(a-3x)^2$
2nd expression $=4a^4+24a^3x+36a^2x^2$
$=4a^2(a^2+6ax+9x^2)$
$=4a^2\{a^2+2.a.3x+(3x)^2\}=4a^2(a+3x)^2$
3rd expression $=6a^4-54a^2x^2$
$=6a^2(a^2-9x^2)$
$=6a^2\{a^2-(3x)^2\}=6a^2(a+3x)(a-3x)$
L.C.M. of numerical co-efficients, 3,4,6 =12
$\therefore$ required L.C.M. $=12a^2(a+3x)^2(a-3x)^2$

Exercise 23

Find L.C.M. of the following expressions (by factorisation) :

1. a^3, a^2+2a
2. x^2+x, x^3-x

3. $xy+y^2, x^2+xy$
4. m^2-4, m^2+2m
5. $a(b+c)^2, a^2(b+c)$
6. $4(x-y)^2, 6(x^2-y^2), 8(x^3-y^3)$
7. $a^2-b^2, a^3-b^3, a^4-b^4$
8. $2x^3-x^2-x, (x-1)^2$
9. a^2-2a-3, a^2+5a+4
10. $a^2-3a+2, (a-1)^2, a^4-1$
11. $a^3-3a^2-10a, a^4-8a^3+15a^2$
12. $8(x^2-4); 12(x^3+8), 36(x^2-3x-10)$
13. $3(x^2+xy), 8(xy-y2), 5(x^2-y^2)$
14. $a^2-ab-2b^2, a^2-5ab+6b^2, a^2-2ab-3b^2$
15. $4a^2b^2, 2a(a^2-b^2), (6a^3b(a^3+b^3)$
16. $a^2-b^2, a^3-b^3, 3a^2-5ab+2b^2$
17. $x^2-xy-2y^2, 2x^2-5xy+2y^2, 2x^2+xy-y^2$
18. $3m^2+5m+2, m^2-m-2, 3m^2-4m-4$
19. $2a^2+3a-2, 3a^2+7a+2, 6a^2-a-1$
20. $3x^2-15x+18, 2x^2+2x-24, 4x^2+36x+80$
21. $a^2-3a+2, a^3+2a^2-3a, a^4+a^3-6a^2$
22. $9a^2-36x^2, 4a^2-4ax+x^2, 2a^2+3ax-2x^2$
23. $15a^3x(a+x)^3, 20ax^3(a-x)^3, 36a^2x^2(a^2-x^2)^2$
24. $x^4+x^2y^2+y^4, x^3y+y^4, (x^2-xy)^3$
25. $ab^4-8ab, a^2b^4+8a^2b, ab^4-4ab^2$
26. $3a^3-18a^2x+27ax^2, 4a^4+24a^3x+36a^2x^2, 6a^4-54a^2x^3$
27. $(a^2+2a)^2, 2a^3+3a^2-2a, 2a^4-3a^3-14a^2$
28. $3a^2-5ab-12b^2, a^5-27a^2b^3, 9a^2+24ab+16b^2$

Chapter 7

Fractions

You know fractions in arithmetic. You also know what is meant by the fraction $\frac{3}{10}$ Here numerator 3 and denominator 10 are two definite whole numbers. If instead of the above we have two symbols a and b of unknown values, then $\frac{a}{b}$ is called algebraic fraction whose numerator is a and denominator is b . The fraction is $a \div b$.

Generally any fraction is expressed in its lowest form. It is desirable to apply the following rules for expressing a fraction in its lowest form.

1. If the numerator and the denominator of an algebraic fraction is divided by a common factor of the numerator and denominator then the fraction remains unchanged. For example, $\frac{ax}{ay} = \frac{x}{y}$ The fraction obtained after replacing all common factors in the above manner is called its lowest form. In many cases it becomes easier to work out a sum if an algebraic fraction is expressed in its lowest form.

2. As in arithmetic, in algebra also for addition and subtraction of fractions the denominators are first made same. For example

$$\frac{a}{b}-\frac{b}{c}+\frac{c}{d}=\frac{acd}{bcd}-\frac{b^2d}{bcd}+\frac{bc^2}{bcd}$$

$$=\frac{acd-b^2d+bc^2}{bcd}$$

3. The process of multiplication is same both in arithmetic and algebra. Product of two or more fractions will be a fraction whose numerator and denominator are respectively equal to the product of the numerators and the denominators of the given fractions. For example,

$$\frac{ad}{b^2} \times \frac{b^3}{c^3} \times \frac{c^2}{d^2} = \frac{a}{1} \times \frac{b}{c} \times \frac{1}{d} = \frac{ab}{cd}$$

This is the lowest form of the product of the three fractions. In order to reduce the product to its lowest form, we are to remove common factors from any numerator and any denominator before actual multiplication. Here, common factor of the numerator of the first fraction and the denominator of the third fraction is d; common factor of the denominator of the first fraction and the numerator of the second fraction is b^2, and common factor of the denominator of the second fraction and the numerator of the third fraction is c^2. These common factors are removed before actual multiplication.

4. As in arithmetic, in algebra also if one fraction is to be divided by another fraction, then the first is multiplied by the reciprocal of the second fraction.

[**N.B.** : Reciprocal of 3 is $\frac{1}{3}$ reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$, reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$ etc.]

For example,

$$\frac{a^2c^2}{b^2d^2} \div \frac{bc}{ad} = \frac{a^2c^2}{b^2d^2} \times \frac{ad}{bc} = \frac{a^2c}{b^2d} \times \frac{a}{b} = \frac{a^3c}{b^3d}$$

Remark : If necessary, the method of removing, common factors from the numerator and denominator is to be repeated as many times as possible to express the fraction in its lowest form.

It should always be remembered that $\frac{x}{x}=1$ and not 0 (zero).

Ex.1. Express $\frac{24a^4b^5c^2}{18a^3b^3c^3}$ in its lowest form.

$$\frac{24a^4b^5c^2}{18a^3b^3c^3}=\frac{4ab^2}{3c}$$

Remark : Here the following common factors of the numerator and the denominator are removed 6, a^3,b^3,c^3.

Ex. 2. Simplify : $\frac{a}{bc}-\frac{b}{ca}+\frac{c}{ab}$

$$\frac{a}{bc}-\frac{b}{ca}+\frac{c}{ab}=\frac{a^2}{abc}-\frac{b^2}{abc}+\frac{c^2}{abc}=\frac{a^2-b^2+c^2}{abc}$$

Ex. 3. Simplify :

$$\frac{x^2}{y^2z^2}\times\frac{yz^3}{xa^3}\times\frac{y^4b^3}{xz}\times\frac{a^3}{b^3y^3}$$

Here the numerator and denominator of the product will be respectively equal to the products of the numerators and denominators of the given fractions.

$$\frac{x^2}{y^2z^2}\times\frac{yz^3}{xa^3}\times\frac{y^4b^3}{xz}\times\frac{a^3}{b^3y^3}$$

$$= \frac{x^2 \times yz^3 \times y^4b^3 \times a^3}{y^2z^2 \times xa^3 \times xz \times b^3y^3}$$

$$= \frac{x^2y^5z^3a^3b^3}{x^2y^5z^3a^3b^3} = 1$$

Ex. 4. Simplify : $\frac{5a^3b^3}{8c^2d^2} \div \frac{15a^2d}{12b^2c} \times \frac{4c^4d^2}{3a^2b^4}$

Given expression $= \frac{5a^3b^3}{\underset{2}{\cancel{8}}c^2d^2} \times \frac{\overset{\cancel{4}\ 2}{\cancel{12}}b^2c}{\underset{\cancel{3}}{\cancel{15}}a^2d} \times \frac{\cancel{4}c^4d^2}{3a^2b^4}$

$$= \frac{2a^3b^5c^5d^2}{3a^4b^4c^2d^3}$$

$$= \frac{2bc^3}{3ad}$$

Remark : This becomes similar to Ex. 3 since we have multiplied by the reciprocal of the second fraction. At the next step common factors are removed.

Ex. 5. Simplify :

$$\frac{x-6}{x+2} - \frac{x-4}{x-1}$$

Given expression $= \frac{(x-6)(x-1)-(x-4)(x+2)}{(x+2)(x-1)}$

$$= \frac{(x^2-7x+6)-(x^2-2x-8)}{x^2+x-1}$$

$$= \frac{x^2-7x+6-x^2+2x+8}{x^2+x-2}$$

$$= \frac{14-5x}{x^2+x-2}$$

Ex 6. Simplify :

$$\frac{a-x}{ax}-\frac{a+x}{x^2}+\frac{x}{a^2}+\frac{4a^2+x^2}{4ax^2}$$

Given expression =

$$\frac{4ax(a-x)-4a^2(a+x)+4x^2.x+a(4a2+x^2)}{4a^2x^2}$$

$$=\frac{4a^2x-4ax^2-4a^3-4a^2x+4x^3+4a^3+ax^2}{4a^2x^2}$$

$$=\frac{4x^3-3ax^2}{4a^2x^2}$$

$$=\frac{x^2(4x-3a)}{4a^2x^2}$$

$$=\frac{4x-3a}{4a^2}$$

Ex. 7. Simplify : $\frac{3a^2-9ab}{a^2-ab}\div\frac{a^2-9b^2}{a^2-b^2}$

Given expression= $\frac{3a^2-9ab}{a^2-ab}\times\frac{a^2-b^2}{a^2-9b^2}$

$$=\frac{3a(a-3b)}{a(a-b)}\times\frac{(a+b)(a-b)}{a^2-(3b)^2}$$

$$=\frac{3(a-3b)}{a-b}\times\frac{(a+b)(a-b)}{(a+3b)(a-3b)}=\frac{3(a+b)}{a+3b}$$

Remark : The process of simplification becomes easier when the numerators and denominators of fractions can easily be factorised.

Ex. 8. Simplify :

$$\left(3a-\frac{1}{3a}\right)\div\left(a-\frac{2}{3}-\frac{1}{3a}\right)$$

Given expression $=\dfrac{9a^2-1}{3a}\div\dfrac{3a^2-2a-1}{3a}$

$$=\frac{(3a+1)(3a-1)}{3a}\times\frac{3a}{3a^2-2a-1}$$

$$=\frac{(3a+1)(3a-1)}{3a^2-2a-1}$$

$$=\frac{(3a+1)(3a-1)}{3a^2-3a+a-1}$$

$$=\frac{(3a+1)(3a-1)}{3a\,(a-1)+1(a-1)}$$

$$=\frac{(3a+1)(3a-1)}{(a-1)(3a+1)}$$

$$=\frac{3a-1}{a-1}$$

Ex. 9. Simplify :

$$\frac{x^3+y^3}{x^3-y^3}\div\frac{(x-y)^2+xy}{(x+y)^2-xy}\div\frac{a^2-ax}{a^2-x^2}$$

Given expression$=\dfrac{x^3+y^3}{x^3-y^3}\times\dfrac{(x+y)^2-xy}{(x-y)^2+xy}\times\dfrac{a^2-x^2}{a^2-ax}$

$$=\frac{(x+y)(x^2-xy+y^2)}{(x-y)(x^2+xy+y^2)}\times\frac{x^2+xy+y^2}{x^2-xy+y^2}\times\frac{(a+x)(a-x)}{a(a-x)}$$

$$=\frac{(x+y)(a+x)}{a\,(x-y)}$$

Ex. 10. Simplify :

$$\frac{a^2+a-6}{a^2-1}\times\frac{a^2-3a-4}{a^2-5a+6}\div\frac{a^2-a-12}{a^2-4a+3}$$

Given expression $= \dfrac{a^2+a-6}{a^2-1}\times\dfrac{a^2-3a-4}{a^2-5a+6}\times\dfrac{a^2-4a+3}{a^2-a-12}$

$$=\frac{(a-2)(a+3)}{(a+1)(a-1)}\times\frac{(a-4)(a+1)}{(a-3)(a-2)}\times\frac{(a-3)(a-1)}{(a-4)(a+3)}$$

$= 1.$

Ex. 11. Simplify :

$$\left(\frac{5x}{2x-1}-\frac{x-2}{x}\right)\times\left(\frac{2x}{x+2}-\frac{1}{x+2}\right)$$

Given expression$=\left\{\dfrac{5x^2-(x-2)(2x-1)}{x(2x-1)}\right\}\times\left(\dfrac{2x-1}{x+2}\right)$

$$=\left\{\frac{5x^2-2x^2+5x-2}{x(2x-1)}\right\}\times\frac{2x-1}{x+2}$$

$$=\frac{3x^2+5x-2}{x(2x-1)}\times\frac{2x-1}{x+2}$$

$$=\frac{(x+2)(3x-1)}{x(2x-1)}\times\frac{2x-1}{x+2}=\frac{3x-1}{x}$$

Ex. 12. Simplify :

$$\frac{a^3+a^2b+ab^2+b^3}{2a(a-b)^2}\times\frac{a^2-b^2}{a+2b}\div\frac{a+b}{a^2+ab-2b^2}\div\frac{a^2+b^2}{4a^2}$$

$$\text{Given expression} = \frac{a^2(a+b)+b^2(a+b)}{2a(a-b)^2}\times\frac{(a+b)(a-b)}{a+2b}$$

$$\times\frac{a^2-b^2+ab-b^2}{a+b}\times\frac{4a^2}{a^2+b^2}$$

$$=\frac{(a+b)(a^2+b^2)}{2a(a-b)^2}\times\frac{(a+b)(a-b)}{a+2b}$$

$$\times\frac{(a-b)(a+2b)}{a+b}\times\frac{4a^2}{a^2+b^2}$$

$$= 2a\,(a+b)$$

Exercise 24

Express in lowest form :

1. $\frac{3a^4b^3}{9a^5b}$
2. $\frac{18p^4q^2}{27q^2}$
3. $\frac{12a^4c^3d^2}{28a^3c^4d^3}$
4. $\frac{39a^3x^2y}{9a^2x^3z}$
5. $\frac{56x^6y^5z^4}{42x^4y^3z^2}$
6. $\frac{74m^3n^3}{111m^2p^4}$
7. $\frac{146b^2c^6}{365abc^4}$
8. $\frac{27x^5y^3z^6}{18x^6y^2z^4}$

Simplify :

9. $\frac{3ab^2}{5b^3c} \times \frac{15b^2c^2}{9a^2b}$

10. $\frac{2a^2b}{3b^2c} \times \frac{c^4}{3a^3} \div \frac{4bc^3}{9a^2}$

11. $\frac{3xy^2}{4yz} \times \frac{5z^3x}{7x^2y} \div \frac{45x^3y^3z}{14xy^3z^2}$

12. $\frac{169m^4}{15n} \div \frac{156m^2np^5}{25p^3} \times \frac{36n^2p^2}{65m^2}$

13. $\frac{x^2}{yz} - \frac{y^2}{zx} + \frac{z^2}{xy}$

14. $\frac{b-c}{bc} + \frac{c-a}{ca} + \frac{a-b}{ab}$

15. $\frac{x-2y}{xy} + \frac{3y-a}{ay} - \frac{3x-2a}{ax}$

16. $\frac{x^2+xy}{xy+y^2}$

17. $\frac{c^2-2c}{4c^3-8c^2}$

18. $\frac{a^2-4b^2}{a^2-2ab}$

19. $\frac{(m+2n)^2}{m^3-4mn^2}$

20. $\frac{5p^2+30p}{p^2+p-30}$

21. $\frac{a+1}{a-2} \times \frac{a^2-a-2}{a^2+a}$

22. $\frac{a^2}{2x+3} \times \frac{6x^2+9x}{4a^4}$

23. $\frac{x^3-y^3}{x^2-y^2} \div \frac{x^2+xy+y^2}{x^2+2xy+y^2}$

24. $\frac{a^2-a}{2a^3+6a^2} \times \frac{a^2+3a}{a^2-1}$

25. $\frac{9m^2-16n^2}{m^2-16} \times \frac{m^2-4m}{3m-4n}$

26. $\frac{a^3-b^3}{a^2+2ab+b^2} \times \frac{a^2-b^2}{a^2+ab}$

27. $\frac{x^2+4x}{x^3-9x} \div \frac{x^2+2x-8}{x^2+x-6}$

28. $\dfrac{x^2+7xy+12y^2}{x^2-y^2} \div \dfrac{x+4y}{x-y}$

29. $\dfrac{a^2-5ab}{a^2-6ab+5b^2} \times \dfrac{a^2-b^2}{a^2+ab}$

30. $\dfrac{p^3+q^3}{p^2-q^2} \times \dfrac{p+q}{p-q} \times \dfrac{p^2-pq}{(p+q)^2}$

31. $\dfrac{m^2+3m}{m^2+4m+3} \div \dfrac{m^2-9}{m^2-2m-3}$

32. $\dfrac{x^2+3x+2}{x^2-4x-12} \times \dfrac{x^2-7x+6}{x^2-4}$

33. $\dfrac{a^3-8}{a+3} \div \left(\dfrac{a-2}{4a} \times \dfrac{8a^4}{a^2+3a}\right)$

34. $\dfrac{x^4-8x}{2x^2+5x-3} \times \dfrac{2x-1}{x^2+2x+4} \div \dfrac{x^2-2x}{x+3}$

35. $\dfrac{1+8x^3}{(2-x)^2} \times \dfrac{4x-x^3}{1-4x^2} \div \dfrac{1-(2x)^2+2x}{2-5x+2x^2}$

36. $\dfrac{a(a^2-b^2)}{a^2+4ab+3b^2} \div \dfrac{a^2-ab}{a+3b}$

37. $\dfrac{1}{x^2+xy+y^2} \times \dfrac{2x}{x^3+y^3} \times \dfrac{x^4+x^2y^2+y^4}{4x^2}$

38. $\dfrac{x(a^2-b^2)}{x^3+y^3} \times \dfrac{x^2-xy+y^2}{ax^2-a^2x} \div \dfrac{2b}{x^2-a^2} \div \dfrac{a^2-b^2}{2ab}$

Chapter 8

Solution of a simple equation with one unknown

In class VII you have learnt how to form a simple equation with one unknown and to find the value of that unknown. We take help of some axioms in solving equations. These axioms are given below :

Axiom 1.

If same or equal quantities are added to equal quantities, their sums will also be equal to each other.

Axiom 2.

If same or equal quantities are subtracted from equal quantities, the difference will also be equal to each other.

Axiom 3.

If equal quantities are multiplied by same or equal quantities, the products will also be equal to each other.

Axiom 4.

If equal quantities are divided by same or equal quantities, the quotients will also be equal to each other.

Apart from all these axioms, some other rules are to followed in finding value of the unknown quantity.

(i) In an equation the unknown quantity is generally kept on the left hand side.

(ii) If a term is taken from left hand side to right hand side or from right hand side to left hand side then its sign has to changed. This is called transportation. Actually this is a different form of axioms 1 and 2.

(iii) If both sides of an equation are fractions, then the results of cross multiplication will be equal. The process of multiplying the numerator of one side with the denominator of the other side is known as cross multiplication. For example —

if $\frac{a}{b}=\frac{c}{d}$ then by cross multiplication $ad=bc$. For if both sides are multiplicated by bd, then $bd\frac{a}{b}=bd\frac{c}{d}$ or $ad=bc$. Here we have multiplied both sides by the product the two denominators. Without the above multiplication we may straight way write $ad=bc$ by multiplying crosswise. This rule can also be applied even if one side is fraction and the other side is a whole number. For, any whole number is also a fraction with its denominator as 1. For Example,

$c=\frac{c}{1}$ If $\frac{a}{b}=c$, then $\frac{a}{b}=\frac{c}{1}$ or, $a=bc$.

Solution of easier simple equations :

Ex. 1. Solve : $14(x-2)+3(x+5)=3(x+8)+5$

$14(x-2)+3(x+5)=3(x+8)+5$

or, $14x-28+3x+15=3x+24+5$

or, $17x-13=3x+29$

or, $17x-3x=29+13$ [by transportation]

or, $14x=42$

or, $x=\frac{42}{14}=3$.

required solution is $x=3$.

Ex. 2. Solve :

$$\frac{x-4}{3}+\frac{2x-3}{35}=\frac{5x-32}{9}-\frac{x+9}{28}$$

$$\frac{x-4}{3}+\frac{2x-3}{35}=\frac{5x-32}{9}-\frac{x+9}{28}$$

or, $\frac{2x-3}{35}+\frac{x+9}{28}=\frac{5x-32}{9}-\frac{x-4}{3}$ [by transportation]

or, $\frac{8x-12+5x+45}{140}=\frac{5x-32-3x+12}{9}$

or, $\frac{13x+33}{140}=\frac{2x-20}{9}$

or, $140(2x-20)=9(13x+33)$ [by cross transportation]

or, $280x-2800=117x+297$

or, $163x=3097$ or, $x=\frac{3097}{163}=19.$

required solution is $x=19$

Ex. 3. Solve : $x(x+1)+(x+2)=2x^2+15$

$x(x+1)+(x+2)=2x^2+15$

or, $x^2+x+x^2+2x=2x^2+15$

or, $2x^2+3x-2x^2=15$ [by transporting $2x^2$]

or, $3x=15$ or, $x=\frac{15}{3}=5$

required solution is $x=5$

Remark : This equation may not appear to be simple (linear) equation, for the higest power of x in the equation is 2. But it is found later that the terms containing x^2 are removed and the equation becomes simple (linear).

Ex. 4. Solve : $\frac{1}{3}\left(x-\frac{5}{2}\right)-\frac{3}{5}\left(x+\frac{4}{3}\right)+\frac{7}{2}=0$

$\frac{1}{3}\left(x-\frac{5}{2}\right)-\frac{3}{5}\left(x+\frac{4}{3}\right)+\frac{7}{2}=0$

or, $\frac{1}{3}\left(\frac{2x-5}{2}\right)-\frac{3}{5}\left(\frac{3x+4}{3}\right)+\frac{7}{2}=0$

or, $\frac{2x-5}{6}-\frac{3x+4}{5}+\frac{7}{2}=0$

or, $\frac{2x-5}{6}-\frac{3x+4}{5}=-\frac{7}{2}$

or, $\frac{10x-25-18x-24}{30}=-\frac{7}{2}$

or, $\frac{-8x-49}{30}=-\frac{7}{2}$ or, $\frac{-(8x+49)}{30}=\frac{-7}{2}$

or, $\frac{8x+49}{30}=\frac{7}{2}$ or, $16x+98=210$

or, $16x=210-98$ or, $16x=112$

or, $x=\frac{112}{16}$ or, $x=7$

required solution is $x=7$

Ex. 5. Solve :

$(x-6)(2x-9)-(11-2x)(7-x)=5x-4-7(x-2)$

$(x-6)(2x-9)-(11-2x)(7-x)=5x-4-7(x-2)$

or, $2x^2-21x+54-77+25x-2x^2=5x-4-7x+14$

or, $4x-23=10-2x$

or, $4x+2x=10+23$ or, $6x=33$

or, $x=\frac{33}{6}=\frac{11}{2}=5\frac{1}{2}$

required solution is $x=5\frac{1}{2}$

Exercise 25

Solve the following equations :

1. $3(2x-1)+7x=-29$ 2. $3(2x+5)-2x=3(x+6)$

3. $50+3(4x-5)+8(x+3)=x+2$

4. $(x-3)(x+5)+2=x^2+x$

5. $3(x+5)-3(2x-1)=32-4(x-5)^2+4x^2$

6. $5x-(4x-7)(3x-5)=6-3(4x-9)(x-1)$

7. $\frac{x+5}{3}+\frac{2x-1}{7}=4$

8. $\frac{6x-2}{9}+\frac{3x+5}{18}=\frac{1}{3}$

9. $\frac{x-3}{3}+\frac{x+2}{8}+\frac{2x-1}{11}=3$

10. $\frac{x+12}{6}-=6\frac{1}{2}-\frac{x}{12}$

11. $\frac{3}{5}(x-4)-\frac{1}{3}(2x-9)=\frac{1}{4}(x-1)-2$

12. $\frac{x+1}{2}-\frac{5x+9}{28}=\frac{x+6}{21}+5-\frac{x-12}{3}$

13. $\frac{5x}{6}-\frac{2x-3}{9}=x+\frac{x-12}{6}-1$

14. $\frac{2}{3}\left(\frac{x}{3}+6\right)=3\frac{3}{4}-\frac{3}{4}\left(\frac{16}{27}-\frac{2x}{3}\right)$

15. $\frac{x-1}{2}-\frac{4x+3}{21}=\frac{5x+9}{28}+\frac{x-12}{3}$

16. $\frac{3x}{4}-\frac{x-7}{51}-(x-3)-\frac{6}{17}(x+10)+\frac{2x+5}{4}=0$

17. $\frac{1}{3}\left(x-\frac{5}{2}\right)-\frac{3}{5}\left(x+\frac{4}{3}\right)+\frac{7}{2}=0$

18. $3(x-4)^2+5(x-3)^2=(2x-5)(4x-1)-40$

19. $\frac{9x+5}{14}+\frac{8x-7}{7}=\frac{18x+11}{28}+\frac{5}{4}$

20. $\frac{1}{4}(x-8)+\frac{x+4}{4}+\frac{23-x}{5}=-7-\frac{x-1}{7}$

Solution of some difficult simple equation : Now we shall discuss some methods of solving relatively more difficult simple equations. These methods are not applications of any rule or formula. Application of these methods make the solution of certain equations relatively easier.

(i) Method of alternends :

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a}{c}=\frac{b}{d}$ For $\frac{a}{b}=\frac{c}{d}$ means $ad = bc$ (by cross multiplication). Now dividing both sides by cd, we get $\frac{a}{c}=\frac{b}{d}$

Note the following.

Solve : $\frac{x^2+3x+4}{x^2+5x+8}=\frac{x+3}{x+5}$

$$\frac{x^2+3x+4}{x^2+5x+8}=\frac{x+3}{x+5}$$

or, $\frac{x^2+3x+4}{x+3}=\frac{x^2+5x+8}{x+5}$ [by the method of alternends]

or, $\frac{x(x+3)+4}{x+3}=\frac{x(x+5)+8}{x+5}$

or, $x+\frac{4}{x+3}=x+\frac{8}{x+5}$

or, $\frac{4}{x+3}=\frac{8}{x+5}$ [removing x from both sides]

or, $8(x+3)=4(x+5)$ [by cross multiplication]

or, $4x=-4$ or, $x=\frac{-4}{4}=-1$

or, required solution is $x=-1$

(ii) Method of transposition :
Note the following example.

Solve : $\frac{9x+5}{21}=\frac{3x-2}{7}+\frac{4x-5}{3x+1}$

In this case it becomes easier to solve the equation if the two terms who denominators do not contain the unknown x are brought to one side of the equation. Thus transporting the first term of the right hand side to the left hand side, we get

$$\frac{9x+5}{21}-\frac{3x-2}{7}=\frac{4x-5}{3x+1}$$

or, $\frac{9x+5-9x+6}{21}=\frac{4x-5}{3x+1}$

or, $\frac{11}{21}=\frac{4x-5}{3x+1}$

or, $21(4x-5)=11(3x+1)$

or, $84x-105=33x+11$

or, $84x-33x=105x+11$

or, $51x=116$

or, $x=\frac{116}{51}=2\frac{14}{51}$

required solution is $x=2\frac{14}{51}$

(iii) Method of break term :
Note the following example.

Solve : $\frac{5}{x+2}+\frac{8}{x-3}=\frac{13}{x-4}$

Note that the unknown term x is absent in the numerators of all the terms. We can express 13, the numerator of the right hand side as sum of the two numerators of the terms in the left hand side and detach the right hand side in two terms.

$$\frac{5}{x+2}+\frac{8}{x-3}=\frac{13}{x-4}$$

or, $\frac{5}{x+2}+\frac{8}{x-3}=\frac{5+8}{x-4}$

or, $\frac{5}{x+2}+\frac{8}{x-3}=\frac{5}{x-4}+\frac{8}{x-4}$

or, $\frac{5}{x+2}+\frac{5}{x-4}=\frac{8}{x-4}+\frac{8}{x-3}$

or, $5\left(\frac{1}{x+2}-\frac{1}{x-4}\right)=8\left(\frac{1}{x-4}-\frac{1}{x-3}\right)$

or, $5\frac{-6}{(x+2)(x-4)}=8\frac{1}{(x-4)(x-3)}$

Multiplying both sides by $(x-4)$ we get

$\frac{30}{x+2}=\frac{8}{x-3}$ or, $\frac{15}{x+2}=\frac{4}{x-3}$

or, $4x+8=-15x+45$ [by cross multiplication]

or, $4x+15x=45-8$

or, $19x=37$ or, $x=\frac{37}{19}=1\frac{18}{19}$

required solution is $x=1\frac{18}{19}$

(iv) Method of division :
Note the following example.

Solve : $\frac{12x-1}{4x-2}+\frac{8x-3}{4x-3}=\frac{20x-17}{4x-5}$

Numerator of each fraction is to be expressed in such form that at least one of its parts becomes divisible by denominator.

$$\frac{12x-1}{4x-2}+\frac{8x-3}{4x-3}=\frac{20x-17}{4x-5}$$

or, $\frac{3(4x-2)+5}{4x-2}+\frac{2(4x-3)+3}{4x-3}=\frac{5(4x-5)+8}{4x-5}$

or, $3+\frac{5}{4x-2}+2+\frac{3}{4x-3}=5+\frac{8}{4x-5}$

or, $\frac{5}{4x-2}+\frac{3}{4x-3}=\frac{8}{4x-5}$ [Since the values of the without x on both sides are equal, they are removed]

Now applying the previous method,

$$\frac{5}{4x-2}+\frac{3}{4x-3}=\frac{5+3}{4x-5}$$

or, $\frac{5}{4x-2}-\frac{5}{4x-5}=\frac{3}{4x-5}-\frac{3}{4x-3}$

or, $5\left(\frac{1}{4x-2}-\frac{1}{4x-5}\right)=3\left(\frac{1}{4x-5}-\frac{1}{4x-3}\right)$

or, $5\,\frac{-3}{(4x-2)(4x-5)}=3\,\frac{-2}{(4x-5)(4x-3)}$

Multiplying both sides by (4x–5), we get

$$\frac{-15}{4x-2}=\frac{6}{4x-3}$$

or, $\frac{-5}{4x-2}=\frac{2}{4x-3}$ [dividing both sides by 3]

or, $8x-4=-20x+15$ or, $8x+20x=15+4$

or, $28x=19$ or, $x=\frac{19}{28}$

required solution is $x=\frac{19}{28}$

Solution of some harder equations :

Ex. 1. Solve :

$$\frac{2+5x}{35}-\frac{2x^2-7}{14(x-1)}=\frac{6-5x}{15}+\frac{10x-11}{30}$$

By transposition, we get

$$\frac{2+5x}{35}-\frac{6-5x}{15}-\frac{10x-11}{30}=\frac{2x^2-7}{14(x-1)}$$

or, $\frac{12+30x-84+70x-70x+77}{210}=\frac{2x^2-7}{14(x-1)}$

or, $\frac{30x+5}{210}=\frac{2x^2-7}{14(x-1)}$ or. $\frac{5(6x+1)}{210}=\frac{2x^2-7}{14(x-1)}$

or, $\frac{6x+1}{42}=\frac{2x^2-7}{14(x-1)}$ or, $\frac{(6x+1)}{3}=\frac{2x^2-7}{14(x-1)}$

or, $6x^2-5x-1=6x^2-21$

or, $-5x=-20$ or, $x=4$

Ex. 2. solve :

$$\frac{24-5x}{x-2}+\frac{8x-49}{4-x}=\frac{28}{x-2}-13$$

By transposition, we get

$$\frac{8x-49}{4-x}+13=\frac{28}{x-2}-\frac{24-5x}{x-2}$$

or, $\frac{8x-49+52-13x}{4-x}=\frac{28-24+5x}{x-2}$

or, $\frac{3-5x}{4-x}=\frac{4+5x}{x-2}$

or, $3x-5x^2-6+10x=16+20x-4x-5x^2$

[by cross multiplication]

or, $13x-16x=16+6$

or, $-3x=22$

or, $x=\frac{\frac{1}{22}}{\frac{1}{-3}}=-7\frac{1}{3}$

Ex. 3. Solve :

$$\frac{1}{x-6}+\frac{1}{x-9}=\frac{1}{x-4}+\frac{1}{x-8}$$

$$\frac{1}{x-3}+\frac{1}{x-9}=\frac{1}{x-4}+\frac{1}{x-8}$$

or, $\frac{1}{x-3}+\frac{1}{x-4}=\frac{1}{x-8}+\frac{1}{x-9}$

or $\frac{-1}{(x-3)(x-4)}=\frac{-1}{(x-8)(x-9)}$

or, $(x-3)(x-4) = (x-8)(x-9)$
or, $x^2-7x+12 = x^2-17x+72$
or, $17x-7x = 72-12$
or, $10x = 60$
or, $x = 6$

Ex. 4. Solve :
$(x+a)(x+b) - c(a+c) = (x-c)(x+c) + ab$ Multiplying, we get

$x^2+ax+bx+ab-ac-c^2 = x^2-c^2+ab$
or, $ax + bx = ac$
or, $(a+b)x = ac$

$$x = \frac{ac}{a+b}$$

Exercise 26

Solve the following equations :

1. $\frac{3}{x+3}=\frac{4}{x+2}$
2. $\frac{5}{3x+4}=\frac{4}{5(x-3)}$
3. $\frac{6x-3}{2x+7}=\frac{3x-2}{x+5}$
4. $\frac{2(x-3)}{3x+1}=\frac{4x-3}{6x+1}$
5. $\frac{3x+1}{x+5}=\frac{6x-7}{2x+1}$
6. $\frac{3x-4}{4x-5}=\frac{6x-7}{8x-9}$

7. $\frac{22x-12}{8x-5}=2+\frac{3x+7}{4x+8}$ 8. $\frac{9x-22}{2x-5}=\frac{3x-5}{2x-7}=3$

9. $\frac{2x}{2x-1}+\frac{2x+1}{2x}=2$ 10. $\frac{x-a}{x+b}=\frac{2x-a}{2x+b}$

11. $-\frac{10x+41}{18}-\frac{12x+38}{13x+23}=\frac{5x+8}{9}$

12. $\frac{2x+3}{x+1}=\frac{4x+5}{4x+4}+\frac{3x+3}{3x+1}$

13. $\frac{3}{x-6}+\frac{7}{x-2}=\frac{10}{x-4}$

14. $\frac{1}{x+1}+\frac{2}{2x+3}=\frac{6}{3x+5}$

15. $\frac{12x+3}{4}=\frac{15x-1}{5}+\frac{3x-2}{2x-3}$

16. $\frac{10}{5x-9}+\frac{14}{2x+9}=\frac{9}{x+8}$

17. $\frac{6x+1}{15}=\frac{2x-4}{7x-16}+\frac{2x-1}{5}$

18. $\frac{8}{x-5}+\frac{7}{x-4}=\frac{15}{x-3}$

19. $\frac{3x+2}{x-1}+\frac{2x-4}{x-2}=5$

20. $\frac{1}{\mathrm{x}}+\frac{1}{\mathrm{x}+3}=\frac{1}{\mathrm{x}+2}+\frac{1}{\mathrm{x}+1}$

21. $\frac{x^2+7+2}{x^2+8x+1}=\frac{x+7}{x+8}$

22. $\frac{(x+2)(x+3)}{(x+1)(x+7)}=\frac{x+5}{x+8}$

23. $\frac{1}{x-2}+\frac{1}{x-6}=\frac{1}{x-3}+\frac{1}{x-5}$

24. $\frac{1}{x-3}+\frac{1}{x-9}=\frac{1}{x-4}+\frac{1}{x-8}$

25. $\frac{x-8}{x-9}+\frac{x-4}{x-5}=\frac{x-9}{x-10}+\frac{x-3}{x-4}$

26. $\frac{x-8}{x-10}+\frac{x-4}{x-6}=\frac{x-7}{x-9}+\frac{x-5}{x-7}$

27. $\frac{2x+1}{2x-1}+\frac{3x+1}{3x-2}=\frac{3x+2}{3x-1}+\frac{6x+1}{6x-5}$

28. $\frac{2x+3}{x+1}+\frac{4x+7}{2x+3}=\frac{6(x+3)}{2x+5}$

29. $\frac{3x+5}{x+1}=\frac{4x+8}{3x+3}+\frac{10x+1}{6x+3}$

30. $\frac{2x+9}{x+4}+\frac{3x-2}{x-1}=\frac{4x+13}{x+3}+\frac{x+1}{x}$

31. $\frac{6+5x}{5}-\frac{3(7-2x^2)}{14(x-1)}=\frac{1+3x}{7}+\frac{11-10x}{10}+\frac{1}{35}$

32. $\frac{11x^2+33x+8}{22x^2+33x+13}=\frac{x+3}{2x+3}$

33. $\frac{a}{x+a}+\frac{b}{x+b}=2$

34. $\frac{x-a}{c-a}+\frac{x-b}{c-b}=2$

35. $\frac{1}{x-a}-\frac{1}{x-a+c}=\frac{1}{x-b-c}-\frac{1}{x-b}$

36. $\frac{a}{x-a}+\frac{b}{x-b}=\frac{a+b.}{x-a-b}$

Answers

Exercise 1

1. (i) $p+q$ (ii) $a-5$ (iii) $10b$ (iv) $3x+5$
(v) $(a+b)+7$ (vi) $\frac{(a+b)}{c}$ (vii) $p-q$

2. (i) y less than x (ii) q more than p or sum of p and q
(iii) 4 times m (iv) n divided by 5 (v) 3 times q more than p (vi) b more than half of a.

3. $x-y$ (i) $(65-x)$yrs. (ii) $(65+y)$yrs.

5. $(1000a+10b)$ mm 6. $x-y$ 7. $\frac{250-x}{y}$ rupees

8. After $\frac{x}{y}$ hrs. 9. $30+x$, $10x+3$, $10y+x$ 10. Brother's age= $(12-x)$yrs, elder sister's age= $(12+y)$yrs; in the second case brother's age= $(12-x+y)$yrs, elder sister's age= $(12+2y)$yrs.

Exercise 2

1. 21 2. -7 3. 10 4. 1 5. 7 6. 22 7. $\frac{4}{5}$

8. 0 9. $15\frac{1}{6}$ 10. 2

Exercise 3

1. (i) Loss of Rs.15 (ii) 20 km above (iii) 10 metres to the south (iv) 10^{o}C decrease in temperature

2. (i) $-$ Rs. 10, profit (ii) 5 km, north (iii) -15 km, to go down (vi) 5°, increase

3.(i) $-8, -3, 0, 1, 4$ (ii) $-9, -7, -5, -1, 0$
(iii) $-4, -2, 3, 6, 8$ (iv) $-5, -3, -1, 1$
(v) $\frac{5}{2}, -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{9}{4}$ (vi) $-4, -1, -\frac{2}{3}, \frac{3}{2}, \frac{5}{2}, 3$

4. (i) 14 (ii) 34 5. No; 10, 6

6. (i) $b-3a$ (ii) 2 (iii) $2a$ (iv) $1-x$

9. Rs. $(2a+bc-2b)$

10. (i) $b-c$ (ii) $y-z$ (iii) $b-c+d$

Exercise 4

1. (i) $a^2+2b^2+3c^2$ (ii) 0 (iii) $9a^2-2a+8$

2. (i) $2x+2y+4z$ (ii) $a+b+c$ (iii) $7q^2$

3.. (i) $a-4b$ (ii) $4x^2-y^2+5z^2$ (iii) $3m^2-3mn$

4. (i) $-16x^2y^2z^2$ (ii) $-8a^4b^4$ (iii) $-a^6b^2c^4$

5 (i) $3x^3-2x^2-9x$ (ii) b^4-c^4 (iii) x^2z-y^2z

6. (i) 0 (ii) 0

7. $al^2+bm^2+cn^2-12-m^2-n^2+alm+bmn+cnl-lm-mn-nl$; 0

8. (i) $4a^2b-5ab^2+3$ (ii) $9xy-6y^4-5x^2y$

(iii) $3p^2qr-4pr-1$ (iv) $3n+4mn^2-5m^2n^3$

Exercise 5

1. (i) $16x^2+24xy+9y^2$ (ii) $64a^2-64ab+16b^2$

(iii) $x^2y^2+2xyc+c^2$ (iv) $16x^2-8xabc+a^2b^2c^2$

(v) $x^2+y^2+2xy+4x+4y+4$

(vi) $4a^2+9b^2+25c^2+12ab-30bc-20ca$

(vii) $x^2+y^2+z^2p^2+2xy+2yzp+2zxp$

(viii) $a^2+b^2+c^2+2ab-2bc-2ca-2a-2b+2c+1$

(ix) $1+4x+4y+6z+8xy+12yz+12zx+4x^2+4y^2+9z^2$

(x) $m^2+4n^2+9p^2+16q^2+4mn-6pm-12pn+16qn-24pq$

2. (i) $25a^2$ (ii) $4x^2$ 3.(i) $(4a+1)^2$ (ii) $(6x-7y)^2$

(iii) $\left(1+\frac{2}{x}\right)^2$ (iv) $\left(2x-\frac{3}{4y}\right)^2$

4. (i) 100 (ii) 9 (iii) 49 (iv) 289 5. 65

9. 624 10.(i) $16x$ (ii) $40ab$.

Exercise 6

1. (i) a^2-36 (ii) $36a^2-25b^2$ (iii) $a^2c^2-b^2d^2$

(iv) $16x^2y^2-25p^2q^2$ (v) $4m^2+n^2-p^2+4mn$

(vi) $a^2+4b^2-9c^2+4ab$ (vii) $4x^2-y^2+9z^2+12xz$

2. (i) 999996 (ii) 2249879 (iii) 302464

3. (i) $-12c(a+2b)$ (ii) $7x^2+21y^2+52xy+6zx+42yz$

(iii) 18128 (iv) 515420 4.(i) $(9a+7)(9a-7)$

(ii) $(11m+13n)(11m-13n)$ (iii) $(5x+3y)(x+5y)$

(iv) $(a+b+1)(a+b-1)$ (v) $(x-2y+3)(x-2y-3)$

5. (i) 500^2-483^2 (ii) 475^2-66^2

(iii) $\left(\frac{5a+4b}{2}\right)^2-\left(\frac{2b-a}{2}\right)^2$ (iv) $(3x)^2-(x+3y)^2$

6. (i) $(2+5a)(2-5a)$ (ii) $(xy+1)(xy-1)$

(iii) $(ab+cd)(ab-cd)$ (iv) $(5m+7n)(5m-7n)$

(v) $(a+b+4)(a+b-4)$ (vi) $(7x+y)(x+5y)$

(vii) $a(15x-14y)(x+14y)$ (viii) $4(x^2+y^2)xy$

(ix) $(a+b-c)(b+c-a)(a+b-c)(a-b+c)$

(x) $(x+y+z)(x+y-z)$ (xi) $(4x+2a-3b)(4x-2a+2b)$

(xii) $(a-b)(a+b-2c)$ (xiii) $(x-y)(x+y-2)$

(xiv) $(p+m-n)(p-m+n)$

(xv) $(a+b+c+d)(a-b-c+d)(a+b-c-d)(a-b+c-d)$

(xvi) $(c-a)(ab+bc+1)$ (xvii) $(1+a)(2-a)$

Exercise 7

1. 0 2. $\frac{3}{4}$ 3. -11 4. 4 5. -2 6. 42 7. 1 8. 15 9. 9 10. 7 11. 9 12. $-16\frac{1}{2}$ 13. 2 14. 4 15. -7 16. 41 17. 8 18. 12 yrs 19. $3x+9=14$ (quantity of rice is x kg). 5 kg 20. $3x+18=162$ (Sulema scored x marks); Nasifa 56, Sulema 48, Mandira 58.

Exercise 8

1. $6a^2+17ab+12b^2+19a+26b+10$

2. $3x^2-4y^2-5z^2+9yz+2zx-xy$

3. $15a^4-8a^3-3a^2+34a-24$

4. $10x^5-5x^4+21x^3-7x^2+11x-6$

5. x^8+x^4+1 6. $6k^4-11k^3-2k^2+4k+1$

7. $24a^5-46a^4+9a^3+13a^2-3a$

8. $15n^4-32m^3+50m^2-32m+15$

9. $-6x^4+7x^4-4x^3-2x^2+9x+2$

10. $x^6-x^5+4x^3+9x^2-25$ 11. $2a^4+a^3b+2ab^3+b^4$

12. $x^5+2x^4y+x^3y^2+x^2y^3+2xy^4+y^5$

13. $p^5-p^4q^2+p^3q^3+p^3+p^2q^3-p^2q^4-p^2q^2+q^3+q^5$

14. $a^5-3a^4b-2a^3b^2-4a^2b^3-b^5$

15. $2a^2x^4+abx^3+2acx^2+3b^2x^2-7bcx-4c^2$

16. $-6x^5+7x^4-4x^3-2x^2+9x+2$

17. $y^6-6y^5+15y^4-20y^3+15y^2-6y+1$

18. $x^3+y^3+z^3-3xyz$

19. $-2a^8+7a^7b+8a^6b^2-31a^5b^3-7a^4b^4+22a^3b^5-6^2b^6$

20. $a^3+2a^2-2a^2b-ab+2ab^2+2a-b^3+1$

21. $p^2x^5+psx^2-q^2x^3-2qrx^2-qsx-r^2x-rs$

22. $a^6b^5-2a^5b^4c+a^4b^3c^2-a^3b^2c^3 \quad ac^5$

23. a^4-b^4 24. x^6-y^6 25. $6x^3-x^2-31x-10$

26. $16x^4-28x^3y-16x^2y^2+43xy^3-15y^4$ 27. a^6-b^6

28. $a^4x^4+ab^3xy^3-a^3bx^3-b^4y^3$

29. $x^8+8x^4y^4-12x^2y^6+27y^8$

30. $2b^2c^2+2c^2a^2+2^2b^2-a^4-b^4-c^4$ 31. $a^8+a^4b^4+b^8$

32. $x^3y^3+3xy^3z^2-x^3yz^2+3x^2y^3z-x^2yz^3+y^3z^3$

33. $8x^6-19x^3y^3-27y^6$ 34. $6x^3-22x^2+32x-24$

35. $15a^4+7a^3-36a^2+76a-48$

36. $6x^5-4x^4-20x^3-7x^2-20x+10$

37. $apx^3+aqx^2+bpx^2+arx+bqx+br$ 38. $2x^3$

39. $2(lm+mn+nl)x+(lm+mn+nl)y+(l^2+m^2+n^2)y+lm+mn$
$+nl+l^2+m^2+n^2$

40. $2a^2b+ab^2+2b^3$

Exercise 9

1. $a+2$ 2. $x+2$ 3. $m+3$

4. $a+2$ 5. $x-4$ 6. $m-7$

7. $x-2$ 8. $a-8$ 9. $p+5$

10. $x+2$ 11. $3y-2$ 12. $2c-1$

13. $-3x+4$ 14. $-7m+3$ 15. $2a-3b$

16. $-6c-d$ 17. $3a \quad 4b$ 18. $3x^2y-5xy^2+4y^3$

20. $a^2c-2abc^2-3b^2c^3$ 21. $2a-1$. 22. $x-3$

23. $2a-7$ 24. $-a^2+a+1$ 25 m^2-m-2,

26. $27x^3+9x^2+3x+1$ 27. $2y+1$

28. $3p^2-2p+5$ 29. $3a^2-4a+5$ 30. $2c-d-3$

31. $3x-7y$ 32. $a-2b-3c$ 33. m^2-2n^2

34. $a+2b+3c$ 35. y^3-x-3 36. $a^2x-2abx^2-3b^2x^3$

Exercise 10

(Quotient $=$ Q, Remainder $=$ R)

1. $Q=4a+2$, $R=4$
2. $Q=2x-\frac{6}{5}$, $R=\frac{1}{5}$.
3. $Q=5a-14$, $R=35a-17$
4. $Q=x-4$, $R=x-1$
5. $Q=3x^2+2x-4$, $R=3x-4$
6. $Q=p^2-3p+2$, $R=2p+3$
7. $Q=3x^2-4x+5$, $R=2x+7$
8. $Q=x^3-7x+9$, $R=8x-15$
9. $Q=2a+3b$, $R=3b^3$
10. $Q=x^3+x-y$, $R=-y^3$
11. $Q=a^2-3ab+2b^2$, $R=-3b^3$
12. $Q=7x-6$, $R=10$
13. $Q=3a-5$, $R=12a$
14. $Q=2a^2+7a+3$, $R=0$
15. $Q=m^2+3m+2$, $R=0$
16. $Q=x^2-2x+3$, $R=-38x^2$
17. $Q=2a^3+4a^2+6a+3$, $R=0$
18. $Q=3x^3+6x^2+4x+2$, $R=0$.
19. $Q=a+4$, $R=0$
20. $Q=27x^3+9x^2+3x+1$, $R=1$
21. $Q=a-4$, $R=a-1$
22. $Q=3x^2+2x-4$, $R=3x-4$
23. $Q=a^3-7a+6$, $R=2a-8$
24. $Q=2a^3-4a^2+4a-2$, $R=0$
25. $R=r$
26. Divisor $=2x^4-x^3-2x^2+5x-1$

Exercise 11

1. $x^3+6x^2+12x+8$
2. $27a^3+108a^2+144a+64$
3. $64c^3+48c^2d+12cd^2+d^3$
4. $27m^3+54m^2n+36mn^2+8n^3$
5. $a^6+6a^4b+12a^2b^2+8b^3$
6. $a^3b^3+3a^2b^2cd+3abc^2d^2+c^3d^3$
7. $27a^3+54a^2bc+36ab^2c^2+8b^3c^3$

8. $x^6+3x^4yz+3x^2y^2z^2+y^3z^3$
9. $x^3+3x^2yz^2+3xy^2z^4+y^3z^6$
10. $p^6+3p^2q^4+3p^3q^4+q^6$
11. $a^3+b^3+8c^3+3a^2b+6a^2c+3ab^3+12ac^2+12abc+12bc^2+6b^2c$
12. $a^3+8b^3+27c^3+6a^3b+9a^2c+12ab^2+27ac^2+36abc+54bc^2+36b^2c$

13. 857375
14. 1331000
15. 1953125
16. 125000
17. 216000
18. 125
19. $216a^3$
20. $27x^3$
21. $64m^3$
22. $64x^3$
23. 1
24. 729
25. 512
26. -27

Exercise 12

1. $a^3-9a^2+27a-27$
2. $8x^3+12x^2y+6xy^2-y^3$
3. $27m^3-27m^2n+9mn^2-n^3$
4. $8p^3-36p^2q+54pq^2-27q^3$
5. $a^6-6a^4b+12a^2b^2-8b^3$
6. $a^3b^3-3a^2b^2cd+3abc^2d^2-c^3d^3$
7. $8x^3-36x^2bc+54xb^2c^2-27b^3c^3$
8. $a^6-3a^4bc+3a^2b^2c^2-b^3c^3$
9. $x^3-3x^2yz^2+3xy^2z^4-y^3z^6$
10. $m^6-3m^4n^2+3m^2n^4-n^6$
11. $1+3b-6c+3b^2+12c^2-12bc-6b^2c+12bc^2+b^3-8c^3$
12. $p^3-8q^3-27r^3-6p^2q+12pq^2-9p^2r-36q^2r+27pr^2-54qr^2+36pqr$

13. $a^3-3a^6+3a^3-1$
14. $8-12b^3+6b^6-b^9$
15. 912673
16. 26463592
17. 99700299
18. 15438249
19. 125
20. 27
21. 8
22. $-b^3$
23. $-27y^3$
24. $216q^3$
25. $m^3-3m^2n+3mn^2-n^3$
26. 1
27. 0
28. 0
29. 216

Exercise 13

1 72
2. 152
3. 98
4. 335
5. 52
6. 0
8. 234
11. 455
12. 1568.

Exercise 14

1. x^3+27 2. $1+27a^3$ 3. $8a^3+27b^3$
4. $m^3n^3+p^3$ 5. $a^3+b^3c^3$ 6. x^6+y^6
7. $64p^3+125q^3$ 8. $a^3+3a^2b+3ab^2+b^3+c^3$ 9. x^6+1
10. $x^3+2y^3+z^3+3x^2y+3xy^2+3y^2z+3yz^2$
11. $2a^3+9a^2+15a+9$ 12. $9x^3-9x^2+9x$

Exercise 15

1. x^3-27 2. $1-64x^3$ 3. $27a^3-125b^3$
4. $a^3b^3-c^3d^3$ 5. x^6-y^6 6. $a^6-8b^3c^3$
7. $64x^6-729y^6$ 8. a^6-64b^6 9. 0
10. 54 11. a^6-b^6 12. $1+8x^3$

Exercise 16

1. $(a+2)(a^2-2a+4)$ 2. $(2x+1)(4x^2-2x+1)$
3. $(a+3b)(a^2-3ab+9b^2)$ 4. $(2m+3n)(4m^2-6mn+9n^2)$
5. $(x+4y)(x^2-4xy+16y^2)$ 6. $(2p+7q)(4p^2-14pq+49q^2)$
7. $(ab+c)(a^2b^2-abc+c^2)$ 8. $3(4a+1)(16a^2-4a+1)$
9. $a(2m+3np)(4m^2-6mnp+9n^2p^2)$
10. $(7x+8y)(49x^2-56xy+64y^2)$
11. $(7abc+10mnp)(49a^2b^2c^2-70abcmnp+100m^2n^2p^2)$
12. $2a(2ax+3y)(4a^2x^2-6axy+9y^2)$ 13. $(1-2a)(1+2a+4a^2)$
14. $(4a-b)(16a^2+4ab+b^2)$ 15. $(5x-6y)(28x^2+30xy+36y^2)$
16. $m(m-2)(m^2+2m+4)$
17. $(a-b)(a+b)(a^2+ab+b^2)(a^2-ab+b^2)$
18. $(bc-2d)(b^2c^2+2bcd+4d^2)$
19. $(3m-4np)(9m^2+12mnp+16n^2p^2)$
20. $(6m-7n)(36m^2+42mn+49n^2)$
21. $(9abc-5)(81a^2b^2c^2+45abc+25)$
22. $(a-2b+c)(a^2+4b^2+c^2-4ab-ca+2bc)$
23. $(m-n+p)(m^2+n^2+p^2-2np+mn-mp)$
24. $2(a-b)(a^2+ab+b^2)(4a^6-2a^3b^3+b^6)$
25. $2(m+n)(4m^2+2mn+n^2)$ 26. $2q(3p^2+q^2)$
27. $(x+2)(x^2+x+1)$

28. $(2x+5y+3z)(4x^2+7y^3+9z^2+2xy+12yz-6zx)$
29. $(1+4x)(1+5x+7x^2)$
30. $(2-a+b)(4+a^2+b^2+2a-2b-2ab)$
31. $(2x-y+4)(4x^2+2xy+y^2-2x-5y+7)$
32. $n(m+n)(m+2n)$

Exercise 17

1. $(x+1)(x+3)$
2. $(x+2)(x+3)$
3. $(x+2)(x+5)$
4. $(a+1)(a+2)$
5. $(a+3)(a+4)$
6. $(a+4)(a+6)$
7. $(m+3)(m+8)$
8. $(p+6)(p+8)$
9. $(4+x)(15+x)$
10. $(x-3)(x-4)$
11. $(z-2)(z-7)$
12. $(x-3)(x-13)$
13. $(a-5)(a-7)$
14. $(m-5)(m-13)$
15. $(x-6)(x-9)$
16. $(a+7)(a-5)$
17. $(c+15)(c-3)$
18. $(x+9)(x-4)$
19. $(p+8)(p-2)$
20. $(x+8)(x-7)$
21. $(x-3)(x+2)$
22. $(a-7)(a+5)$
23. $(x-7)(x+3)$
24. $(x-7)(x+4)$
25. $(m-24)(m+3)$
26. $(a-11)(a+10)$
27. $(b-17)(b+3)$
28. $(x-13)(x+9)$
29. $(a-9)(a+7)$
30. $(x-7)(x-9)$
31. $(m+9)(m-8)$
32. $(12-a)(13-a)$
33. $(x+15)(x-7)$
34. $(p+13)(p-11)$
35. $(a-3)(a-20)$
36. $(1-9x)(1+4x)$
37. $(x+27)(x-3)$
38. $(a-18)(a+5)$
39. $(x+2)(x-51)$
40. $(5+x)(16-x)$
41. $(a-1)(a+1)(a^2+5)$
42. $(x-2)(x+2)(x^2-5)$
43. $(x^2+3)(x^2+3)$
44. $(x-2)(x^3-2)(x^2+2x+4)$
45. $(a-1)(a+2)(a^2+a+1)(a^2-2a+4)$
46. $(m^2-7)(m^2+5)$
47. $(a+4b)(a-3b)$
48. $(x-15y)(x+14y)$
49. $(m+7n)(m-5n)$
50. $(p-3q)(p-13q)$
51. $(x+24y)(x-4y)$
52. $(a+8b)(a-5b)$
53. $(x+y-3)(x+y+2)$
54. $(a+1)(a+1)(a^2+2a-2)$
55. $(x^2-2x+9)(x^2-2x-4)$
56. $(m^2+2m+3)(m-4)(m+6)$
57. $(p^2-3q-7)(p^2-3q-9)$
58. $(x-10)(x+2)(x-9)(x+1)$
59. $(x^2+10x+18)(x+8)(x+2)$
60. $(x+2)(x-5)(x^2-3x-6)$

Exercise 18

1. $(a+2)(2a+1)$ 2. $(x+5)(3x+5)$ 3. $(a+4)(3a+2)$
4. $(a+4)(2a+1)$ 5. $(m+2)(2m-3)$ 6. $(c+3)(2c+1)$
7. $(p+3)(2p-5)$ 8. $(2x+3)(3x-5)$ 9. $(a+1)(5a-9)$
10. $(x-1)(9x+8)$ 11. $(k+1)(3k-25)$ 12. $(x-1)(2x+1)$
13. $(2a-7)(5a+4)$ 14. $(2m-5n)(3m+2n)$
15. $(3x-1)(3x+11)$ 16. $(a+b)(5a-13b)$
17. $(4p-3q)(3p+8q)$ 18. $(a-5b)(5a-b)$
19. $(a+7b)(7a-b)$ 20. $(5x-4y)(2x+5y)$
21. $(a-b)(11a+13b)$ 22. $3(x+y)(x-3y)$
23. $3(p-3q)(3p+q)$ 24. $(a+8b)(8a-b)$
25. $(x-2y)(12x+5y)$ 26. $(3a-7)(7a-3)$
27. $(5-x)(5+3x)$ 28. $(5-a)(1+3a)$
29. $(3-2x)(4+3x)$ 30. $(3-2a)(2+3a)$
31. $(ab+3)(ab-13)$ 32. $(xy+8)(xy-9)$
33. $(a^2+1)(3a^2-5)$ 34. $(x-1)(x^2+x+1)(8x^3-9)$
35. $(m^2-11)(m^2+12)$ 36. $(a^3-3)(a^3+16)$
37. $(p^2+8)(p^2-19)$ 38. $(2a^3+9)(3a^3-5)$
39. $(a+b+1)(3a+3b+2)$ 40. $(2x-2y+3)(3x-3y-5)$
41. $(x^2+x+1)(3x^2+3x-25)$ 42. $(3x+2y-1)(6x+4y-3)$
43. $4(a+2b)(2b-a)$ 44. $(x^2+xy-y^2)(5x^2-13xy-5y^2)$
45. $(x+a)(ax+1)$ 46. $(x-a)\left(x-\frac{1}{a}\right)$

Exercise 19

1. b 2. xy 3. m^2n^2 4. $4a^2b^2$
5. $2pq^2$ 6. 7 7. $3ab^2c$ 8. $2xy$
9. $3xy^2z^3$ 10. $7m^2np^3$ 11. $5aby$ 12. $4a^3x^4y^5$
13. $2b$ 14. m 15. ab 16. $3a$
17. p^2 18. 5 19. $3x^2y^2z^2$ 20. $8a^2b^2x^3y^4$
21. $4mn$ 22. $8ac$ 23. $5c^3$ 24. $7p^2q^2r^3$
25. $7abc$ 26. $37a^5b^3c^2$ 27. $9a^2c^2$ 28. $15x^3y^3z^3d^3$

Exercise 20

1. $a(b+c)$ 2. $x+y$ 3. $a+b$ 4. $a+b$
5. $a-b$ 6. $c-d$ 7. $2x+1$ 8. $3a(b+c)$
9. $a-2b$ 10. $a-3b$ 11. $x-1$ 12. $ax(a-x)^2$
13. $a-b$ 14. $x-y$ 15. $a+1$ 16. $a-1$
17. $4(x+2)$ 18. $x(x-5)$ 19. $x+1$ 20. $c-1$
21. $p+9$ 22. $m+3$ 23. $x(x-7)$ 24. $a+3$
25. $x+2y$ 26. $a(a+2)$ 27. $8(x+a)$ 28. $9x^2(x+2a)$
29. $a+b-c$ 30. $x(x+4)$

Exercise 21

1. $a+1$ 2. $x+1$ 3. $2a-3$ 4. $x-3$
5. $a-2$ 6. x^2-3x+2 7. $a+5$ 8. x^2-2x+1
9. $3a-7$ 10. a^2-3a+7 11. y^2-y+1
12. a^2-3a+2 13. $x-2$ 14. a^2+6a+5
15. $x+3$ 16. $2x+1$ 17. $a+4$
18. x^2+3x+1 19. $3x-1$ 20. $a-2$

Exercise 22

1. $2a^3b$ 2. x^2y^2z 3. a^3b^3 4. $24m^3n^2p^3$
5. $15x^2yz^3$ 6. abc 7. $x^2y^2z^2$ 8. $6a^2b^2c^2$
9. $a^2b^2c^2$ 10. $45m^3n^2p^3$ 11. $24x^3y^3z^4$ 12. $78c^5d^4$
13. $81a^3b^2c^2x^2$ 14. $30x^2y^3z^4$ 15. $96a^4b^3c^5$
16. $420a^4x^4y^4$ 17. $36a^2bc^3$ 18. $72a^3b^3c^3x^3y^2z^4$
19. $36a^5b^3c^2x^2y^4z^5$ 20. $210a^5b^4c^2$

Exercise 23

1. $a^3(a+2)$ 2. $x(x^2-1)$ 3. $xy(x+y)$
4. $m(m^2-4)$ 5. $a^2(b+c)^2$ 6. $ac(a+b)$
7. $(x-1)^2(x^2+x+1)$ 8. $24(x-y)^2(x+y)(x^2+xy+y^2)$
9. $(a^4-b^4)(a^2+ab+b^2)$ 10. $x(x-1)^2(2x+1)$
11. $(a+1)(a-3)(a+4)$ 12. $(a-1)^2(a+1)(a-2)(a^2+1)$
13. $a^2(a+2)(a-3)(a-5)$

14. $72(x-2)(x+2)(x-5)(x^2-2x+4)$

15. $120xy(x^2-y^2)$

16. $(a+b)(a-2b)(a-3b)$

17. $12a^3b^2(a^2-b^2)(a^2-ab+b^2)$

18. $(a-b)(a+b)(3a-2b)(a^2+ab+b^2)$

19. $(x+y)(x-2y)(2x-y)$ 20. $(m+1)(m-2)(3m+2)$

21. $(a+2)(2a-1)(3a+1)$ 22. $12(x-2)(x-3)(x+4)(x+5)$

23. $a^2(a-1)(a-2)(a+3)$ 24. $9(a+2x)(a\quad 2x)(2a-x)^2$

25. $180a^3x^3(a+x)^3(a-x)^3$

26. $x^3y(x+y)(x-y)^3(x^2+xy+y^2)(x^2-xy+y^2)$

27. $a^2b^2(b-2)(b+2)(b^2-2b+4)(b^2+2b+4)$

28. $12a^2(a-3x)^2(a+3x)^2$ 29. $a^2(a+2)^2(2a-1)(2a-7)$

30. $a^2(a-3b)(3a+4b)^2(a^2+3ab+9b^2)$

Exercise 24

1. $\frac{b^2}{3a}$ 2. $\frac{2p^4}{3}$ 3. $\frac{3a}{7cd}$ 4. $\frac{13ay}{3xz}$

5. $\frac{4}{3}x^2y^2z^2$ 6. $\frac{2mn^3}{3p^4}$ 7. $\frac{2bc^2}{5a}$ 8. $\frac{3yz^2}{2x}$

9. $\frac{c}{a}$ 10. $\frac{a}{2b^2}$ 11. $\frac{z^3}{5x^2}$ 12. 1

13. $\frac{x^3-y^3+z^3}{xyz}$ 14. 0 15. 0 16. $\frac{x}{y}$

17. $\frac{1}{4c}$ 18. $\frac{a+2b}{a}$ 19. $\frac{m+2n}{m(m-2n)}$ 20. $\frac{5p}{p-5}$

21. $\frac{a+1}{a}$ 22. $\frac{3x}{4a^2}$ 23. $x+y$ 24. $\frac{1}{2(a+1)}$

25. $\frac{m(3m+4n)}{m+4}$ 26. $\frac{a^2+ab+b^2}{a}$ 27. $\frac{1}{x-3}$ 28. $\frac{x+3y}{x+y}$

29. 1 30. $\frac{p(p^2-pq+q^2)}{p^2-q^2}$ 31. $\frac{m}{m+3}$ 32. $\frac{x^2-1}{x^2-4}$

33. $\dfrac{a^2+2a+4}{2a^2}$ 34. 1 35. $x(2+x)$ 36. 1

37. $\dfrac{1}{2x(x+y)}$ 38. $\dfrac{x+a}{x+y}$.

Exercise 25

1. -2 2. 3 3. -3 4. 13 5. 2 6. 2 7. 4
8. $\frac{1}{3}$ 9. 6 10. -6 11. 9 12. 15 13. 6
14. $2\frac{1}{2}$ 15. 15 16. 7 17. $-2\frac{5}{8}$ 18. 4 19. 2
20. 8.

Exercise 26

1. -6 2. 7 3 $\frac{1}{10}$ 4. $-\frac{3}{29}$ 5. 2 6. 1 7. 19
8. $2\frac{2}{5}$ 9. $\frac{1}{4}$ 10. 0 11. 1 12. 5 13. 9 14. $-\frac{7}{5}$
15. $-\frac{17}{22}$ 16. -1 17. -2 18. $4\frac{7}{23}$ 19. 6 20. $-\frac{3}{2}$
21. -9 22. 13 23. 4 24. 6 25. 7 26. 8
27. $\frac{7}{12}$ 28. $-1\frac{3}{10}$ 29. $-\frac{3}{4}$ 30. $-\frac{3}{2}$ 31. 4 32. 5
33. $-\dfrac{a+b}{2}$ 34. c 35. $\dfrac{a+b}{2}$ 36. $\dfrac{a+b}{2}$.

GEOMETRY

Chapter 1

Revision on previous lessons

Exercise 1

1. Some possible answers of the following questions are given in brackets. Choose right answer and write it. Draw the diagram where possible in support of your answer :

(i) How many points are there in a plane ?
(one, two, many)

(ii) How many lines are there in a plane ?
(one, two, many)

(iii) How many points are there in a straight line ?
(one, two, many)

(iv) How many straight lines can be drawn through a point ? (one, two, many)

(v) What is the name by which we call the points lying on a line ? (collinear, concurrent)

(vi) What is the name by which we call the straight lines passing through a point ? (collinear, concurrent)

(vii) What is the minimum number of points required for drawing a definite straight line ? (one, two, more than two)

(viii) How do two straight lines lie on a plane ?
(intersecting or parallel, intersecting and parallel, non-intersecting and non-parallel)

(ix) In how many points do two straight lines in a plane intersect each other ? (one, two, many)

(x) What do you say about the nature of two straight lines on a plane if they do not intersect ?
(intersecting, prependicular, parallel).

2. Show the following by drawing a circle :

(i) radius, (ii) centre, (iii) diameter, (iv) chord, (v) arc of the circle.

3. Draw the following figures, name them and state the names of the vertices and sides in each case :

(i) angle, (ii) triangle, (iii) quadrilateral.

4. Give definitions of the plane figures having the following names. Show with the help of figures the properties obtained from definitions :

(i) isosceles triangle, (ii) equilateral triangle, (iii) scalene triangle, (iv) acute-angled triangle, (v) right-angled triangle, (vi) obtuse-angled triangle, (vi) parallelogram, (viii) rectangle (ix) square, (x) rhombus, (xii) trapezium.

5. (a) How many classifications are there according to the magnitude of angles ? What are these ?
[magnitude of the angles are greater than 0°, but less than 180°.]

(b) Which of the following angles are in which class ?

(i) 30°, (ii) 90°, (iii) 175°, (iv) 89°, (v) 180°, (vi) 91°.

6. Which properties of a figure remain unchanged under reflection, translation and rotation ?

7. Name the geometric transformation which is applicable in case of each of the following properties :

(i) Only one point fixed, (ii) some points fixed, (iii) some straight lines fixed, (iv) some points fixed and some straight lines fixed, (v) none of the points fixed.

8. Complete the following table :

name of the transformation	name(s) of fixed point(s)	name(s) of fixed straight line(s)
Reflection		
Translation		
Rotation		

9. What is (are) the geometric transformation (transformations) under which the transformed figure (image) of a straight line remains the same straignt line or its parallel straight line ?

10. When do we call two plane figures congruent ?

11. Answer the following questions with the help of the properties of figures already known to you :

(i) When are two straight lines parallel to each other ?

(ii) Under what conditions are two triangles congruent ?

(iii) Two chords of a circle are given to be equal. What can you conclude from this ?

(iv) Two arcs of a circle are given to be equal. What can you conclude from this ?

(v) In a circle two arcs subtend equal angles at the centre. What can you conclude from this ?

Exercise 2

1. Draw a line segment AB and produce AB to D and BA to E.

2. From the straight line AB cut off line segments AX $= 3{\cdot}2$ cm, XY $= 2{\cdot}6$ cm, YZ $= 1{\cdot}9$ cm in succession, on one side of the straight line.

3. Draw a circle with radius 3 cm.

4. Draw a line segment PQ of length $8{\cdot}1$ cm. Draw a circle with PQ as diameter.

5. Divide the line segment of length $6{\cdot}4$ cm into four equal parts.

6. Draw two perpendiculars at the two end points of a line segment of length $8{\cdot}2$ cm.

7. Draw a traingle having sides of lengths 8 cm, 6 cm, 10 cm and draw the prependiculars from the vertices on the opposite sides. Mention the characteristics you observe ?

8. ABC is a triangle whose sides are AB $= 5$ cm, BC $= 7$ cm and included angle $\angle ABC = 55^\circ$; Draw the triangle and bisect the angles of the triangle.

9. Draw a rhombus each of whose sides is of length $4{\cdot}8$ cm and one angle is 60°. Draw any diagonal and also draw its perpendicular bisector. Mention the characteristics you observe ?

10. ABCD is a quadrilateral whose sides are AB $= 3$ cm, BC $= 4$ cm, CD $= 3{\cdot}8$ cm, DA $= 4{\cdot}3$ cm and $\angle ABC = 90^\circ$; Draw the quadrilateral.

Chapter 2

List of properties previously known and a few more definitions

2·1. We have come to know about some geometric properties by drawing figures and measuring line segments, angles etc. in class VI and with the help of transformation (reflection, translation, rotation) in class VII. Now we shall know about few more geometrical properties of some plane geometical figures and we shall establish (prove) them with reasons using previously known geometrtcal properties. The method of proving or of establishing in this manner was first introduced by Greek Mathematicians, particularly Euclid. This was as follows ; some initial statements are made which can be understood easily from experience or intuition. These are called axioms. Some definitions are also given. After this, using reasoning step by step we reach to a conclusion. These are called formal geometrical proofs. We prove essentially the properties of euclidean geometry. This is why we call the geometry which we study as euclidean geometry.

2·2. I ist of previously known properties :

Before we proceed to establish new properties let us recall the previously known properties.

1. A straight line can always be drawn through any point. In figure 1, straight line AB is drawn through a point O.

fig. No. 1

You know that one can draw really many such straight lines (fig. 2)

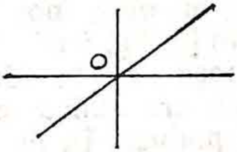


Fig No. 2

2. Trough two points there can be drawn one and only one straight line. In fig. 3, only one straight line is drawn through the points A, B.

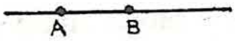


Fig. No. 3

3. A straight line is extended indefinitely on both the sides (fig. 4).

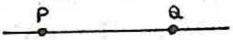


Fig. No. 4

4. Any line segment is bounded on both the sides. In fig. 5, the two end points of the line segment PQ are P and Q.

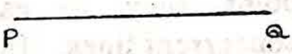


Fig. No. 5

5. A line segment can be extended in any direction as required. In fig 6(i), QP is extended to R and in 6(ii), PQ is extended to S.

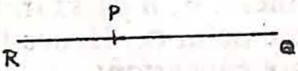


Fig. No. 6(i)

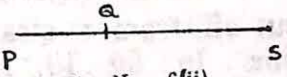


Fig. No. 6(ii)

6. Two straight lines in a plane will intersect at a point or will be parallel.

In fig. 7(i), straight lines AB and CD intersect each other at the point O and in fig. 7(ii) straight lines PQ and RS are parallel.

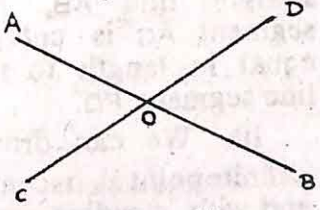


Fig. No. 7(i)

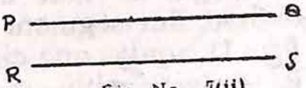
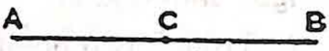


Fig. No. 7(ii)

7. A straight line may be drawn through more than two points [in fig. 8(i)] or may not be drawn [in fig. 8(ii)] If it is possible to draw. points are called collinear points. In fig. 8(i), the three points A, C, B are collinear.



8(i)

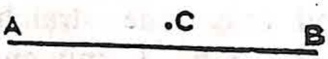


Fig. No. 8(ii)

8. More than two straight lines may pass through the same point [fig. 9(i)], or may not pass [fig. 9(ii)]. If they pass through the same point, they are called concurrent lines. In fig. 9(i), the three straight lines *l*, *m*, *n* pass through the point O. Hence they are concurrent.

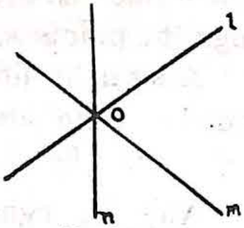


Fig. No. 9(i)

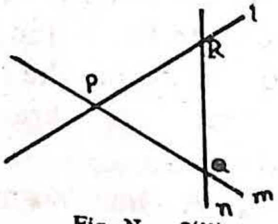


Fig. No. 9(ii)

9. A line segment equal to any length can be cut off from a straight line. In fig. 10, from straight line AB, the segment AC is cut off equal in length to the line segment PQ.

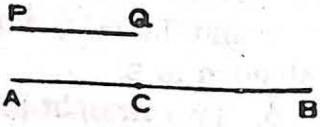


Fig. No. 10

10. We can draw only one circle with one definite point as its centre and with a radius equal in length to that of a definite line segment. In fig. 11, only one circle is drawn with O as centre and radius equal to the line segment *a*.

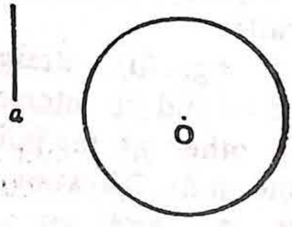


Fig. No. 11

11. **Congruence :** A geometrical figure and its transformed figure under one or more transformations are congruent.

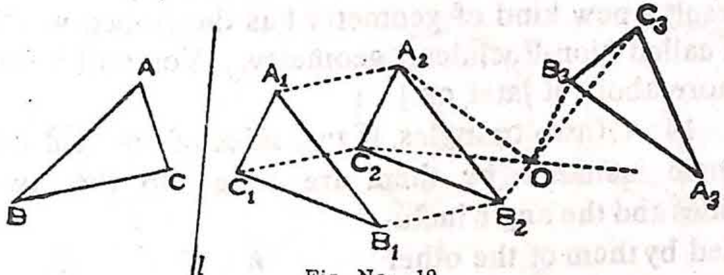


Fig. No. 12

In fig. 12, the triangle ABC is transformed to triangle $A_1B_1C_1$ under a reflection, then to $A_2B_2C_2$ under a translation and finally transformed to the triangle $A_3B_3C_3$ under a rotation.

[**Remark :** We get this idea of congruence through transformation. This idea contains in the Fuclidean geometry (transformation geometry). You will know more about it later on.]

12. When a straight line cuts two other straight lines if a pair of corresponding angles be equal, then the two straight lines are parallel. In fig. 13, the corresponding angles $\angle EGB$ and $\angle GHD$ formed by the intersections of the straight lines AB and CD with EF are equal. Hence AB $\parallel$ CD.

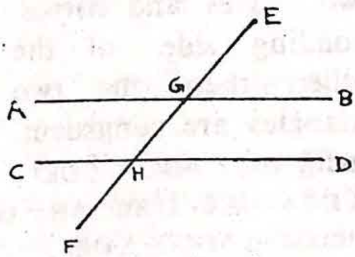


Fig. No. 13

13. None of the two intersecting straight lines can be parallel to a third line.

In fig. 14, AB and CD are two intersecting straight lines. if EF is parallel to AB, then CD will not be parallel to EF.

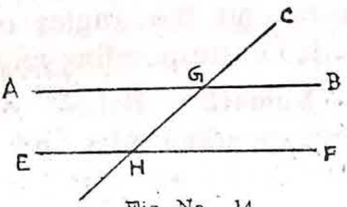


Fig. No. 14

[Remark : This axiom is called the playfair's axiom. This axiom is one form of Euclid's fifth postulate. Many Geometers did not accept it ; as a result a new kind of geometry has developed which is called Non-Euclidean geometry. You will know more about it later on.]

14. Of two triangles, if two sides of one and the angle included by them are equal to the two sides and the angle included by them of the other then the two triangles are congruent. In fig. 15, in $\triangle ABC$ and $\triangle DEF$ $AB = DE$, $AC = DF$ and $\angle ABC = \angle EDF$. Hence $\triangle ABC \cong \triangle DEF$.

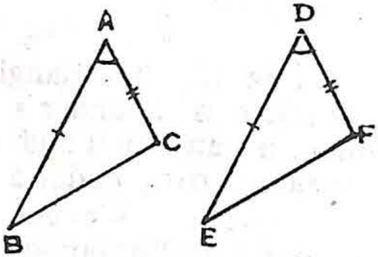


Fig. No. 15

15. Of two triangles, if two angles of one and one side opposite to one angle are equal to the two angles and corresponding side of the other, then the two triangles are congruent. In fig 16, $\angle ABC = \angle DEF$. $\angle BCA = \angle EFD$ and $AB = DE$. Hence $\triangle ABC \cong \triangle DEF$.

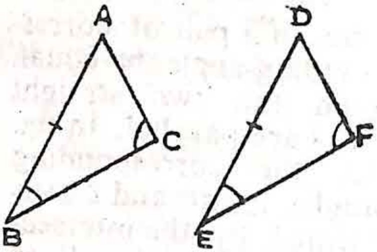


Fig. No. 16

Definition : In the case of triangles, the sides opposite to equal angles are called corresponding sides and the angles opposite to equal sides are called corresponding angles.

Remark : If two triangles be congruent then correspouding sides and corresponding angles are equal and areas of two triangles are also equal.]

16. In the same circle (or in two congruent circles) equal chords cut off equal arcs and subtend equal angles at the centre.

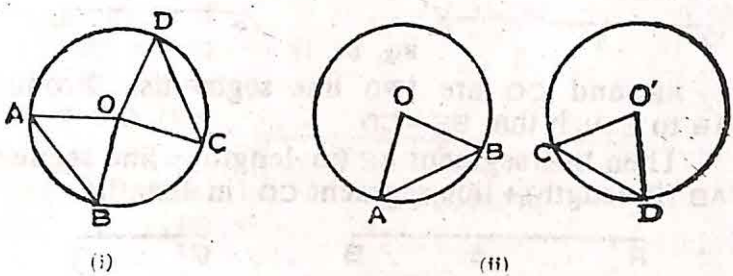


Fig. No. 17

In fig 17(i), in circle with O as centre, chord AB = chord CD : hence arc AB = arc CD and ∠AOB = ∠COD. Again in fig. 17(ii), in two congruent circles with O and O′ as centres, arc AB = arc CD, hence chord AB = chord CD and ∠AOB = ∠CO′D.

17. In the same circle (or in two congruent circles) the two chords that cut off equal arcs are equal and subtend equal angles at the centre. In fig 17(i) it arc AB = arc CD, then chord AB = chord CD and ∠AOB = ∠COD.

Again, in fig. 17(ii) if arc AB = arc CD, the chord AB = chord CD and ∠AOB = ∠CO′D.

18. In the same circle (or in two congruent circles) if two chords subtend equal angles at the centre then they are equal and the two arcs they cut off are also equal in fig 17(i), if ∠AOB = ∠COD, then chord AB = chord CD and arc AB = arc CD.

Again in fig. 17(ii) if ∠AOB = ∠CO′D then chord AB = chord CD and arc AB = arc CD.

[Note that two circles having radii equal in length are congruent.]

2·3. In subsequent chapters we need apply some results for establishing some geometrical properties. We discuss them below with the help of figures :

(i) Addition and subtraction of line segments :

Fig. No. 18

AB and CD are two line segments. Produce AB to E such that $BE = CD$.

Then line segment AE (in length) = line segment AB (in length) + line segment CD (in length)

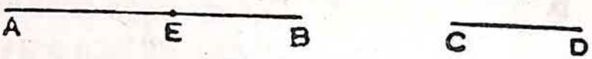


Fig. No. 19

AB and CD are two line segments. From BA, cut off BE equal to CD.

Then line segment AE (in length) = line segment AB (in length) − line segment CD (in length).

(ii) Addition and subtraction of an angle :

ABC and DEF are two angles. Let us draw an angle equal to $\angle DEF$ with B as vertex and BC as a side such that the other side say BG is on that side of BC which is opposite to BA.

Then $\angle ABG = \angle ABC + \angle DEF$

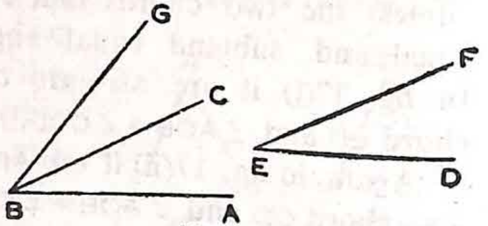


Fig. No. 20

ABC and DEF are two angles. Let us draw an angle $\angle CBG$ equal to $\angle DEF$ with B as vertex such that side BG and BA are on the same side of BC. Then $\angle ABG = \angle ABC - \angle DEF$

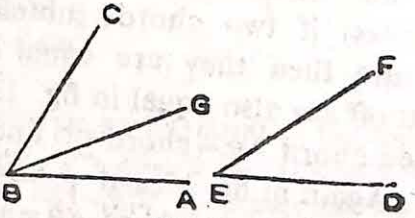


Fig. No. 21

[Remark : If we do not consider the direction then the addition of two line segments can be obtained even if we produce BA to E, so that AE is equal to CD (fig. 22).

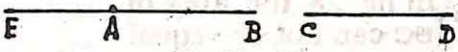


Fig. No. 22

Again the difference of two segments can be obtained by cutting off AE equal to CD from AB (fig. 23).

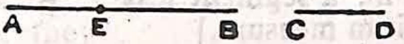


Fig. No. 23

If the orientation is not considered in the case of angles, the addition of two angles (as shown in fig 24) and difference of two angles also could have been obtained by drawing figures as in Fig. 25.]

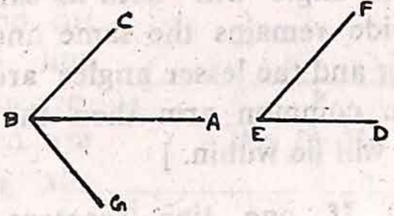


Fig. No. 24

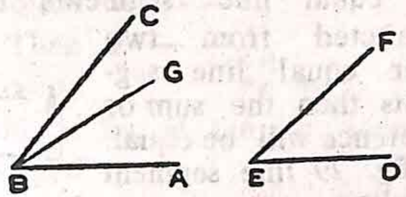


Fig. No. 25

(iii) Any segment and part of that segment are not equal In fig. 26 segment PQ and segment PR are not equal.

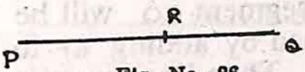


Fig. No. 26

(iv) Any angle and part of that angle are not equal. In fig. 27 ∠ABC and ∠ABD are not equal.

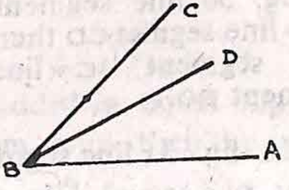


Fig. No. 27

The area of any triangle cannot be equal to the area of part of that.

In fig. 28, the area of ΔABC can not be equal to the area of ΔBDC.

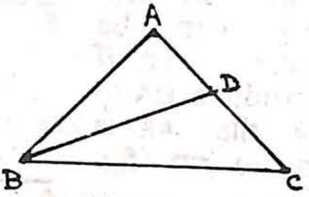


Fig. No. 28

[Remark : (1) We can cut off from a line segment, a segment less than it in measure.]

(2) If we draw an angle less in measure than another angle with both as same vertex such that one side remains the same and other sides of the greater and the lesser angles are on the same side of the common arm then the arm of the lesser angle will lie within.]

(v) If one line segment be added to or Subtracted from two equal line segments or two equal line segments be added to or subtracted from two other equal line segments then the sum or difference will be equal. In fig. 29 line segment AB = line segment CD. The line segment AP and line segment CQ will be formed by adding EF to both. Then line segment AP = line segment CQ. In fig. 30, line segment AB = line segmet CD then line segment AC = line segment BD.

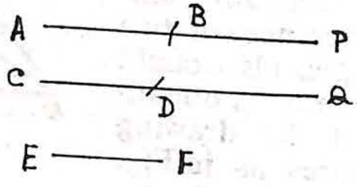
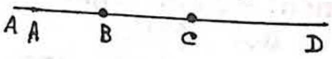


Fig. No. 29



FiS. No. 30

In fig. 31 line segment EF has been subtracted from two equal line segments AB and CD. As a resutlt, two line segments AP and CQ are formed. Then two line segments AP and CQ are equal.

In fig. 32, the line segments AC and BD are formed by subtracting line segment CB from two equal line segments AB and CD, then they will be equal.

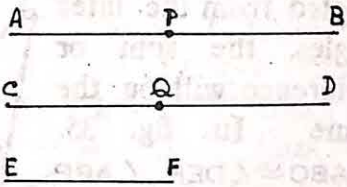


Fig. No. 31

In fig. 33, AB and CD are two equal line segments and EF and GH other two equal line segments. The line segments AP and CQ are formed by adding the line segment EF to the line segment AB and the line segment GH to the line segment CD. Then line segment AP = line segment CQ. Again, in fig. 34, the line segments AP and CQ are formed by subtracting two equal line segments EF and GH from two equal line segments AB and CD respectively Then line segment AP = line segment CQ.

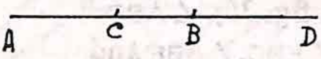


Fig. No. 32

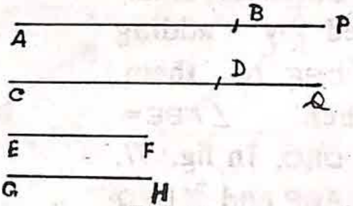


Fig. No. 33

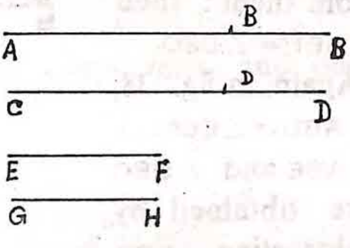


Fig. No. 34

(vi) If one angle is added to equal angles or one angle is subtracted from them then the sum or difference will be equal.

If two equal angles be added to another two equal angles or the former angles be subtracted from the later angles, the sum or difference will be the same In fig. 35, ∠ABC = ∠DEF. ∠ABP and ∠DEQ are obtained by adding ∠XYZ to them ; then ∠ABP = ∠DEQ.

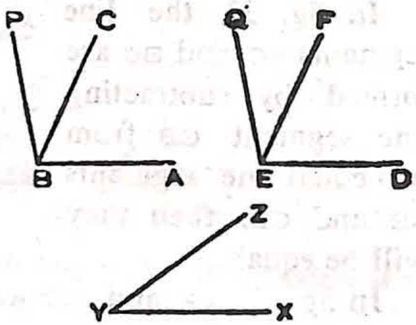


Fig No. 35

In fig. 36, ∠ABD = ∠EBC. ∠ABE and ∠DBC are obtained by adding ∠DBE to them. Then, ∠ABE = ∠DBC. In fig. 37, ∠ABP and ∠DEQ are obtained by subtracting ∠XYZ from them : then ∠AEP = ∠DEQ.

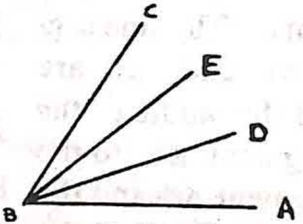


Fig No. 36

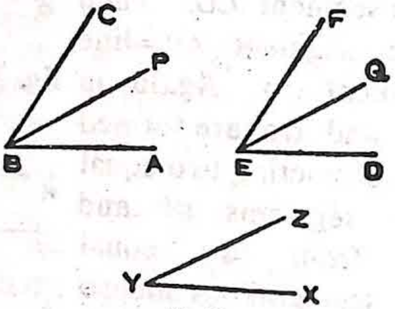


Fig No. 37

Again, in fig. 38, ∠ABD = ∠EBC. ∠ABE and ∠DBC are obtained by subtracting ∠EBD from them : then ∠ABE = ∠DBC.

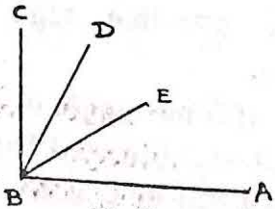


Fig No. 38

In fig. 39, ∠ABC = ∠DEF and ∠PQR = ∠XYZ.

$\angle ABS$ is obtained by adding $\angle PQR$ to $\angle ABC$ and $\angle DET$ is obtained by adding $\angle XYZ$ to $\angle DEF$, then $\angle ABS = \angle DET$.

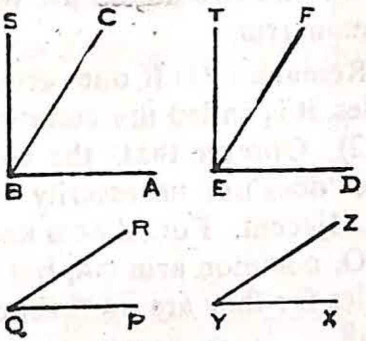


Fig No. 39

Again in fig. 40 $\angle ABC = \angle DEF$ and $\angle PQR = \angle XYZ$.

$\angle SBC$ and $\angle TEF$ are obtained by subtracting $\angle PQR$ and $\angle XYZ$ from $\angle ABC$ and $\angle DEF$ respectively. Then, $\angle SBC = \angle TEF$.

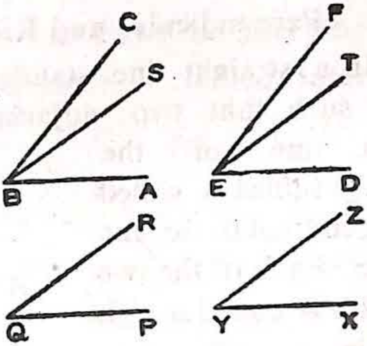


Fig No. 40

2·4 Few more definition :

You have already become familiar with few geometrical figures and you know some formal definitions of some figures. Here definitions of some more figures are given below

Adjacent angles :

If two angles have the same vertex and one common arm and if the two angles are on opposite sides of the common arm then the two angles are called adjacent angles.

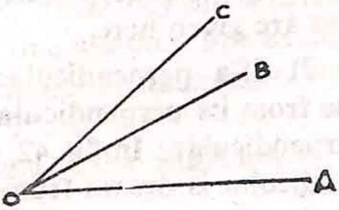


Fig No. 41

In fig 41. $\angle AOB$ and $\angle BOC$ are adjacent angles, because they have same vertex O a common arm OB and two angles are on opposite sides of the common arm.

[Remark : (1) If one arm belongs to both the angles, it is called the common arm.

(2) Observe that, the common arm with same vertex does not necessarily imply that the angles are adjacent. For, ∠AOB and ∠AOC have the vertex O, common arm OA, but they are not adjacent angles for they are on thesame side of the common arm.]

2. Perpendicular and Right angle.

If a straight line stands on another straight line such that two adjacent angles are equal then one of the straight lines is called perpendicular to the other. Each of the two angles is called a right angle.

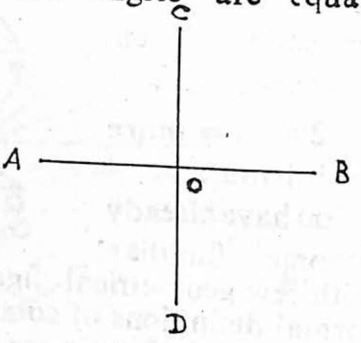


Fig No. 42

In fig. 42 CD ⊥ AB
∠AOC = ∠BOC =1 right angle.

[You have known previously, 1 right angle=90°]

[Remark : (1) You are already acquainted with right angle and perpendicular. Their formal definitions are given here.

(2) If a perpendicular is drawn on a straight line from its perpendicular is called the foot of the perpendicular In fig. 42, O is the foot of the perpendicular is drawn from the point C. The length of he segment CO is called the perpendicular-length.]

3. External angle :

If the arm of an angle be extended in other direction, the angle thus formed is called the external angle.

In fig. 43 the arm B of ∠BAC is produced so that ∠CAD is formed. Hence ∠CAD is an external angle.

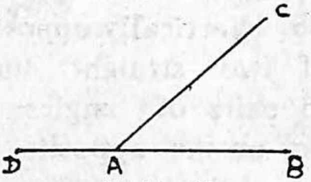


Fig No. 43

4. Internal bisector and External bisector :

The bisector of an angle is called internal bisector and bisector of its external angle is called external bisector.

In fig. 44, AE is the internal bisector and AF is the external bisector of ∠BAC.

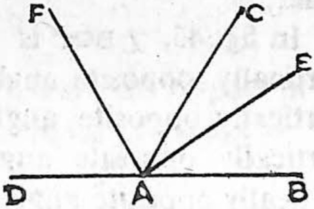


Fig No. 44

What is the magnitude ∠EAF ?

5. Complementary angles and supplementary angles.

Complementary angles : If the sum of two angles be one right angle, then they are said to be complementary angles of one another. For example 30° is complement of the angle 60°, the angle 81° is complement of the angle 9° etc.

[Remark : complements of the same angle are equal to each other.]

Supplementary angles : If the sum of two angles be two right angles they are said to be supplementary angles of one another. For example, the angle 135° is supplement of the angle 45°, the angle 85° is supplement of the angle 95° etc.

[Remark : supplements of the same angle are equal to each other]

6. Vertically opposite angles :

If two straight lines intersect each other, two pairs of angles form on the opposite sides of the interseting point. Then one of any pair of angles is said to be vertically opposite angle of the other.

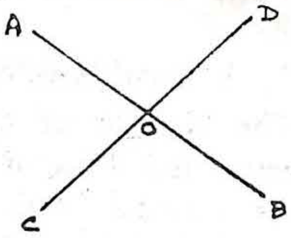


Fig No. 45

In fig. 45, ∠BOC is vertically opposite angle of ∠AOD or ∠ OD is vertically opposite angle of ∠BOC ; again ∠DOB is vertically opposite angle of ∠COA or ∠COA is vertically opposite angle of ∠DOB.

7. Transversal, Exterior angles, Interior angles, Interior opposite angles, Alternate angles :

Transversal : If a straight line cuts two straight lines, the former straight line is called transversal of later two straight lines.

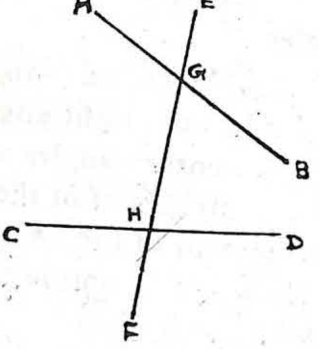


Fig No. 46

In fig. 46, the straight line EF is the transversal of two straight lines AB and CD.

Interior angles and Exterior angles :

If a straight line cuts two straight lines, eight angles are formed, four of them are in the inside region of the two straight lines. These four angles are called interior angles and the other four angles are called exterior angles.

In fig. 46, ∠HGB, ∠AGH, ∠GHC and ∠DHG are interior angles. Again, ∠BGE, ∠EGA, ∠CHF and ∠FHD are exterior angles.

Further observe that, ∠HGB is adjacent and ∠DHG is further off in respect of exterior angle ∠BGE. We shall call ∠HGB the adjacent interior angle and ∠DHG the further off interior angle.

Interior opposite angles :

The further off interior angle in respect of an exterior angle is called interior opposite angle.

In fig. 46, ∠DHG in respect of ∠BGE, ∠GHC in respect of ∠EGA, ∠AGH in respect of ∠CHF and ∠HGB in respect of ∠FHD are interior opposite angles.

Alternate angles :

The interior angle adjacent to one external angle and the interior angle adjacent to further off interior angle in respect of same external angle are called alternate angle to each other.

In fig. 46, ∠AGH is alternate angle of ∠DHG, etc.

[Remark : You have known earlier, which are called corresponding angles. Observe that, an exterior angle and its interior opposite angle are corresponding angles to each other.]

8. Exterior angles and interior opposite angles of a triangle :

If one side of a triangle be produced, the angle so formed is called the exterior angle of the triangle and each of the two interior angles which are not adjacent to the exterior angle is called interior opposite angles.

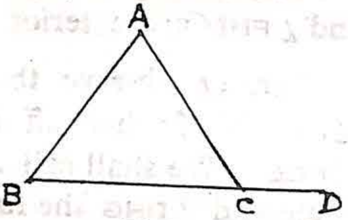


Fig No. 47

In fig. 47, the side BC of $\triangle$ABC is produced to D. In this case, $\angle$ACD is exterior angle, and $\angle$CAB and $\angle$ABC are interior opposite angles

9. Median :

The line segment formed by joining the middle point of any side of a triangle to the opposite vertex is called median of the triangle.

In fig. 48, AD is median of $\triangle$ABC.

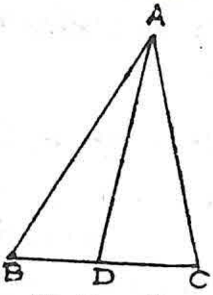


Fig. No. 48

[Remark : Three medians can be drawn in any triangle.]

10. Isosceles trapezium :

If two oblique sides of a trapezium be equal. then the trapezium is called isosceles trapezium.

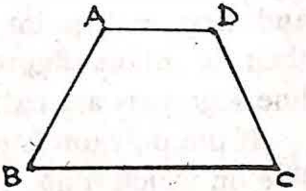


Fig. No. 49

In fig. 49, two oblique sides AB and DC of a trapezium ABCD are equal to each other. Hence the trapezium ABCD is an isosceles trapezium.

11. Polygon :

A closed plane figure formed by some line segments is called a polygon.

In fig. 50, ABCDE is a polygon. AB, BC, CD, DE, EA are the sides of the polygon.

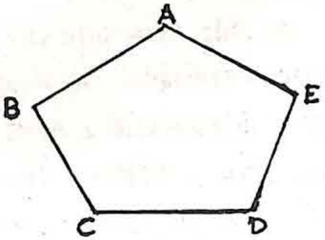


Fig. No. 50

A polygon consisting of three sides is called triangle, polygon consisting of four sides is called quadrilateral, polygon consisting of five sides is called pentagon and polygon consisting of six sides is called hexagon.

In a polygon, the line segments formed by joining the different vertices other than two adjacent vertices are called the diagonals.

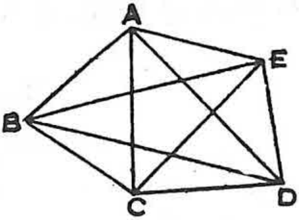


Fig. No. 51

In fig. 51, AC, AD, BD, BE, CE are diagonals of the polygon ABCDE.

[**Remark :** If a plane figure is formed by some line segments such that first end point of a segment is connected with the last end point of the previous segment and the last end point of the last segment and first end point of the first segment coincide then the plane figure is called a polygon. The line segments are called the sides of the polygon.

If the polygon is on the same side of the straight line on which one of its sides lies, it is called a convex polygon. By the term polygon we shall mean convex polygon.]

12. Regular polygon :

If each side and each angle of a polygon be equal, the polygon is called a regular polygon.

In fig 52, ABCDEF is a regular hexagon.

Its side AB = side BC = side CD = side DE = side EF = side FA and ∠ ABC = ∠BCD = ∠CDE = ∠ DEF = ∠EFA = ∠ FAB.

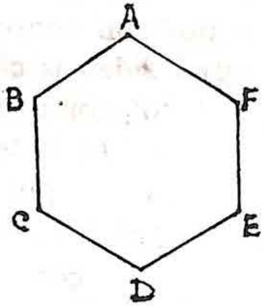


Fig. No. 52

[**Remark :** Regular polygon consisting of three sides is an equilateral triangle and regular polygon consisting of four sides is a square.]

Exercise 3

1. Write down definitions with figures :

(i) Adjacent angles, (ii) Perpendicular, (iii) vertically opposite angles, (iv) Median, (v) Isosceles trapezium, (vi) Polygon, (vii) Regular polygon.

2. Show or explain the followings by drawing figures :

(i) An external angle with respect to an angle. (ii) The exterior angle and interior opposite angles of a triangle. (iii) Transversal. (iv) Alternate angles. (v) Corresponding angles.

3. What do you mean by complementary angles ? Write down the measures of the complements of the following angles :

3°, 5°, 18°, 32°, 45°, 50°, 60°, 72°, 80°, 88°.

4. What do you mean by supplementary angles ? Find the measures of the supplements of the following angles :

10°, 30°, 45°, 60°, 90°, 110°, 143°, 155°, 167°.

5. (i) What is the measure of complement of an angle equal to its measure ?

(ii) What is the measure of supplement of an angle equal to its measure ?

(iii) What is the measure of complement of an angle equal to half of that angle ?

(iv) What is the measure of supplement of an angle equal to thrice that angle ?

6. Give sign '✓' against the correct sentence :

(i) The complementary angle of any angle must be acute.

(ii) The complementary angle of any angle must be obtuse.

(iii) The complementary angle of any angle may be acute, obtuse or right.

(iv) The supplementary angle of any angle must be acute.

(v) The supplementary angle of any angle must be obtuse.

(vi) The supplementary angle of any angle may be acute, obtuse or right.

7. An angle is given, how will you construct its complementary and supplementary angles ?

Chapter 3

Proofs of geometrical properties

3.1 Theorem : You have come to know that we adapt, to establish some geometrical properties, methods which are otherwise called proofs. What is to be proved, the method of the proof (including construction) and arriving at the conclusion—all the stages taken as a whole is what we call a theorem. A theorem has four parts. The statement which gives the geometrical property is called the general enunciation. For example, if two sides of a triangle be equal then their opposite angles are also equal ; this geometrical property along with the proof is a theorem. The statement is the general enunciation of that theorem. A part of general enunciation is what is given or the condition which is given ; for example, in this case this part is, 'The two sides of a triangle are equal'. The other part is what is to be proved, in other words, the property to be established for instance, in this case, 'their opposite angles are equal'.

The second part of the theorem is called the particular enunciation.

The figure is to be drawn following the general enunciation. The statement according to the naming of the figure is called the particular enunciation.

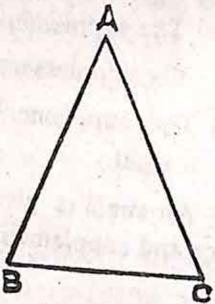


Fig No. 53

As mentioned earlier, the particular enunciation of the theorem will be :

Let us suppose, ABC is a triangle whose side AB = side AC (fig. 53).

It is to be proved that ∠ACB = ∠ABC.

The next part of it is called construction. To prove the theorem it may be necessary to have some more constructions in the figure This statement is called construction. You wlll find later on that for the above theorem it will be necessary to bisect ∠BAC (fig. 54).

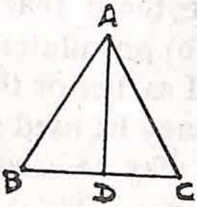


Fig. No. 54

[Remark : In proof of some theorem, construction may not be necessary.]

The final part is the proof, the statement by which the theorem is proved through a strict sequence of reasoning.

[Remarks : While proving the theorem the following points are to be kept in view.]

(i) Construction of the figure is always necessary (you should try to construct next figure from the very beginning)

(ii) The figure is to be drawn clearly with the pencil. The letters with which you name the figure are to be written in ink

(iii) There should be agreement between the names of the figures and the corresponding statements. You may be asked questions by naming the figure in a way different from what you may be used to in your book. This is why you should get into the habit of naming differently on your own in each theorem.

(iv) Make in the figure those constructions which are required for the purpose of the proof and their mention should be made.

(v) Whatever conclusions be made for the purpose of the proof the reasoning of those must be mentioned.

For these reasonings (a) the axioms, may be used, (b) postulates may be used, (c) the properties proved earlier or the properties depending only on them may be used and (d) either some or all of the above may be used.

Theorem 1

If a line segment stands on another straight line the sum of two adjacent angles so formed is equal to two right angles.

Let us suppose, the line segment DC stands on a straight line AB at the point D. As a result, the two adjacent angles $\angle BDC$ and $\angle CDA$ are formed.

It is to be proved that, $\angle BDC + \angle CDA = 2$ right angles.

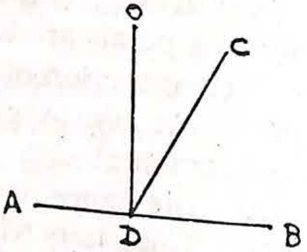


Fig. No. 55

Construction : Draw DO perpendicular to the straight line AB at a point D on it.

Proof : $\angle BDC + \angle CDA = \angle BDC + \angle CDO + \angle ODA$

$= \angle BDO + \angle ODA$

$= 1$ right angle $+ 1$ right angle [$\because OD \perp AB$]

= right angles.

[Remark : We know that if a perpendicular is drawn to a line at a point on it, we get two right angles. So perpendicular is drawn.]

Corollary 1. The internal and external bisectors of an angle are perpendicular to each other. BP and BQ are respectively internal and external bisectors of $\angle ABC$.

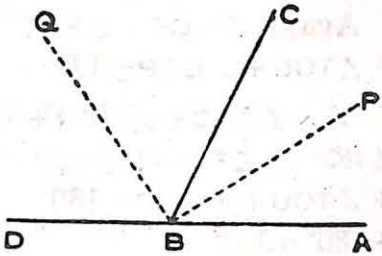


Fig. No. 56

$\therefore \angle PBC = \frac{1}{2} \angle ABC$

and $\angle CBQ = \frac{1}{2} \angle CBD$

$\therefore \angle PBC + \angle CBQ = \frac{1}{2} (\angle ABC + \angle CBD)$

or, $\angle PBQ = \frac{1}{2} \times 180° = 90°$

$\therefore PB \perp BQ$.

Corollary 2. The four angles formed by the intersection of two mutually intersecting straight lines are together equal to four right angles.

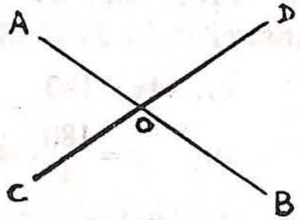


Fig. No. 57

AB and CD are two straigt lines intersecting at the point O.

$\because$ OC stands on the straight line AB, $\therefore \angle AOC + \angle COB =$ 2 right angles Again since OD stands on the straight line AB, $\therefore \angle BOD + \angle DOA = 2$ right angles.

$\therefore \angle AOC + \angle COB + \angle BOD + \angle DOA = 4$ right angles.

Corollary 3. The sum of the consecutive angles formed by some line segments emerged from the point O, is equal to four right angles.

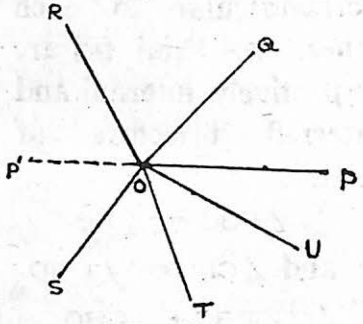


Fig. No. 58

Produce PO to P'

$\therefore \ \angle POQ + \angle QOR + \angle ROP' = 180^\circ$

Again $\angle P'OS + \angle SOT + \angle TOU + \angle UOP = 180^\circ$

$\therefore \ \angle POQ + \angle QOR + \angle ROP' + \angle P'OS + \angle SOT + \angle TOU + \angle UOP = 180^\circ + 180^\circ = 360^\circ$

$\therefore \ \angle POQ + \angle QOR + \angle ROS + \angle SOT + \angle TOU + \angle UOP = 360^\circ = 4$ right angles.

[**Corollary :** If some conclusions are arrived at easily from the conclusion of a theorem they are called corollaries.]

Some applications :

1. From fig. 59. find the value of x.
Answer : $x^\circ + 3x^\circ = 180^\circ$

or, $4x = 180$

or, $x = \frac{180}{4} = 45$

$\therefore$ the value of x is 45.

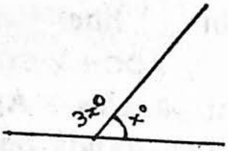


Fig. No. 59

2. In fig. 60, $\angle BEF = \angle ECD$; show that $\angle FEC = \angle DCA$

Proof : The line segment EF stands on the straight line AB i.e , CB.

$\therefore \angle BEF + \angle FEC = 180^\circ$

Again CD stands on the straight line AB i.e , AE.

$\therefore \angle ECD + \angle DCA = 180^\circ$

$\therefore \angle BEF + \angle FEC = \angle ECD + \angle DCA$

But $\angle BEF = \angle ECD$ (by assumption)

$\therefore \angle FEC = \angle DCA$.

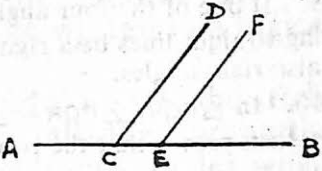


Fig. No. 60

Exercise 4

1. The straight line RS stands on the straight line PQ. If $\angle PRS = 42^\circ$, what is the measure of $\angle SRQ$?

2. A boy measured, by a protractor, the two angles formed by one line segment standing on a straight line and obtained the measures as $56{\cdot}2^\circ$ and $123{\cdot}3^\circ$.. What is the mistake in his measure as a whole ?

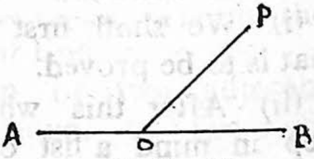


Fig No. 61

3. In fig. 61, if $\angle AOP$ be greater than $\angle BOP$ by 50° find the measures of each angle.

4. If one of the two angles formed by a line segment standing upon a straight line twice the other, what will be the magnitude of two angles ?

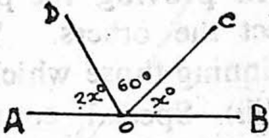


Fig No. 62

5. Find the value of x from the fig. 62.

6. Find the value of x from the fig. 63 ? What is the relation between AB and the bisector of $\angle DOC$?

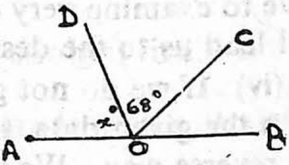


Fig No. 63

7. Prove that, not more than one perpendicular can be drawn to a straight line at a point on it.

8. In a circle with centre O. AB and AC are two equal chords. Prove that, AO produced will bisect $\angle BOC$.

9. If one of the four angles formed by two mutually intersecting straight lines be a right angle, prove that the other three are also right angles.

10. In $\triangle PQR$, $\angle PQR = \angle PRQ$. If QR be produced on both sides, then prove that the two external angles formed are equal.

11. In $\triangle XYZ$, $\angle XYZ = \angle XZY$; If XY and XZ be produced prove that two external angles formed are equal.

[Applications : To establish some more geometrical properties we shall apply the geometrical properties already we have (axioms or postulates) or we will obtain later (theorem). To establish these additional properties we shall proceed as follows :

(i) We shall first analyse what are given and what is to be proved.

(ii) After this whe have to prepare a list or keep in mind a list of properties we get from the given data. We shall consider those that will help us in proving the property under discussion and reject the others. We are to choose from the beginning those which will be necessary for us.

(iii) Special constructions are to be made if necessary. Before we proceed for constructions, we have to examine very carefully which construction will lead us to the desired objective.

(iv) If we do not get easily the method of proof from the given data, then we shall often proceed in the reverse way. We shall think how to get at what is to be proved. Now we shall again examine what will give us that property which is required to be proved. We shall proceed in this way till we get at the given data. If we now proceed again in the reverse way, the property can be proved.

(v) Having understood the method of the proof, all these steps are to be written in a logical sequence.]

[Remark : (1) In order to apply the right property at the right place, all the properties necessary for this and the conditions under which they are established are to be committed to memory.

(2) While establishing a property, you must keep in view that we do not use any such property with which we have not been acquainted earlier or it has been necessary to use the property under discussion to establish it.]

Theorem 2

If the sum of two adjacent angles be equal to two right angles, the two exterior arms of the angles are in the same straight line.

Let us suppose, the sum of two adjacent angles ∠BDC and ∠CDA is equal to two right angles and the exterior arms of the two adjacent angles are DA and DB.

It is to be proved that DA and DB are in the same straight line. Let us suppose that DA and DB are not in the same straight line. BD is produced to E.

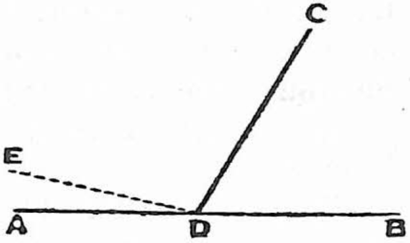


Fig. No. 64

∵ The line segment DC stands on the straight line BE,

∴ ∠BDC + ∠CDE = 2 right angles.

Again ∠BDC + ∠CDA = 2 right angles
(by hypothesis)

$\therefore \angle BDC + \angle CDE = \angle BDC + \angle CDA$

$\therefore \angle CDE = \angle CDA.$

Since one arm of these two equal angles is common and other two arms are in the same side, those two arms will coincide. That is the assumption that DA and DB are not in same straight line, is wrong.

$\therefore$ DA and DB will be in same straight line.

[Remark : Observe that, what is to be proved in this theorem was hypothesis for theorem 1 and what has been assumed here, had to be proved in theorem 1. Of the two theorems, the second is called the converse of the first.

Keep in mind that if a theorem holds, its converse does not necessarily hold. You have read in class seven about the congruence of two triangles. If two triangles are congruent, the three angles of one triangle must be equal to the three angles of the other. The converse theorem will be, of two triangles, if three angles of one are equal to the three angles of the other then the two triangles are congruent. But this theorem does not hold. Hence the truth of the converse theorem has to be proved.

Generally, the converse theorem is proved as follows :

We assume that the result to be proved is not true and with this assumption we arrive at an incorrect or absurd conclusion with thh help of the previous theorem. This means that what we have assumed, is not correct ; or in other words, what is to be proved, is correct. Such a proof is called an indirect proof.

Some applications :

1. The magnitude of two adjacent angles is 33°

and 147°. How do the exterior sides of those two angles lie ?

Answer : Since the sum of two angles $= 33^\circ + 147^\circ = 180^\circ$; hence two exterior arms lie on the same straight line.

2. The two bisectors of two adjacent angles $\angle AOB$ and $\angle BOC$ are prependicular to each other. Prove that three points A, O, C are collinear.

Let us suppose, OD and OE are respectively two bisectors of $\angle AOB$ and $\angle BOC$.

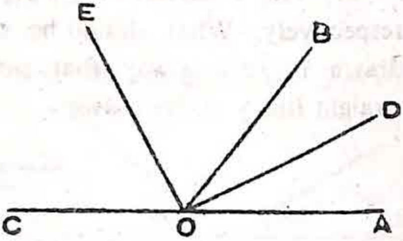


Fig. No. 65

$\therefore \angle AOB = 2\angle BOD$

and $\angle BOC = 2\angle BOE$

$\therefore \angle AOB + \angle BOC = 2(\angle BOD + \angle BOE)$

$2\angle EOD$

$\because OD \perp OE, \quad \therefore \angle EOD = 1$ right angle

$\therefore \angle AOB + \angle BOC = 2$ right angles

$\therefore$ OA and OC lie on the same straight line.

$\therefore$ three points A, O, C are collinear.

Exercise 5

1, The measure of two adjacent angles are 77° and 103°. How do the exterior sides of two adjacent angles lie ?

2. A boy drew to angles $\angle AOB = 105^\circ$ and $\angle BOC = 75^\circ$ by a protractor in such a way that OA and OC are in opposite side of OB. By producing AO he found that it did not coincide with OC. Did he draw the angles correctly ? Give reasons.

3. Four consecutive angles ∠AOB = 20°, ∠BOC = 55°, ∠CAD = 81° and ∠DQE = 240° are drawn at a point O in same direction. How do OA and OE lie ?

4. Prove Theorem 2 by drawing perpendicular on BD, not by producing BD in the fig. 64.

5. An angle of measure 55° is drawn by protractor, Now (without using protractor how) will you draw an angle of 125° with the help of a scale only ? Give reasons.

6. The measures of ∠POR and ∠ROS are 100 and 50° respectively. What should be the measure of an angle ∠SOT drawn in such a way that OP and OT will lie in the same straight line ? Give reasons.

Theorem 3

If two straight lines intersect each other, the vertically opposite angles will be equal.

Let us suppose that two straight lines AB and CD intersect at the point O

It is to be proved that,

(i) ∠AOD = vertically opposite ∠BOC and

(ii) ∠DOB = vertically opposite ∠COA.

Proof : Since the line segment OC stands on the straight line AB,

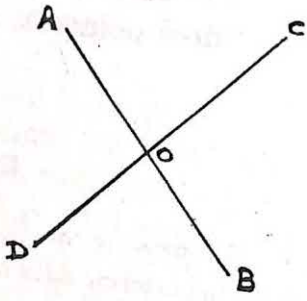


Fig. No. 66

∴ ∠BOC + ∠COA = two right angles. Again, since line segment OA stands on the straight line DC.

$\therefore\ \angle COA + \angle AOD =$ two right angles.

$\therefore\ \angle COA + \angle AOD = \angle BOC + COA$

$\therefore\ \angle AOD = \angle BOC$

similarly, it can be proved that

$\angle DOB = \angle COA$.

[Remark : You have read properties of rotation in class VII. This theorem can also be proved by using a rotation of magnitude $\angle BOA$ about the point O in the anti clock-wise direction. You can verify its truth.]

Some Application :

1. From the fig. 67, find the measure of $\angle FOD$.

Answer : $\angle AOC + \angle COE + \angle EOB = 180^\circ$

$\therefore\ 40^\circ + \angle COE + 30^\circ = 180^\circ$

$\therefore\ \angle COE = 180^\circ - (40^\circ + 30^\circ) = 180^\circ - 70^\circ = 110^\circ$

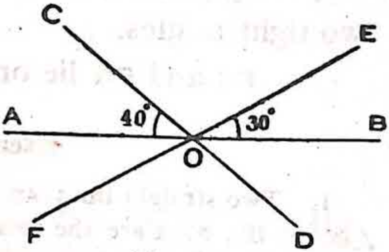


Fig. No. 67

Again, the straight lines CD and EF intersect each other at the point O.

$\therefore\ \angle FOD =$ vertically opposite $\angle COE = 110^\circ$.

2. P is a point on the straight line AB. From the point P, two segments PQ and PR are drawn on opposite sides of AB. If $\angle APQ$ and $\angle BPR$ be equal, show that PQ and PR lie on the same straight line.

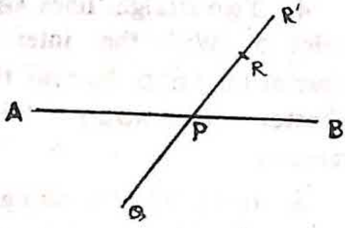


Fig. No. 68

Proof : QP is produced to R

$\therefore\ \angle APQ =$ vertically opposite $\angle BPR$

But, $\angle AOP = \angle BPR$ (by hypothesis)

∵ $\angle BPR = \angle BPR'$

∵ PR will coincide with PR'

∴ PQ and PR will lie on the same straight line.

Alternative proof :

The line segment PQ stands on the straight line AB.

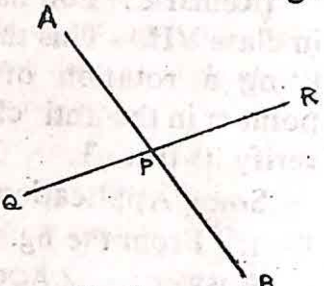


Fig. No. 69

∴ $\angle APQ + \angle QPB =$ two right angles.

But $\angle APQ = \angle BPR$ (by hypothesis)

∴ $\angle QPB + \angle BPR =$ two right angles.

∴ PQ and PR lie on the same straight line.

Exercise 6

1. Two straight lines AB and CD intersect at a point O. If $\angle BOC = 70^\circ$, what are the measures of the other angles ?

2. Two straight lines interest each other in such a way that the sum of one pair of vertically opposite angles is 110°. Find the measures of each angle.

3. Two straight lines AB and CD intersect each other at the point O. If sum of angles $\angle DOB$, $\angle BOC$, $\angle COA$ is 240°, then can (i) the measure of $\angle DOB$ be equal to 50° (ii) the measure of $\angle DOB$ be 80° ? Give reasons.

4. Two straight lines AB and CD intersect each other at the point O. Will the internal bisector of $\angle BOD$ be also the bisector of $\angle AOC$? Give reasons.

5. In fig. 70. two straight lines AB and CD intersect each other at the point O. Find the values of x, y, z.

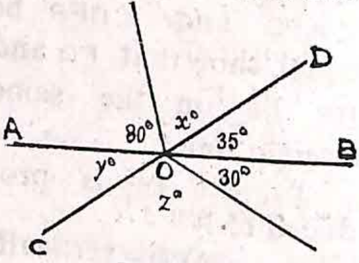


Fig No. 70

6. Two straight lines AB and CD intersect each other

at the point O. The internal bisectors of $\angle AOD$ and $\angle BOC$ are OX and OY. Prove that OX, OY lie on the same straight line.

7. Two segments AB and CD bisect each other at the point O. Prove that $\triangle AOD \cong \triangle BOC$.

8. Line segment AB = line segment AC ; BA and CD are produced to D and E respectively in such a way that AD = AE. Prove that $\triangle CAD \cong \triangle BAE$.

9. Two line segments PQ and RS intersect each other at the point X in such a way that XP = XR. If $\angle PSX$ equals to $\angle RQX$, prove that $\triangle PXS \cong \triangle RQS$.

Theorem 4

When a straight line cuts two other straight lines. those other two straight lines are parallel if either (i) a pair of alternate angles are equal,

or, (ii) a pair of interior angles on the same side of the cutting line are together equal to two right angles.

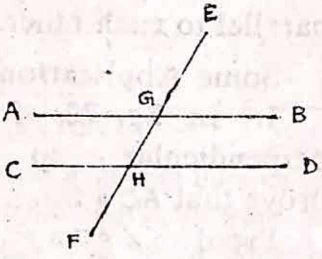


Fig No. 71

Let the straight line EF intersect the straight lines AB and CD at G and H respectively.

(i) Let, $\angle AGH$ = alternate $\angle DHG$.

It is to be proved that, AB and CD are parallel to each other.

Proof : *Two straight lines AB and EF intersect to each other at the point G.*

$\therefore \angle AGH =$ vertically opposite $\angle BGE$.

But $AGH = \angle DHG$ (by hypothesis)

$\therefore AGE = \angle DHG$

But they are corresponding angles.

$\therefore$ AB and CD are parallel to each other [by axiom]

(ii) Let, two interior angles on the same side of the cutting line EF,

$\angle DHG + \angle HGB = 2$ right angles.

It is to be proved that, AB and CD are parallel to each other.

Proof : Since the line segment GB stands on the straight line EF

$\therefore \angle HGB + \angle BGE = 2$ right angles

$\therefore \angle HGB + \angle BGE = \angle DHG + HGB$

$\therefore \angle BGE = \angle DHG$

But they are corresponding angles

$\therefore$ AB and CD are parallel to each other [by axiom]

Corollary 1. Two perpendiculars drawn to the same straight line are parallel to each other.

Some Applications :

1. In fig. 72, if BE is perpendicular to AB, prove that AC ∥ DB.

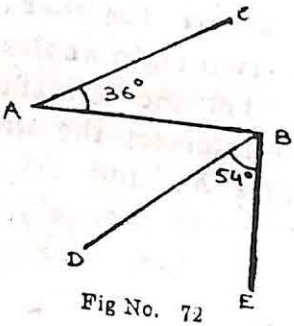


Fig No. 72

Proof : $\angle ABD + \angle DBE$

$= 90°$ [$\because$ $BE \perp AB$]

But $\angle DBE = 54°$

$\therefore \angle ABD = 90° - 54° = 36°$

$\therefore \angle ABD = \angle BAC$, because measures of each is 36°.

But they are alternate angles

∴ AC ∥ DB.

2. The straight line EF intersects two straight lines AB and CD at G and H respectively. A and C lie on same side of EF. If ∠AGH and ∠FHD are supplementary to each other, prove that AB and CD are parallel to each other.

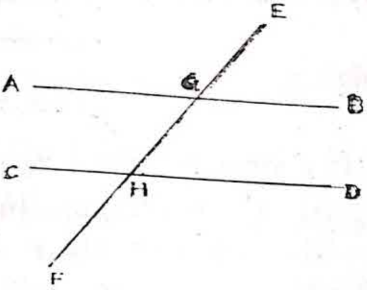


Fig. No. 73

Proof : CD and EF intersect each other at the point H.

∴ ∠FHD = vertically opposite ∠GHC

But ∠AGH and ∠FHD are supplementary to each other [by hypothesis]

∴ ∠AGH + ∠FHD = 180°

∴ ∠AGH + ∠GHC = 180°

In other words, the sum of two interior angles on the same side of the cutting line EF is 2 right angles. ∴ AB ∥ CD.

Exercise 7

1. The straight line EF intersects two straight lines AB and CD at G and H respectively. What is the relation between two straight lines AB and CD subject to any one of the following conditions ? Give reason in each case.

(i) ∠AGE = 130°, ∠GHD = 50°

(ii) ∠AGE = ∠DHF

(iii) ∠EGB = 35°, ∠CHF = 45°

(iv) ∠AGH = 35°, ∠DHF = 150°

(v) ∠BGH and ∠CHF are supplementary to each other.

2. $\angle BAX$ and $\angle ABY$ drawn on opposite sides of AB at the end points A and B of the line segmet AB are equal. Prove that the internal bisectors of two angles are parallel to each other.

3. The line segments AB and CD bisect each other at the point O. Prove that, AC and BD are parallel to each other What kind of quadrilateral is the figure ABCD ?

Theorem 5

If a straight line cuts two parallel straight lines
(i) the corresponding angles are equal
(ii) the alternate angles are equal
(iii) the interior angles on the same side of the cutting line are together equal to two right angles.

Let, the straight line EF intersect the two straight lines AB and CD at the point G and H respectively.

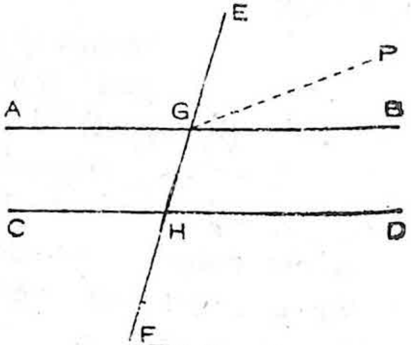


Fig. No. 74

It is to be proved that

(i) $\angle BGE$ = corresponding $\angle DHG$

(ii) $\angle AGH$ = alternate $\angle DHG$

(iii) $\angle HGB + \angle DHG = 2$ right angles

Proof : (i) Let us suppose that $\angle BGE \neq \angle DHG$.

Then $\angle PGE$ is drawn at the point G making equal to $\angle DHG$.

Now, the straight line EF intersects the two straight lines GP and CD at G and H respectively and $\angle PGE$ = corresponding $\angle DHG$ (assumed).

$\therefore \quad GP \parallel CD$

But by hypothesis $AB \parallel CD$

That is, two intersecting straight lines GP and AB are parallel to another straight line CD, which is impossible. Hence, our assumption is not correct : that is $\angle BGE = \angle DHG$

(ii) $\because$ $\angle BGE = \angle DHG$ (proved)

and $\angle BGE =$ vertically opposite $\angle AGH$

$\therefore$ $\angle AGH = DHG$

(iii) $\angle BGE = \angle DHG$ (proved)

Again the line segment GB stands on the straight line EF.

$\because$ $\angle HGB + \angle BGE = 2$ right angles.

$\therefore$ $\angle HGB + \angle DHG = 2$ right angles.

[Remarks : (1) This theorem is also converse of theorem 4. So indirect proof is given.

(2) You can prove the properties (ii) and (iii) without using property (i) by applying the procedure you have followed in proving property (i).

(3) You know the properties of translation. The property (i) can be proved by applying translation of magnitude equal to the length of the segment GH along the straight line EF.]

Corollary : Two straight lines parallel to same straight line are also parallel to each other.

Each of two straight lines AB and CD is parallel to straight line EF.

It is to be proved that AB $\parallel$ CD.

Construction : The straight line PQ intersects three straight lines AB, CD and EF at the points R, S, T respectively.

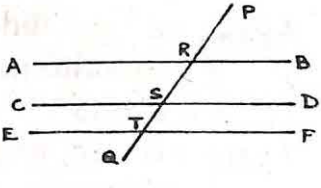


Fig. No. 75

Proof : $\because$ AB $\parallel$ EF, PQ is the transversal

$\therefore$ $\angle PRB = \angle PTF$

Again, $\because$ CD $\parallel$ EF, PQ is the transversal

$\therefore$ $\angle PSD = \angle PTF$

$\therefore$ $\angle PRB = \angle PSD$

But they are corresponding angles,

∴ AB ∥ CD

Alternative proof :

If AB and CD are not parallel to each other, they will intersect each other.

That is, each of two intersecting straight lines is parallel to the straight line EF, which is impossible.

Hence AB and CD can not intersect each other.

Hence AB ∥ CD

Some Applications :

1. In fig. 76, AB ∥ CD, PB ∥ QR ;
$\angle ABP = 30^\circ$, $\angle FED = 105^\circ$.
Find the values of x, y, z.

Answer : ∵ AB ∥ CD

BF is the transversal

∴ $\angle ABE + \angle BEC = 180^\circ$

But $\angle BEC$ = vertically opposite $\angle FED = 105^\circ$

∴ $30^\circ + x^\circ + 105^\circ = 180^\circ$

or, $x^\circ + 135^\circ = 180^\circ$

∴ $x^\circ = 180^\circ - 135^\circ = 45^\circ$

∴ $x = 45$

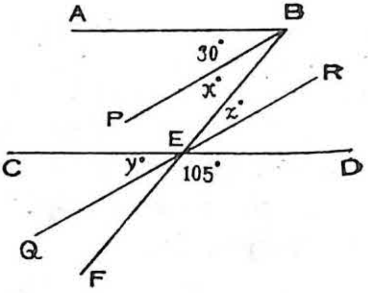


Fig. No. 76

Again, PB ∥ QR and BF is the transversal

∴ $\angle REB$ = alternate $\angle PBE$

∴ $z^\circ = x^\circ = 45^\circ$ ∴ $z = 45$

Again, AB ∥ CE, BF is the transversal

∴ $\angle CEF$ = corresponding $\angle ABE = 30^\circ + 45^\circ = 75^\circ$

Again, $\angle QEF$ = vertically opposite $\angle REB = 45^\circ$

∴ $y^\circ + 45^\circ = 75^\circ$

∴ $y^\circ = 75^\circ - 45^\circ = 30^\circ$

∴ $y = 30$

∴ The values of x, y, z are 45, 30, 45 respectively.

2. O is a point within two parallel straight lines AB and CD. OP and OQ are perpendiculars on AB and CD respectively. prove that the three points P, O, Q are collinear.

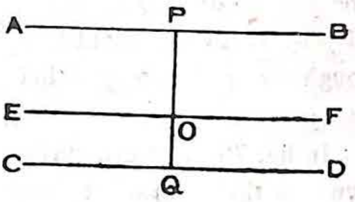


Fig. No. 77

[Analysis : ∵ AB ∥ CD and each of ∠OPB and ∠OQD is right angle ;

∴ If a line is drawn through the point O parallel to AB, then it will also be parallel to CD. As a result, two adjacent angles will be formed, each of which is 90°]

Construction : A straight line EF is drawn through the point O parallel to AB.

Proof : Both CD and EF are parallel to AB.

∴ FE and CD are also parallel to each other.

Again, AB ∥ EF and PO is the transversal

∴ ∠FOP = alternate ∠OPA = 90°

But, ∠OPA = 90° (∵ OP ⊥ AB)

Again, EF ∥ CD and OQ is the transversal

∴ ∠QOF = alternate ∠OQC = 90°

∴ OQ ⊥ CD

∴ ∠QOF + ∠FOP = 180°

∴ Three points P, O, Q are collinear.

[Remark : It can also be pròved by drawing a perpendicular to PO or QO from the point O without drawing a parallel to EF through O. Try to prove in this way.]

Exercise 8

1. The straight line EF intersects the two straight lines AB and CD at G and H respectively. If $\angle EGB = 50^\circ$, find the measures of other angles.

2. In fig. 78, $AB \parallel CD$; $\angle APQ = 35^\circ$, $\angle CRQ = 60^\circ$ what is the magnitude of $\angle FQR$?

[Hint : Draw a straight line through the point O parallel to AB.]

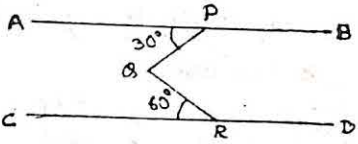


Fig. No. 78

3. In fig. 79, $AB \parallel CD$, write down within bracket the names of angles whose measures are equal.

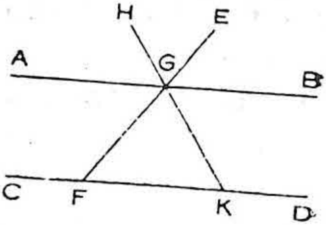


Fig. No. 79

4. In fig. 80, $PQ \parallel RS$, $\angle BFQ = 40^\circ$, $\angle BPR = 155^\circ$. If $\angle CRS = 70^\circ$, what are the measures of the angles of $\triangle APR$?

[Hint : Draw a straight line through the point A parallel to PQ.]

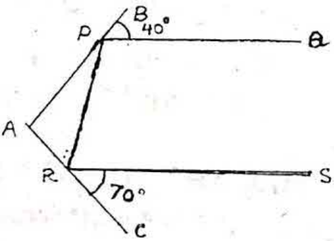


Fig. No. 80

5. In $\triangle ABC$, sides BA and CA are produced to E and D respectively in such a way that $DE \parallel BC$. Prove that, $\triangle ABC$ and $\triangle ADE$ are equiangular. [If three angles of one of two triangles are equal to three angles of other, then two triangles are called equiangular.]

6. AB and CD are two parallel straight lines. Prove that, if PQ is perpendicular on AB, it will also be perpendicular on CD.

7. E and F are two points on two parallel straight lines AB and CD respectively. O is the mid point of the line segment EF. Prove that, any line segment through the point O and extended upto two parallel straight lines is bisected at the point O.

8. PQ and RS are two parallel straight lines. The end points A and B of the line segment AB lie on PQ and RS respectively. Two end points C and D of any other line segment CD, through the mid point X of AB, lie on PQ and RS respectively. Prove that $AC = BD$.

9. Each pair of sides of two angles is parallel. Prove that two angles are equal or supplementary to each other.

10. AB and CD are two equal and parallel line segments. Two end points A, C of same side and the two end points B, D of other side are joined. By joining AD, prove that $\triangle ABD \cong \triangle ACD$. Hence prove that AC and BD are equal and parallel.

Theorem 6

If one side of a triangle be produced, the exterior angle so formed, is equal to the sum of two interior opposite angles.

Let the side BC of $\triangle ABC$ be produced to D.

It is to be proved that, Exterior angle $\angle ACD$ = Interior opposite angle $\angle CAB$ + interior opposite angle $\angle ABC$.

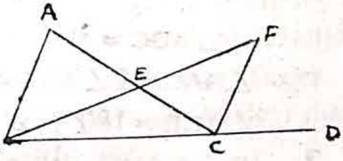


Fig. No. 81

Construction : Suppose, E is mid point of AC ; BE is joined and produced to F in such a way that $BE = EF$. FC is joined.

Proof : In $\triangle ABE$ and $\triangle CEF$,

$AE = EC$ (given)

$BE = EF$ (by construction)

$\angle AEB$ = vertically opposite $\angle CEF$

$\therefore \quad \triangle AEB \cong \triangle CEF$

$\therefore \quad \angle EAB = \angle ECF$

But they are alternate angles

∴ $BA \parallel CF$

∴ $\angle ABC$ = corresponding $\angle FCD$

∴ $\angle EAB + \angle ABC = \angle ECF + \angle FCD$

that is, $\angle CAB + \angle ABC = \angle ACD$

that is, $\angle ACD = \angle CAB + \angle ABC$.

Corollary : If one side of a triangle be produced the exterior angle so formed will be greater than any one of the interior opposite angles.

Some applications :

1. In fig. 82, if $\angle ACD = 114^\circ$ and $\angle B = \frac{1}{2}\angle A$, what are measures of angles of $\triangle ABC$?

Answer : In $\triangle ABC$ exterior $\angle ACD = \angle ABC + \angle BAC$ (the sum of interior opposite angles)

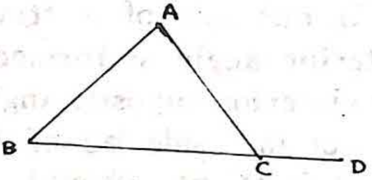


Fig. No. 82

Again, $\angle ABC = \frac{1}{2}\angle BAC$

∴ $\angle BAC = 2\angle ABC$

∵ $3\angle ABC = \angle ACD$
$= 114^\circ$

∴ $\angle ABC = 38^\circ$

∴ $\angle BAC = 2\angle ABC = 2 \times 38^\circ = 76^\circ$

∴ $\angle ACB = 180^\circ - \angle ACD = 180^\circ - 114^\circ = 66^\circ$

2. In $\triangle ABC$, internal bisector of $\angle B$ and external bisector of $\angle C$ meet at the point D. Prove that, $\angle BDC = \frac{1}{2}\angle A$.

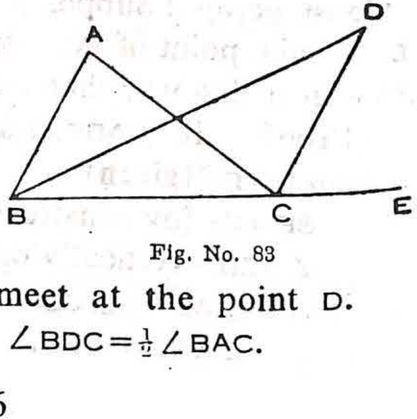


Fig. No. 83

The internal bisector of $\angle B$ and external bisector of $\angle C$ of $\triangle ABC$, that is the internal bisector CD of $\angle ACE$, meet at the point D.

It is to be proved that $\angle BDC = \frac{1}{2}\angle BAC$.

Proof : The exterior angle $\angle DCE$ of $\triangle BDC$

$$= \angle CBD + \angle BDC$$

Again, exterior angle $\angle ACE$ of $\triangle ABC$

$$= \angle CBA + \angle BAC$$

$$\therefore \quad \tfrac{1}{2}\angle ACE = \tfrac{1}{2}\angle CBA + \tfrac{1}{2}\angle BAC$$

$$\because \quad \angle DCE = \angle CBD + \tfrac{1}{2}\angle BAC$$

$$\therefore \quad \angle CBD + \angle BDC = \angle CBD + \tfrac{1}{2}\angle BAC$$

$$\therefore \quad \angle BDC = \tfrac{1}{2}\angle BAC$$

Exercise 9

1. The side BC of $\triangle ABC$ is produced to D. If $\angle ACD = 130^\circ$. $\angle ABC = 80^\circ$, what is the measure of $\angle CAB$?

2. In fig. 84, AB || CD. From the figure, find the measure of $\angle FEG$.

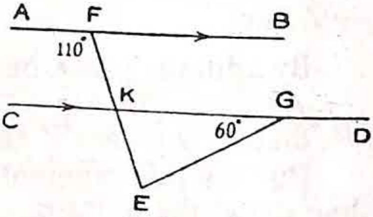


Fig. No. 84

3. The side BC of $\triangle ABC$ is produced to D. $\angle ACD = 100^\circ$ and $\angle ABC = \frac{2}{3}\angle CAB$. Find the measures of the angles of the triangle.

4. Prove that, only one staight line through an exterior point of a straight line intersects the later one at right angles.

[Only one perpendicular can be drawn on a straight line through an exterior point of that straight line.]

5. In $\triangle ABC$, $\angle ABC = \angle CAB$. BC is produced to D. Prove that $\angle ACD = 2\angle ABC$.

6. O is a point within $\triangle ABC$. By producing BC to OC. prove that $\angle BOC > \angle BAC$.

7. Prove that the sum of two exterior angles, formed by producing side BC of $\triangle ABC$ in both directions, is greater than two right angles.

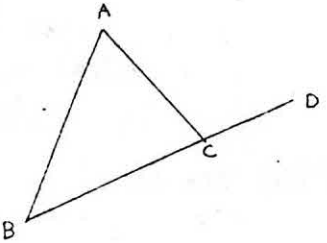
8. S is a point within $\triangle PQR$. By producing PS prove that. $\angle QSR > \angle QPR$.

Theorem 7

The sum of three angles of a triangle is equal to two right angles.

Let ABC be a triangle.

It is to be proved that $\angle ABC + \angle BCA + \angle CAB = 2$ right angles



Construction : Side BC is produced to D.

Proof : Exterior angle $\angle ACD =$ sum of interior opposite angles $= \angle CAB + \angle ABC$.

Fig. No. 85

By adding $\angle BCA$ on both sides, we get

$\angle ACD + \angle BCA = \angle CAB + \angle ABC + \angle BCA$

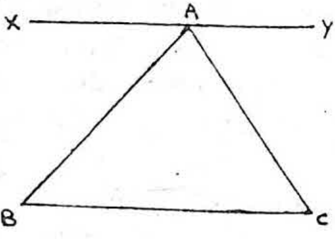
that is, $\angle CAB + \angle ABC + \angle BCA = \angle BCA + \angle ACD$

But the line segment CA stands on the straight line BD at the point C.

$\therefore \quad \angle BCA + \angle ACD = 2$ right angles

$\therefore \quad \angle CAB + \angle ABC + \angle BCA = 2$ right angles.

Alternative proof :



Let, ABC be a triangle.

It is to be proved that $\angle CBA + \angle ACB + \angle BAC = 2$ right angles.

Construction : Through the point A, XY is drawn parallel to BC.

Fig. No. 86

Proof : $\because$ BC ∥ XY and AB is the transversal

$\therefore \quad \angle CBA =$ alternate $\angle XAB$

Again, $\because$ BC ∥ XY and AC is the transversal

$\therefore \quad \angle ACB =$ alternate $\angle CAY$

$\therefore \quad \angle CBA + \angle ACB = \angle XAB + \angle CAY$

Adding $\angle BAC$ on both sides.

$$\angle CBA + \angle ACB + \angle BAC = \angle XAB + \angle CAY + \angle BAC$$
$$= \angle XAB + \angle BAC + \angle CAY$$
$$= 2 \text{ right angles}$$

[Remarks : (1) In this case also, the construction has been made for the same reason.

(2) In fig. 87, points D and E are the mid points of AB and AC respectively and DP, EQ are perpendiculars on BC from the points D and E respectively. Now, draw this figure on a piece of paper and cut it out. Fold along DE, DP and EQ. Then the points B, A, C will meet at a point on BC. You may verify this property in this way.]

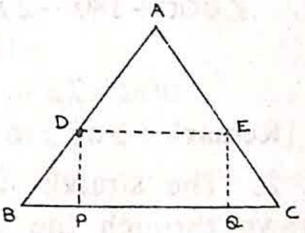


Fig. No. 87

Corollary 1. If two angles of a triangle are equal to two angles of another triangle then the third angle of both the triangles must be equal.

Corollary 2. In a right angled triangle, the sum of two angles other than the right angle is equal to one right angle.

Corollary 3. In a right-angled triangle, besides the right angle, each of the other angles must be an acute angle.

Corollary 4. In an obtused-angled triangle, besides the obtuse angle, each of the other angles must be an acute angle.

Some Applications :

1. From the figure 88, express the measure of $\angle BOC$ in terms of $\angle A$.

Answer : $\angle BOC + 2x^\circ = 180^\circ$

or $2(x^\circ + y^\circ + z^\circ) = 180^\circ$

$\therefore \quad x^\circ + y^\circ + z^\circ = 90^\circ$

Again $y^\circ + z^\circ = \angle A$

$\therefore \quad x^\circ = 90^\circ - (y^\circ + z^\circ)$

$= 90^\circ - \angle A$

$\therefore \quad \angle BOC + 180^\circ - 2\angle A = 180^\circ$

$\therefore \quad \angle BOC = 2\angle A.$

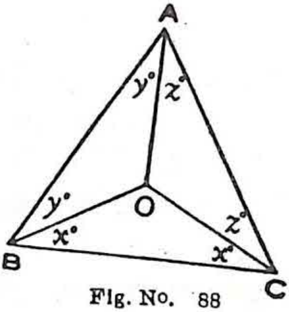


Fig. No. 88

[**Remark :** You can prove also by producing AO.]

2. The straight lines parallel to BC and BA drawn through the vertices A and C of $\triangle ABC$ respectively intersect at a point D. Prove that,

$\angle ABC = \angle CDA$

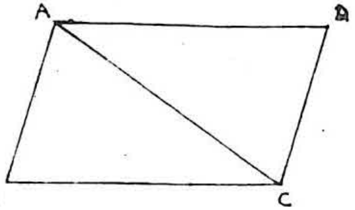


Fig No. 89

Proof : $\because$ BC $\parallel$ AD and AC is the transversal

$\therefore \quad \angle CAD = \angle ACB$

Again, $\because$ BA $\parallel$ AB and AC is the transversal

$\therefore \quad \angle ACD = \angle BAC$

$\therefore \quad \angle ACB + \angle BAC = \angle CAD + \angle ACD$

Again, in $\triangle ADC$, $\angle CAD + \angle ACD + \angle CDA = 180^\circ$

and in $\triangle ABC$, $\angle ACB + \angle BAC + \angle ABC = 180^\circ$

$\therefore \quad \angle ACB + \angle BAC + \angle ABC = \angle CAD + \angle ACD + \angle CDA$

$\therefore \quad \angle ABC = \angle CDA$

[**Remark :** You can also prove it by proving $\triangle ABC$ and $\triangle ACD$ congruent.]

3. prove that the sum of three exterior angles, formed by producing the sides of a triangle in same order is 360°.

Let the sides AB, BC and CA of $\triangle ABC$ be produced to the points D, E and F respectively.

It is to be proved that, $\angle DBC + \angle ECA + \angle FAB = 360^\circ$

Proof : Exterior $\angle DBC = \angle BAC + \angle ACB$ (sum of two interior opposite angles)

Exterior $\angle ECA = \angle CBA + \angle BAC$

(Sum of two interior opposite angles)

and exterior $\angle FAB = \angle BAC + \angle CBA$

(Sum of two interior opposite angles)

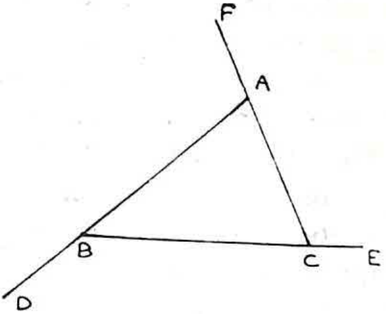


Fig. No. 90

$\therefore \angle DBC + \angle ECA + \angle FAB$
$= 2(\angle BAC + \angle CBA + \angle ACB)$
$= 2 \times 180^\circ = 360^\circ$

[$\because$ Sum of three angles of a triangle is 180°]

Exercise 10

1. The measures of the two angles of a triangle are 51° and 68° ; what is the measure of the third angle ?

2. The measures of the angles of a triangle are x°, $2x^\circ$ and $3x^\circ$; find numerical values of the angles of the triangle.

3. Of the two acute angles of a right-angled triangle, one is double the other ; find the measures of each of three angles of the triangle.

4. $\angle ABC$ of $\triangle ABC$ is right angle. AD is perpendicular on BC. If $\angle ABC = 35^\circ$, find the measures of $\angle BAD$, $\angle CAD$, $\angle BCA$, $\angle ADB$ and $\angle ADC$.

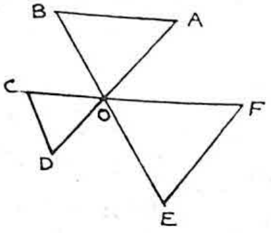


Fig. No. 91

4. From fig. 91, find the sum of $\angle A$, $\angle B$, $\angle C$, $\angle D$, $\angle E$ and $\angle F$.

6. Through the end points A and B of a line segment AB two parallel straight lines AC and BD are drawn on same side of AB. Prove that, internal bisectors of $\angle CAB$ and $\angle ABD$ intersect each other at right angles.

7. The internal bisectors of $\angle ABC$ and $\angle ACB$ of $\triangle ABC$ meet at the point O. Prove that $\angle BOC = 90^\circ + \frac{1}{2}\angle BAC$.

8. The external bisectors of $\angle ABC$ and $\angle ACB$ of $\angle ABC$ meet at the point O. Prove that $\angle BOC = 90^\circ - \frac{1}{2}\angle BAC$.

9. The external bisector of $\angle ABC$ of $\triangle ABC$ intersects a straight line through the point A, parallel to BC, at the point D. Prove that, $\angle ADB = 90^\circ - \frac{1}{2}\angle ABC$.

10. ABC is a triangle. Prove that, the triangle formed by meeting the straight lines parallel to BC, CA and AB drawn through the vertices A, B, C respectively is equi-angular to $\triangle ABC$.

Theorem 8

The sum of the interior angles of a polygon of *n* sides is equal to $2(n-2)$ right angles.

Let, ABCDEFG be a polygon of *n* sides.

It is to be proved that, the sum of the interior angles of this polygon = $2(n-2)$ right angles

Construction : A, C ; A, D ; A, E : A, F are joined.

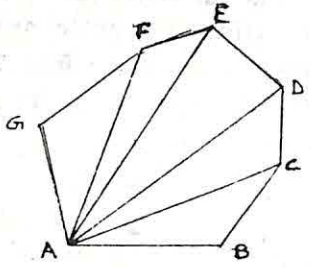


Fig. No. 92

Proof : $(n-2)$ numbers of triangles are formed with A as vertex by adding the other angular points to the point A.

The sum of interior angles of the polygon

$=$ the sum of the angles of $(n-2)$ numbers of triangles

$=2(n-2 \times 2$ right angles

$=2(n-2)$ right angles

[Remark : We know, the sum of three angles of a triangle is two right angles. So we divide the polygon into some triangles by adding the angular points]

Alternative Proof : Let, ABCDEFG be a polygon of n sides.

It is to be proved that, the sum of the interior angles of the polygon $= 2(n-2)$ right angles.

Construction : the angular points A, B, C, D, E, F, G are joined to any point O within the polygon.

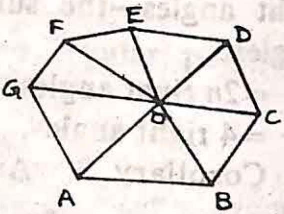


Fig. No. 93

Proof : By joining the points A, B, C, D, E, F, G with the point O, the number of triangles formed is equal to the number of sides of the polygon, or in other words, n number of triangles are formed The sum of angles of the triangles is $2n$ right angles. Now the sum of interior angles of the polygon $=$ the sum of angles of the triangles—the sum of angles formed at the point O $=2n$ right angles $-$ 4 right angles.

[$\because$ the sum of angles formed at any point is 4 right angles.]

$=2(n-2)$ right angles

Corollary 1. The sum of four angles of any quadrilateral is 4 right angles.

Corollary 2. The sum of exterior angles formed by producing the sides of any polygon of n sides in same order in same direction is 4 right angles.

By producing each side in same direction, in same order, the sum of each interior angle and exterior angle adjacent to it will be right angles.

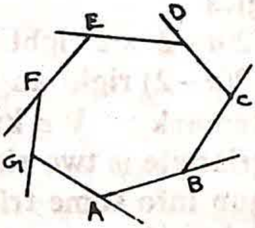


Fig. No. 94

$\therefore$ the sum of n number of interior angles + the sum of n number of exterior angles $= 2n$ right angles.

$\therefore$ the sum of n number of exterior angles $= 2n$ right angles—the sum of n number of interior angles.

$= 2n$ right angles $- (2n - 4)$ right angles

$= 4$ right angles.

Corollary 3. Any of the interior angles of a regular polygon $= \dfrac{2n-4}{n} \times 90^\circ$

Corollary 4. Any of the exterior angles of a regular polygon $= \dfrac{360^\circ}{n}$

[Remark : If the measure of each interior or exterior angle of a regular polygon is known from corollary 3 and corollary 4, the number of sides of that polygon can be determined.]

Some Application :

1. Find the sum of the interior angles of a polygon of nine sides.

We know, the sum of interior angles of a polygon of n sides $= 2(n-2)$ right angles.

$\therefore$ The sum of interior angles of the polygon of nine sides $= 2(9-2)$ right angles $= 2 \times 7$ right angles $= 14$ right angles.

2. What are the measures of each exterior angle and each interior angle of a regular polygon whose number of sides is 15 ?

Total measures of the interior angles $= 2(15-2)$ right angles $= 2\times 13$ right angles $= 26\times 90^\circ$.

$\therefore$ Measure of each interior angle $= \dfrac{26\times 90^\circ}{15} = 156^\circ$

$\therefore$ Measure of each exterior angle $= \dfrac{360^\circ}{15} = 24^\circ$

Remark. First find the measure of each exterior angle and then find the measure of each exterior angle.

3. If each exterior angle of a regular polygon is 60°, what is its number of sides ?

$$\text{number of sides} = \frac{360}{60} = 6$$

4. If each interior angle of a regular polygon is 135°, what is its number of sides ?

The sum of interior angles of the polygon $= 2(n-2)\times 90^\circ$, if the number of sides of the polygon be n.

$\therefore\quad 2(n-2)\times 90 = 135n$

or, $\quad 180n - 360 = 135n$

or, $\quad 180n - 135n = 360$

or, $\quad 45n = 360$

or, $\quad n = \dfrac{360}{45} = 8.$

Alternative Method :

Measure of each interior angle $= 135^\circ$

Measure of each exterior angle $= 180^\circ - 135^\circ$

$= 45^\circ$

$\therefore$ Required number of sides $= \dfrac{360}{45} = 8.$

5. ABCDE is a regular pentagon. Prove that, ACD is an isosceles triangle.

In the regular pentagon ABCDE, A, C ; A, D are joined.

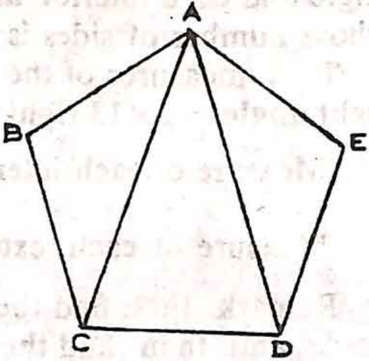


Fig. No. 95

It is to be proved that, $\triangle ACD$ is isosceles.

∵ ABCDE is a regular pentagon,

∴ all its sides and all its angles are equal.

∴ In $\triangle ABC$ and $\triangle AED$

$AB = AE$

$BC = DE$

and included $\angle ABC$ = included $\angle AED$

∴ $\triangle ABC \cong \triangle AED$

∴ $AC = AD$

∴ $\triangle ACD$ is isosceles.

6. Prove that the sum of the angles formed by intersection of sides of a hexagon produced both ways is equal to 4 right angles.

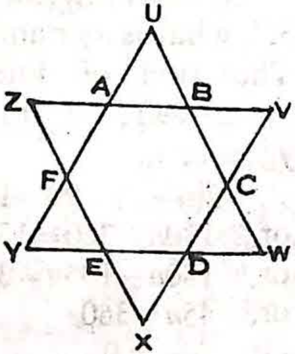


Fig. No. 96

Let ABCDEF be a hexagon. The sides are produced both ways so that they meet and form the angles $\angle U$, $\angle V$, $\angle W$, $\angle X$, $\angle Y$, $\angle Z$.

It is to be proved that $\angle U + \angle V + \angle W + \angle X + \angle Y + \angle Z = 4$ right angles.

We know that the sum of the exterior angles formed by producing the sides of and polygon in same direction and order is 360°

∴ $\angle ZAF + \angle UBA + \angle VCB + \angle WDC + \angle XED$
$+ \angle YFE = = 4$ right angles

Similarly, $\angle UAB + \angle VBC + \angle WCD + \angle XDE +$
$\angle YFF + \angle ZAF = 4$ right angles.

$\angle UAB + \angle VBC + WCD + \angle XDE + \angle YEF + \angle ZFA$
$= 4$ **right angles**

$\therefore$ **The sum of these 12 angles = 8 right angles**

Again, the sum of the angles of $\triangle AZF$, $\triangle FYE$ $\triangle EXD$, $\triangle DWC$, $\triangle CVB$, $\triangle BUA = 6 \times 2$ **right angles**
$= 12$ **right angles**

$\therefore \angle U + \angle V \angle W + \angle X + \angle Y + \angle Z$
= **The sum of angles of the 6 triangles—sum of the 12 angles**
= **12 right angles – 8 right angles**
= **4 right angles.**

Exercise 11

1. Find the sum of the interior angles of the following polygons :
(i) Pentagon (ii) Hexagon (iii) Heptagon (iv) Octagon (v) the polygon whose number of sides is 10 (Decagon) (vi) the polygon whose number of sides is 12 (Dodecagon).

2. The measures of three angles of a quadrilateral are 104·5° 65°, 72·5° respectively ; what is the measure of fourth angle ?

3. the measures of four angles of a pentagon are 65°, 89°, 132° and 116° respectively; find the measure of the fifth angle

4. Two angles of a quadrilateral are equal. Of the other two angles, one is half of any of the former two angles and the other is double. Find the measures of the angles.

5. Can the measures of three angles of a convex quadrilateral be 68°, 70° and 35° ?

[Each interior angle of a convex polygon is less than two right angles]

6. Can the measures of five angles of convex hexagon be 120°, 170°, 95°, 178°, 160° ?

7. In fig 97, find the sum of $\angle A$, $\angle B$, $\angle C$, $\angle D$ and $\angle E$.

8. Find the measure of each interior and exterior angles of the following regular polygons :

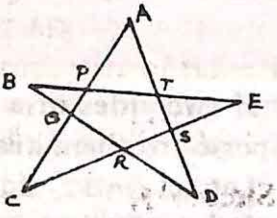


Fig. No. 97

(i) Pentagon (ii) Hexagon (iii) Octagon (iv) whose number of sides is 9 (v) whose number of sides is 10 (vi) whose number of sides is 12 (vii) whose number of sides is 18 (viii) whose number of sides is 20.

9. Can an exterior angle of any regular polygon be of the following measures ?

(i) 6° (ii) 10° (iii) 13° (iv) 18° (v) 24° (vi) 30° (vii) 35° (viii) 40° (ix) 42° (x) 45° (xi) 50° (xii) 70° (xiii) 72°.

Find the number of sides of the polygons in the possible cases.

10. Can an interior angle cf any regular polygon be of the following measures ?

(i) 80° (ii) 100° (iii) 120° (iv) 144° (v) 155° (vi) 160° (vii) 165° (viii) 168° (ix) 170° (x) 171° (xi) 172° (xii) 175°.

Find the number of sides of the polygons in the possible cases.

11. The bisectors of $\angle P$ and $\angle Q$ of the quadrilateral PQRS intersect each other at right angle. Prove that the bisectors of $\angle R$ and $\angle S$ will also Intersect each other at right angle.

12. Prove that the angle formed by the internal bisectors of two adjacent angles of any quadrilateral is half of the sum of other two angles of the quadrilateral.

13. The internal bisectors of four angles of a quadrilateral meet at the point O. If $\angle POQ = 75°$, what is the measure of $\angle ROS$?

Theorem 9

If two sides of a triangle be equal, the angles opposite to them are also equal.

Let, in $\triangle ABC$, side $AB = AC$.

It is to be proved that, $\angle ACB = \angle ABC$.

Construction :

The bisector AD of $\angle BAC$ is drawn which meets BC at the point D.

Proof : In ΔACD and ΔBCD,

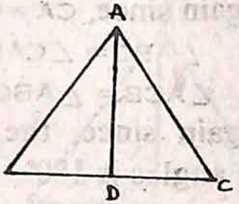


Fig No. 98

$AC = AB$ (by hypothesis),

AD is common,

Included $\angle CAD$ = included $\angle BAD$ (by construction)

$\therefore\ \angle ACD \cong \angle ABD$

$\therefore\ \angle ACD = \angle ABD$ that is $\angle ACB = \angle AEC$.

[Remark : In this case, it is to be proved that two angles are equal. We know from the properties we got earlier that (1) if a line segment is perpendicular on another line segment then two angles are equal (a right angle) (2) the corresponding angles of two congruent triangles are equal (3) the vertically opposite angles are equal and (4) corresponding angles or alternate angles are equal in the case of parallel straight lines. We can reject (1), (3) and (4) in view of what is given. So we have to get two congruent triangles so that the two angles to be proved equal become corresponding angles of those two congruent triangles. There being no such triangles we have to make such a construction, by which we get two congruent triangles. With this objective we draw the bisector AD.

(2) You have known the properties of reflection. The theorem can also be proved by applying reflection in the straight line AD and using the properties of reflection.]

Corollary 1. Three angles of an equilateral triangle are equal and the measure of each is 60°.

In the equilateral triangle ABC

$\therefore\ AB = AC$

$\therefore\ \angle ACB = \angle ABC$

Again since, $CA = CB$

$\therefore\ \angle ABC = \angle CAB$

$\therefore\ \angle ACB = \angle ABC = \angle CAB$

Again since, the sum of three angles is 180°

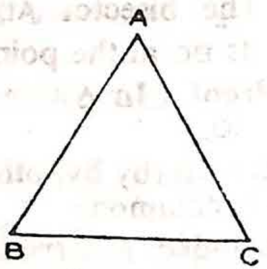


Fig. No. 99

$\therefore$ the measure of each angle $= \dfrac{180^\circ}{3} = 60^\circ$

Corollary 2. The measure of each acute angle of an isosceles right-angled triangle is 45°

Some Applications :

1. In $\triangle ABC$, $AB = AC$; BC is produced to D. If $\angle ACD = 112^\circ$, find the measures of the angles of the triangle.

Answer : Since, AC stands on the straight line BD

$\therefore\ \angle ACB + \angle ACD$

$= 180^\circ$

$\therefore\ \angle ACB = 180^\circ - 112^\circ$

$= 68^\circ$

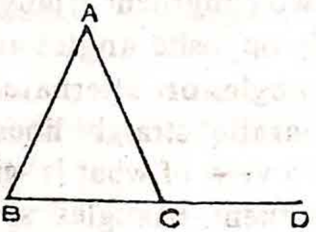


Fig. No. 100

Again, since, in $\triangle ABC$, $AB = AC$

$\therefore\ \angle ACB = \angle ABC \quad \therefore\ \angle ABC = 68^\circ$

$\therefore\ \angle BAC = \angle ACD - \angle ABC = 112^\circ - 68^\circ = 44^\circ$

2. **Prove that, the bisector of the vertical angle of an isosceles triangle bisects the base at right angles.**

Let ABC be an isosceles triangle whose $AB = AC$.

The bisector AD of the vertical angle BAC intersects the base BC at point D.

It is to be proved that (i) BD = DC and (ii) AD ⊥ BC

Proof : In △ABD and △ACD, AB = AC (by hypothesis)

AD is common

And included ∠BAD = included ∠CAD (by hypothesis)

∴ ∠ABD ≅ △ACD

∴ BD = CD and

∠ADB = ∠ADC

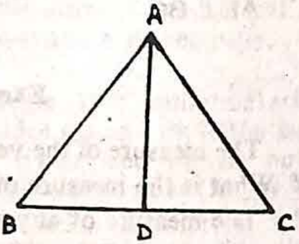


Fig. No. 101

But since AD stands on the straight line BC

∴ ∠ADB + ∠ADC = 180°

∴ Each of them is 90°

∴ AD ⊥ BC.

3. Prove that, the external bisector of the vertical angle of an isosceles triangle is parallel to the base.

Let ABC be an isosceles triangle whose AB = AC. BA is produced to D. AE is external bisector ∠BAC.

It is to be proved that AE ∥ BC

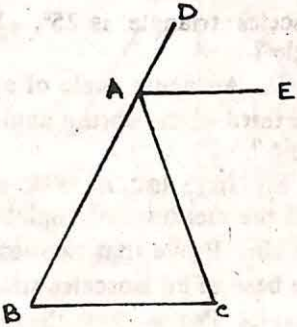


Fig. No. 102

Proof : In △ABC. the exterior angle ∠CAD = ∠ABC + ∠ACB (interior opposite angles)

Again, since, in △ABC

AB = AC

∴ ∠ACB = ∠ABC ∴ ∠CAD = 2∠ABC

∴ AE is internal bisector of ∠CAD

∴ ∠CAD = 2∠DAE

∴ ∠DAE = ∠ABC

But they are corresponding angles

∴ AE ∥ BC

Exercise 12

1. The measure of the vertical angle of an isosceles triangle is 80°. What is the measure of each base angle ?

2. The measure of any angle adjacent to the base of an isosceles triangle is 70°. What is the measure of the vertical angle ?

3. The sum of two base angles of an isosceles triangle is 100°, find the measure of the angles of the triangle.

4. The sum of the vertical angle and one base angle of an isosceles triangle is 150°, find the measures of the angles of the triangle.

5. Any of the base angles of an isosceles triangle is (i) double the vertical angle (ii) half the vertical angle (iii) four times the vertical angle (iv) one fourth of the vertical angle. Find the measures of the angles of the triangle in each case,

6. The measure of obtuse angle of an obtuse-angled isosceles triangle is 110°; find the measures of other angles.

7. The measure of an acute angle of an obtuse-angled isosceles triangle is 25°. What is the measure of the obtuse angle ?

8. An acute angle of an obtuse-angled isosceles triangle is one third of the obtuse angle. What is the measure of each angle ?

9. In △ABC, AB = AC: BA is produced to E. If ∠CAE = 150°, find the measures of angles of the triangle.

10. Prove that two exterior angles, formed by producing the base of an isosceles triangle both ways, are equal.

11. Prove that the angles, formed with the base by the equal sides of an isosceles triangle when produced are equal.

12. Prove that, the perpendicular drawn from the vertex on the base of an isosceles triangle bisects the vertical angle and the base.

13. Prove that, in an isosceles triangle

(i) Medians drawn from the vertices opposite to two equal sides are equal.

(ii) The lengths of two perpendiculars drawn from the vertices adjacent to the base angles to opposite sides are equal.

14. In $\triangle ABC$, $AB=AC$; the bisectors of $\angle ABC$ and $\angle ACB$ intersect. AC and AB at the points D and E respectively. Prove that, $BD=CE$.

15. PQR is an equilateral triangle. Two equilateral triangles APQ and BPR are drawn on the sides PQ and PR in the external side of the triangle. Prove that (i) AP and BP lie on same straight line (ii) $ABRQ$ is an isosceles trapezium (iii) $AR=BQ$.

16. ABC is an equilateral triangle. Equilateral triangles XAB. YBC and ZCA are drawn on the sides of this triangle. Prove that, XYZ is an equilateral triangle.

17. ABC is a right-angled isosceles triangle whose $AB=AC$: say which angle of the triangle is right angle. Two equilateral triangles PAB, QAC are drawn on sides AB and AC. Prove that $PC=BQ$.

Theorem 10

If two angles of a triangle be equal, then the sides opposite to those angles will also be equal.

Let in $\triangle ABC$, $\angle ABC = \angle ACB$

It is to beproved that $AC=AB$

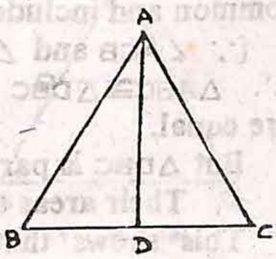


Fig. No. 103

Constraction : The bisector AD of $\angle BAC$ is drawn. AD intersects BC at the point D.

Proof : In $\triangle ABD$ and $\triangle ACD$

$\angle ABD = \angle ACD$ (by hypothesis)

$\angle BAC = \angle CAD$ (by construction)

AD is the common side

$\therefore$ $\triangle ABD \cong \triangle ACD$, $\therefore$ $AB=AC$ That is. $AC=AB$

[Remark : (1) Try to understand why AD has been drawn. By drawing AD we get two congruent triangles.

Now if AD be perpendicular on BC without being the bisector of ∠BAC, can the theorem be proved ? State with reasons.

Again if AD be such a line segment that D is the mid point of BC then can't the theorem be proved ?

(2) In this case also you can try to prove by reflecting in the line AD.]

Aternative Proof : Let in △ABC, ∠ABC = ∠ACB

It is proved that, AC = AB.

Proof : If AC and AB be not equal, then any one of the sides AC and AB will be greater than the other. Let AC > AB. Then the line segment CD is cut off from CA making equal to BA, B, D are joined.

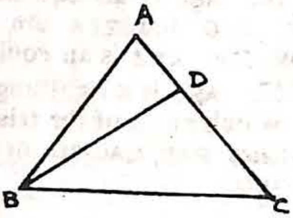


Fig. No. 104

Now, in △ABC and △BDC, BA = CD, BC is common and included ∠ABC = included ∠DCB

[∵ ∠ACB and ∠DCB are same angles.]

∴ △ABC ≅ △DBC ∴ Area of two triangles are equal.

But △DBC is part of △ABC

∴ Their areas can not be equal.

This shows that the conclusion to which we reached from what we assumed is impossible. Therefore, AC and AB are not unequal ∴ AC = AB

[Remark : This is the converse of the theorem 9. That is why, in this case also, we proceeded by assuming a property which is not the conclusion we like to reach at and by that we reached to a wrong conclusion. Therefore we can say the conclusion to which we like to reach is correct.]

Corollary : If three angles of a triangle be equal to each other, the triangle will be equilateral.

Some Applications :

1. In fig No. 105, prove that, $AC = BC$.

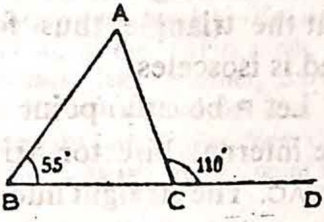


Fig. No. 105

Proof : Side BC of $\triangle ABC$ is produced to D.

$\therefore$ exterior angle $\angle ACD = \angle ABC + \angle BAC$

$\therefore \quad \angle BAC - \angle ACD = \angle ABC = 110° - 55° = 55°$

$\therefore \quad \angle ABC = \angle BAC$

$\therefore \quad AC = BC.$

2. Each side of an equilateral triangle ABC is 4 cm. The mid points of AB and AC are D and E respectively. What is the length of DE ?

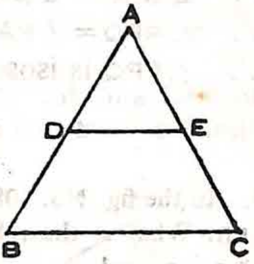


Fig. No. 106

Answer : Since D and E are the mid points of AB and AC respectively,

$\therefore \; AD = \frac{1}{2}AB, \; AB = \frac{1}{2}AC$

$\because \quad AD = AC$

$\therefore \quad AD = AE$

$\therefore \quad \angle AED = \angle ADE$

Again, since $\triangle ABC$ is equilateral

$\therefore \quad \angle DAE = 60°$

$\therefore \quad \angle ADE + \angle AED = 120°$

$\therefore \quad \angle ADE = \angle AED = 60°,$

$\therefore \quad \triangle ADE$ is equilateral

$\therefore \quad DE = AD = \frac{1}{2}AB = \frac{1}{2} \times 4$ cm $= 2$ cm.

A straight line, passing through any point on the internal bisector of an angle, parallel to any side of the angle intersects the other side. Prove that the triangle thus formed is isosceles

Let P be any point on the internal bisector AD of ∠BAC. The straight line RS drawn through the point P parallel to BA intersects AC at the point Q.

It is to be proved that, △AQP is isosceles.

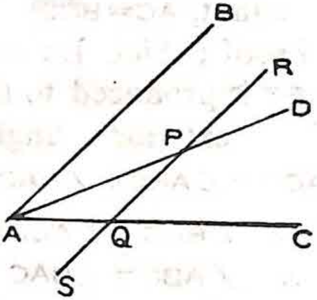


Fig. No. 107

Proof : Since BA ∥ RS and AP is transversal.

∴ ∠BAD = alternate ∠APQ

Again AD is the bisector of ∠BAC

∴ ∠BAD = ∠CAD, that is, ∠PAQ

∴ ∠APQ = ∠PAQ ∴ AQ = PQ

∴ △APQ is isosceles.

Exercise 13

1. In the fig. No. 108, CE is the bisector of ∠ACD and CE ∥ PR. What is the relation between CP and CQ ? Give reasons.

2. In a right-angled triangle ABC, ∠A is right angle. Take a point D on BC such that BA = AD. Prove that, D is the mid point of BC.

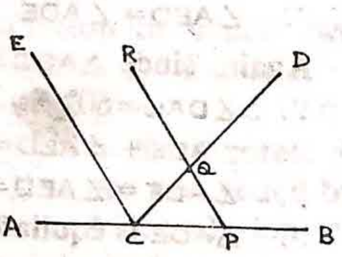


Fig. No. 108

3. Prove that, four triangles formed by joining the mid points of the three sides of an equilateral triangle are congruent equilateral triangles.

4. ABC is a right-angled triangle whose $\angle B$ is right angle. $\angle ABD$ is constructed at the point B making equal to $\angle BAC$. Prove that, D is mid point of AC.

5. In ΔABC, $\angle BAC = 90^\circ$ and $\angle BCA = 30^\circ$; Prove that. $AB = \frac{1}{2}BC$.

6. In ΔABC, side $AB =$ side AC. The straight line DE parallel to side BC intersects AB and AC at the points D and E respectively. Prove that, ΔADE is isosceles. Further, prove that, $DB = EC$. State what kind of the quadrilateral $DBCE$ is.

7. In ΔAQR, $PQ = QR$. The straight line drawn from the mid point D of QR parallel to PQ intersects PR at the point E. prove that $DE = \frac{1}{2}PQ$.

8. In a right-angled triangle XYZ, side $XY = \frac{1}{2}$ hypotenuse XZ, Prove that, $\angle YXZ = 60^\circ$.

9. In trapezium $ABCD$, $AD \parallel BC$ and $\angle ABC = \angle BCD$. Prove that, $ABCD$ is an isosceles trapezium.

10. The internal bisector of $\angle BAC$ of the triangle ABC and the straight line through the mid point D of AC parallel to AB intersect each other at the point E. Prove that, $\angle AEC = 1$ right angle.

11. In ΔABC, $AB = AC$ and $\angle B = 2\angle A$; the internal bisector of $\angle B$ intersects AC at the point D. Prove that ΔBCD is isosceles.

Theorm 11

In two triangles, if three sides of one be equal to three sides of the other, then the two triangles will be congruent.

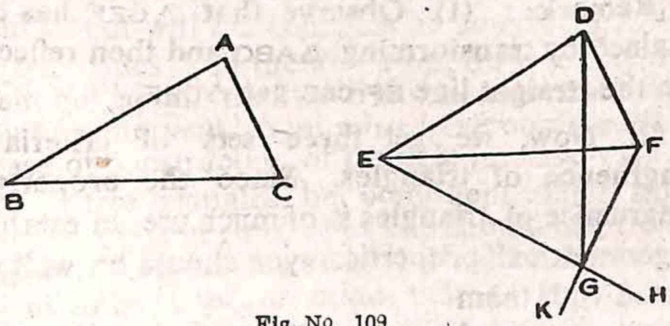


Fig. No. 109

Let in ΔABC and ΔDEF, $AB = DE$,

$BC = EF$ and $CA = FD$.

It is to be proved that, $\Delta ABC = \Delta DEF$.

Construction : $\angle FEH$ and $\angle EFK$ are drawn at the points E and F making equal to $\angle CBA$ and $\angle ACB$ respectively on that side of EF which is opposite to the side in which $\angle FED$ and $\angle EFD$ lie. Two sides EH and FK intersect each other at the point G. G, D are joined.

Proof : In $\triangle ABC$ and $\triangle GEF$,

$\angle ABC = \triangle GEF$ (by construction)

$\angle BCA = \angle EFG$ (by construction)

$BC = EF$ (by hypothesis)

$\therefore \triangle ABC \cong \triangle GEF$ $\therefore AB = GE$ and $CA = FG$

But $AB = DE$ and $CA = FD$.

$\therefore GE = DE$ and $FG = FD$

Again, $\because$ in $\triangle DEG$, $GE = DE$, $\therefore \angle EGD = \angle EDG$

and $\because$ in $\triangle DFG$, $FG = FD$ $\therefore \angle FGD = \angle FDG$

$\therefore \angle EGD + \angle FDG = \angle EDG + \angle FDG$

That is, $\angle EGF = \angle EDF$

Now, in $\triangle GEF$ and $\triangle DEF$. $GE = DE$, $GF = DF$ and included $\angle EGF =$ included $\angle EDF$

$\therefore \triangle EGF \cong \triangle DEF$

But $\triangle ABC \cong \triangle EGF$ $\therefore ABC \cong DEF$,

[**Remarks :** (1) Observe that $\triangle GEF$ has been obtained by transforming $\triangle ABC$ and then reflecting it in the straight line EF can get $\triangle DEF$.

(2) Now, we get three sets of criteria for congruence of triangles. Since the property of congruence of triangles is of much use in establishin geometrical properties, you should be well conversed with them.

The three sets of criteria are,

(i) Side angle-side (SAS), or in other words, for two triangles to be congruent, the two sides ane the included angle of one triangle must be equal to two sides and included angle of the other;

or, (ii) angle-angle-side (AAS), or in other words, for congruence of two triangles, any two angles of one and the side opposite to one angle must be equal to any two angles and corresponding side of the other. We have already learnt that if two angles be equal then the third angle be also equal. Therefore, we can also called the criteria angle side-angle (ASA), or in other words, of two triangles, two angles and their adjacent side of one triangle must equal to two angles and adjacent side of the other.

or, (iii) Side-side-side (SSS), or in other words, for congruence of two triangles, the three sides of one of them must be congruent to the three sides of the other.

Observe that, for two triangles to be congruent. three of its elements must be equal: but the two triangles will not be congruent if any three of its elements be equal. We can easily see that three angles of the two triangles are equal by drawing an equilateral triangle on line segments of two different measures: but will the triangles be congruent ?

If two sides and the angle opposite to any one of them be equal, the two triangles may not necessarily be congruent. You must have observed it in one case of construction of triangle in class VII.

(3) If two triangles be congruent, their sides and angles must be equal; but one should determine carefully which angles and which sides are equal. It is also to be kept in mind that corresponding sides or in other words, the sides opposite to the equal angles and the corresponding angles, or in other words, angles opposite to equal sides must be equal.

(4) It may be understood that the criteria of the congruence of theorem 11 will not be valid in case of proving theorem 9, because for proving theorem 11, the property of theorem 9 has been applied. Therefore, observe that, if one proves theorem 9 in the following way then he will make mistake.]

In $\triangle ABC$, $AB=AC$; A is joined to the mid point D of BC. Now, in $\triangle ABD$ and $\triangle ACD$,

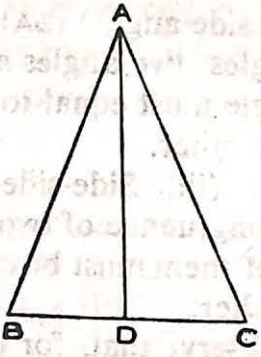


Fig No. 110

$AB=AC$

AD is common side

and $BD=CD$

$\therefore \triangle ABD \cong \triangle ACD$

$\therefore \angle ABD = \angle ACD$

or in other words,

$\angle ABC = \angle ACB$ |

Some Applications :

1. Of the quadrilateral ABCD, $AB=AD$ and $BC=DC$. Prove that, the diagonal AC bisects the diagonal BD at right angles.

Construction :

Suppose, AC and BD intersect each other at the point O.

Proof : In $\triangle ABC$ and $\triangle ADC$,

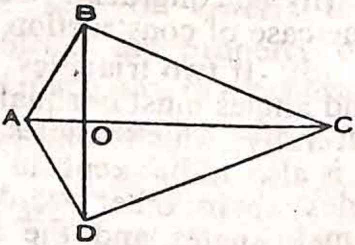


Fig No. 111

$AB=AD$ (by hypothesis)

$BC=DC$ (by hypothesis)

AC is common

$\triangle ABC \cong \triangle ADC$

$\therefore \angle BAC = \angle DAC$

or in other words, $\angle BAO = \angle DAO$, Again, in $\triangle ABO$ and $\triangle ADO$

$AB = AD$ (by hypothesis)

AO is common

included $\angle BAO$ = included $\angle DAO$ (proved)

$\therefore$ $\triangle BOA \cong \triangle DOA$

$\therefore$ $BO = DO$

and $\angle AOB = \angle AOD$

But these two adjacent angles are formed as the line segment OA stands on the line segment BD.

$\therefore$ Each of them is a right angle.

$\therefore$ $BD \perp AC$

$\therefore$ AC bisects BD at right angles.

2. The adjoining figure is related to bisection of the line segment AB.

See the procedure of the construction given in mathematics book of class VI and analyse whether it is vaild.

Construction :

A, P ; A, Q ; B, P ; B, Q are joined.

Proof : In $\triangle APQ$ and $\triangle BPQ$

$AP = BP$ (by construction)

$AQ = BQ$ (by construction)

PQ is common

$\therefore$ $\triangle APQ \cong \triangle BPQ$

$\therefore$ $\angle APR = \angle BPR$

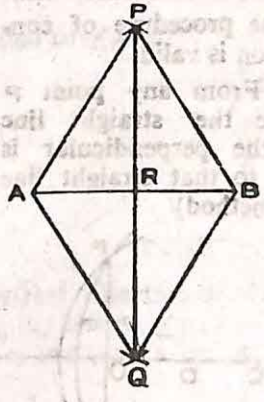


Fig No. 112

Again, in $\triangle APR$ and $\triangle BRP$, $AP = BP$

PR is common,

included $\angle APR$ = included $\angle BPR$

$\therefore$ $\triangle APR \cong \triangle BPR$ $\quad\therefore$ $AR = BR$.

Exercise 14

1. In fig. no. 113, some relations between sides and angles of $\triangle ABC$ and $\triangle PQR$ are given. State the other conditions for which two triangles will be congruent ;

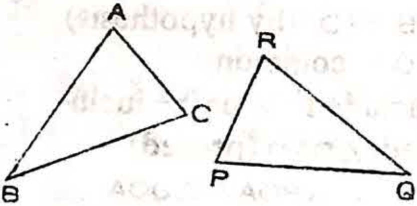


Fig No. 113

(i) $AB = PQ$ and $AC = PR$

(ii) $AB = PQ$ and $\angle ABC = \angle PQR$

(iii) $\angle ABC = \angle PQR$ and $\angle ACB = \angle PRQ$

2. ABCD is a rhombus. Prove that, diagonal AC bisects $\angle BAD$ and $\angle DCB$ and diagonals AD, BD bisect each other at right angles.

3. Two equal line segments BD and CE are cut off from the sides BA and CA of $\triangle ABC$. If $BE = CD$, prove that $\triangle ABC$ is isosceles.

4. $\triangle ABC$ and $\triangle DBC$ lie on same side of BC. If $BA = CD$, $BD = CA$. prove that $AD \parallel BC$.

5. You have done some constructions in class VI and VII. For your advantage, figures of each constructions with traces are given here. Analyse, in each case, whether the procedure of construction is valid.

(i) From any point P outside the straight line AB, the perpendicular is drawn to that straight line (first method)

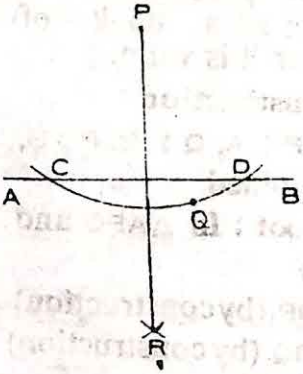


Fig No. 114

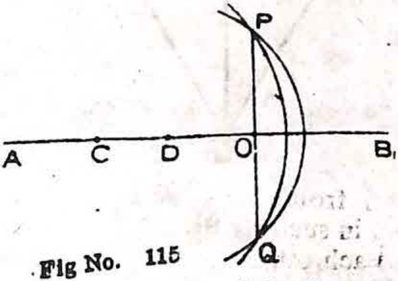


Fig No. 115

(ii) From any point P outside the straight line AB, the perpendicular PQ is drawn to that straight line (second method).

(iii) The perpendicular PQ is drawn to AB at the point P on the straight line AB (first method).

(iv) The perpendicular PQ is drawn to AB at the point P

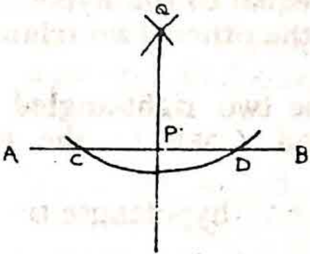


Fig No. 116

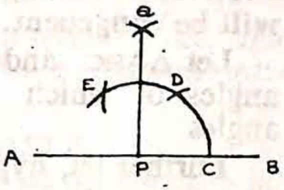


Fig No. 117

on the straight line AB (second method.)

(v) The perpendicular PQ is drawn to AB at the point P on the straight line AB (third method).

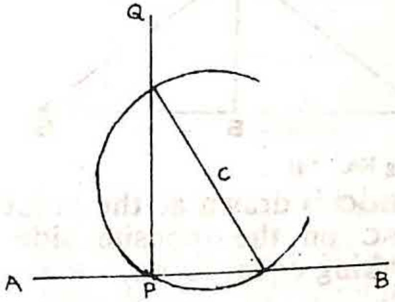


Fig No. 118

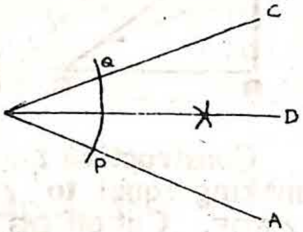


Fig No. 119

(vi) ∠ABC is bisected by BD.

vii) ∠PQR is drawn making equal to ∠ABC.

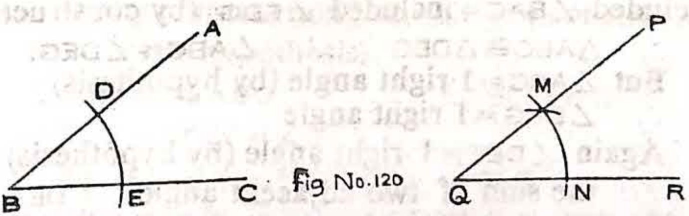


Fig No.120

6. In △PQR, PQ = PR, the internal bisectors of ∠Q and ∠R meet at the point O. Prove that, PO bisects ∠P.

7. In △ABC, AB = AC. D and E are two points on BC such that BD = CE. Prove that AD = AE.

8. Cut off line segments AP, AQ from side AB and AR, AS respectively from side AC of ∠BAC in such a way that AP = AR and AQ = AS. PS and QR intersect each other at the point T. Prove that (i) ∠APS = ∠ARQ, (ii) QT = RT and (iii) AT is bisector of ∠BAC.

Theorem 12

If the hypotenuse and a side of one of the two right-angled triangles be equal to the hypotenuse and corresponding side of the other, two triangles will be congruent.

Let $\triangle ABC$ and $\triangle DEF$ be two right-angled triangles of which $\angle ABC$ and $\angle DEF$ be the right angles.

Further let, hypotenuse $AC =$ hypotenuse DF and $AB = DE$

It is to be proved that $\triangle ABC \cong \triangle DEF$

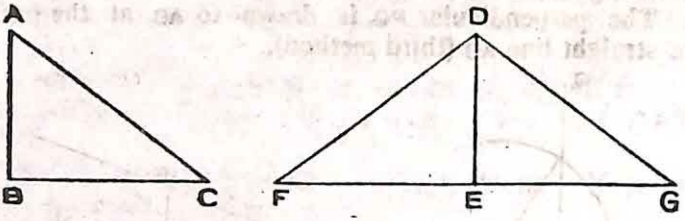


Fig No. 121

Construction : $\angle EDG$ is drawn at the point E making equal to $\angle BAC$ on the opposite side of $\angle EDF$. Cut off DG making equal to AC. E, G are joined.

Proof : In $\triangle ABC$ and $\triangle DEG$,
$AB = DE$ (by hypothesis), $AC = DG$ (by construction)
included $\angle BAC =$ included $\angle EDG$ (by construction)
$\therefore \triangle ABC \cong \triangle DEG$ $\therefore \angle ABC = \angle DEG$.
But $\angle ABC = 1$ right angle (by hypothesis)
$\therefore \angle DEG = 1$ right-angle

Again $\angle DEF = 1$ right angle (by hypothesis)
$\therefore$ the sum of two adjacent angles $\angle DEF$ and $\angle DEG$ is two right angles
$\therefore$ EF and EG lie on same straight line
$\therefore$ DFG is a triangle whose $CF = DG$ ($\because AC = DF$)
$\therefore \angle DFE = \angle DGE = \angle ACB$ $\because \triangle ABC \cong \triangle DEG$
Again, in $\triangle ABC$ and $\triangle DEF$,
$\angle ABC = DEF$ (both are right angles)
$\angle ACB = \angle DFE$ (proved)

$AC = DF$ (by hypothesis)

$\therefore \quad \triangle ABC \cong \triangle DEF.$

Alternative Proof :

Let $\angle ABC$ be right angle of the right-angled triangle ABC and $\angle DEF$ be right angle of the right-angled triangle DEF. Further, let hypotenuse $AC =$ hypotenuse DF and $AB = DE$.

It is to be proved, $\triangle ABC \cong \triangle DEF$

Construction : $\triangle DGF$ is drawn on opposite side of $\triangle DFE$ is such a way that $AB = DG$ and $BC = GF$ G, E are joined.

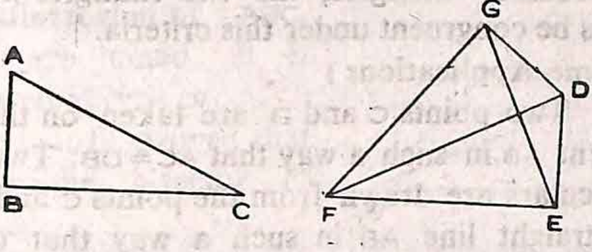


Fig No. 122

Proof : $DG = AB$ (by construction)

But $AB = DE$ (by hypothesis)

$\therefore \quad DG = DE, \quad \therefore \quad \angle DEG = \angle DGE$

Again, in $\triangle ABC$ and $\triangle DGF$,

$AB = DG$ (by construction), $BC = GF$ (by construction)

$AC = DF$ (by hypothesis)

$\therefore \quad \triangle ABC \cong \triangle DGF, \quad \therefore \quad \angle ABC = \angle DGF$

$\therefore \quad \angle DEF = \angle DGF$

$\therefore \angle DEF - \angle DEG = \angle DGF - \angle DGE$

or in other words, $\angle FEG = \angle FGE, \quad \therefore \quad FG = FE$

Again, in $\triangle DEF$ and $\triangle DGF$,

$DE = DG, \quad FE = FG,$ FD is common

$\therefore \quad \triangle DEF \cong \triangle DGF$

But is proved earlier that,

$\triangle DGF \cong \triangle ABC \qquad \therefore \triangle DEF \cong \triangle ABC$

[**Remark** : (1) Observe that, the construction has been made only to obtain a triangle congruent to $\triangle ABC$.

(2) In the first proof, the side-angle-side property of congruence and in the second, side-side-side property have been used.

(3) The criteria for congruence of right-angled triangles and to be noted carefully. In this case, the two sides an the opposite angle of another (hypotenuse) being equal, the two triangles are congruent. For ordinary triangles, the two triangles will not always be congruent under this criteria.]

Some Applications :

1. Two points C and D are taken on the line segment AB in such a way that $AC=DB$. Two perpendiculars are drawn from the points C and D on the straight line AB in such a way that one of the two perpendiculars lies on that side of AB which is opposite to the side on which the other perpendicular lies. If $BE=AF$, prove that, $BE \parallel AF$.

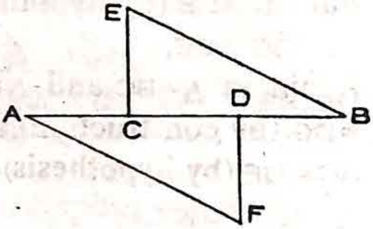


Fig No. 123

Proof : $\because$ $AC=DB$

$\therefore$ $AC+CD=DB+CD$

or in other words.

$AD=BC$

Again, since $CE \perp AB$ and $DF \perp AB$

$\therefore$ $\triangle ADF$ and $\triangle BCE$ are right-angled triangles.

Now, in two right-angled triangles ADF and BCE.

$AD=BC$ (proved)

hypotenuse $AF=$ hypotenuse BE (by hypothesis)

$\therefore$ $\triangle ADF \cong \triangle BCE$

$\therefore \angle EAD = \angle CBE$

But they are alternate angles. $\therefore BE = AF$

2. D is such a point within $\angle ABC$ that the length of two perpendiculars drawn from the point D on BA and BC are equal. Prove that, BD is the bisector of $\angle ABC$.

Construction :

Perpendiculars DE and DF are drawn on BA and BC respectively from the point D lying within the angularregion of $\angle ABC$. B, D are joined. It is given that $DE = DF$.

It is to be proved that BD is the bisector of $\angle ABC$.

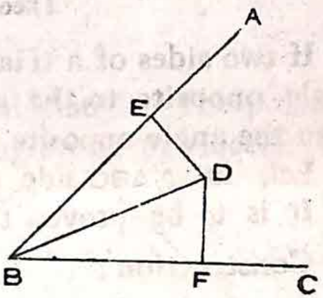


Fig No. 124

Proof : $\because DE \perp AB$

$\therefore \triangle BDF$ is a right-angled triangle.

Again, since $DF \perp BC$,

$\therefore \triangle BDF$ is a right-angled triangle.

Now, in the right-angled triangles BDE and BDF, hypotenuse BD is common. $DE = DF$ (by hypothesis)

$\therefore \triangle BDE \cong \triangle BDF$, $\therefore \angle DBE = \angle DBF$,

or, in other words, BD is the bisector of $\angle ABC$.

Exercise 15

1. By producing FE of $\triangle DEF$ to EG making equal to BC and joining D, G, prove theorem 12.

2. Construct $\angle FDH$ at the point D making equal to $\angle BAC$ on that side of DF which is opposite to the side on which point E lies and cut off DG from DH making equal to AB. Now by joining E, G ; prove theorem 12.

3. If the lengths of perpendiculars drawn from any two vertices of a triangle be equal, then prove that the triangle is isosceles.

4. If lengths of two perpendiculars drawn from mid point of any side of a triangle on the two other sides be equal, prove that the triangle is isosceles.

5. In the quadrilateral ABCD, AB = AD and AC is perpendicular on BD. Prove that, BC = DC.

Theorem 13

If two sides of a triangle be unequal, then the angle opposite to the greater side will be greater than the angle opposite to the less.

Let, in $\triangle ABC$ side $AB >$ side AC.

It is to be proved that $\angle ACB > \angle ABC$

Construction :

The segment AD is cut off from AB making equal to AC. D, C are joined.

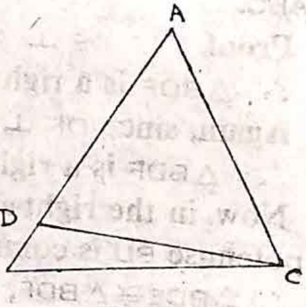


Fig No. 125

Proof : In $\triangle ACD$,

$AC = AD$ (by construction)

$\therefore \angle ADC = \angle ACD$,

Again, in $\triangle BCD$,

exterior angle $\angle ADC >$ interior opposite $\angle DBC$

or, in other words. $\angle ADC > \angle ABC$

But $\angle ACB > \angle ACD$

$\therefore \angle ACB > \angle ADC$, $\angle ACB > \angle ABC$.

[Remark : Here, one angle is to be shown greater than another angle. We have already observed that the exterior angle of a triangle is greater than either of the interior opposite angles. Keeping this point in view the construction has been made.]

Corollary 1. The angle opposite to the greatest side of a triangle is the greatest.

Corollary 2. If the angle opposite to the greatest side of a triangle is acute, then the triangle is an acute-angled triangle,

[Since the angle opposite to the greatest side is greatest (cor. 1), therefore each of the other two angles is less than this. Therefore, they are also acute angles Therefore the triangle is an acute-angled triangle.]

Some Applications :

1. ABC is an equilateral triangle. The point A is joined to the point D lying on BC produced.

Prove that $\angle BAD > \angle ADB$

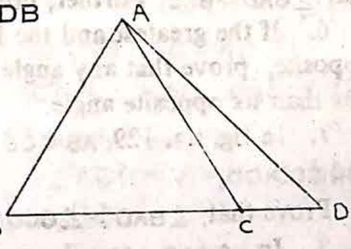


Fig No 126

Proof : $\therefore$ $\triangle ABC$ is equilateral

$\therefore$ $BC = AB$

$\because$ $DB = BC + CD$

$\therefore$ $DB > AB$

[$\because$ $BC = AB$]

$\therefore$ $\angle BAD > \angle ADB$

2. In the quadrilateral ABCD, $BC = BA$ and $CD > AD$,

Prove that $\angle BAD > \angle BCD$

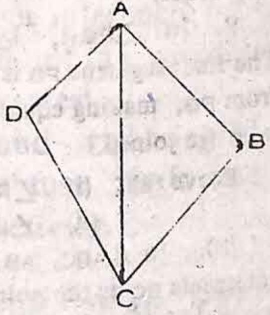


Fig No. 127

Construction : A, C are joined

Proof : $\because$ $BC = BA$

$\therefore$ $\angle BAC = \angle BCA$

Again, $\because$ $CD > AD$

$\therefore$ $\angle DAC > \angle DCA$

$\therefore$ $\angle DAC + \angle BAC$
$> \angle DCA + \angle BCA$

or, in other words, $\angle BAD > \angle BCD$.

Exercise 16

1. In fig. no. 128, $AC > AB$ and x, y are related by one of the relations given below :

(i) $x = 2y$ (ii) $x = y$

iii) $x = \frac{3}{5}y$. Which relation is correct ?

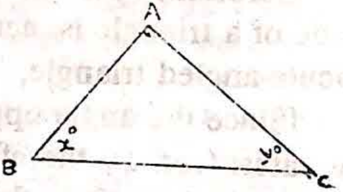


Fig No. 128

2. Prove theorem 13 by producing AC and making $AD = AB$.

3. In $\triangle ABC$, $AC > AB$. D is such a point on AC that $\angle ADB = \angle ABD$. Prove that, $\angle ABC > \angle ACB$.

4. In $\triangle ABC$, $AC = AB$; the bisector of $\angle ACB$ intersects AB at the point D, Prove that $BD < CD$.

5. In $\triangle ABC$, AD is perpendicular on BC. If $AC > AB$, prove that $\angle CAD > BAD$. Further, prove that $DC > BD$.

6. If the greatest and the least sides of a quadrilateral be opposite, prove that any angle adjacent to greatest side will be less than its opposite angle.

7. In fig. no. 129, $AB < OB$ and $CD > OD$.

Prove that, $\angle BAO > \angle OCD$

8. In $\triangle ABC$. $AB = 7$ cm, $BC = 5$ cm and $CA = 9$ cm. Arrange the measures of the angles of the triangle in ascending order.

9. In $\triangle PQR$, $PQ > PR$. The line segment PS is cut off from PQ, making equal to PR, R, S are joined.

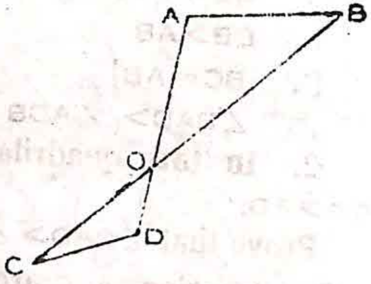


Fig. No. 129

Prove that (i) $\angle PSR = \frac{1}{2}(\angle PQR + \angle PRQ)$

(ii) $\angle QRS = \frac{1}{2}(\angle PRQ - \angle PQR)$

10. In $\triangle ABC$, $AB > AC$. The internal bisector of $\angle BAC$ intersects BC at the point D. Cut off from AB the line segment AE making equal to AC. Join DE. Prove that (i) $\triangle ACD \cong \triangle AED$. Hence prove that $\angle ACB > \angle ABC$.

Theorem 14

If two angles of a triangle be unequal, then the side opposite to the greater angle is greater than the side opposite to the less.

Let us suppose that in $\triangle ABC$.

$\angle ACB > \angle ABC$

It is to be proved that, side $AB >$ side AC.

Proof. Any of the following three relations between AB and AC may hold :

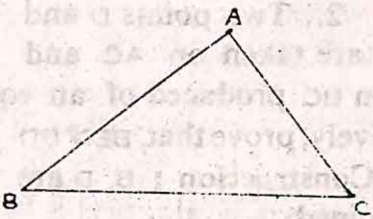


Fig No. 130

(i) $AB < AC$ (ii) $AB = AC$ or (iii) $AB > AC$

(i) If $AB < AC$, then $\angle ACB < \angle ABC$, but it is given that $\angle ACB > \angle ABC$, so $AB < AC$.

(ii) If $AB = AC$, then $\angle ACB = \angle ABC$, but by hypothesis this is also impossible.

$\therefore$ $AB \neq AC$ $\therefore$ AB must be $> AC$.

[**Remark** : This is converse of theorem 13. So this theorem is proved indirectly. Here it is seen that two of three relations are not correct, so the third relation is taken as the correct inference.

Corollary 1. The opposite side of the greatest angle of any triangle will be the greatest

Corollary 2. In obtuse-angle and right-angled triangles, opposite sides of the obtuse angle and right angle are greatest sides respectively.

Some Application :

1. In fig. no. 131, in $\triangle ABC$, $\angle ABC = 42$ $\angle ACB = 108°$. What is the relation between the sides AC and BC ?

Answer : In $\triangle ABC$. $\angle ABC = 42^\circ$, $\angle ACB = 108^\circ$

$\therefore \quad \angle BAC = 180^\circ$

$-(42^\circ + 108) = 180^\circ - 150^\circ$

$= 30^\circ$

$\therefore \quad \angle ABC > \angle BAC$

$\therefore \quad AC > BC.$

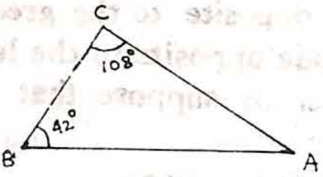


Fig No. 131

2. Two points D and E are taken on AC and on BC produced of an equilateral triangle respectively, prove that, $BE > BD$

Construction : B, D are joined.

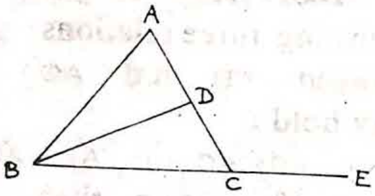


Fig No. 132

Proof : In $\triangle ABD$, exterior $\angle BDC >$ interior opposite $\angle BAD$; But $\angle BAC = \angle BCA$

[$\because$ angles of an equilateral triangle are equal to each other.]

$\therefore \quad \angle BDC > \angle BCD \quad \therefore \quad BC > BD$

$\therefore \quad BE = BC + CD > BD.$

Exercise 17

1. In $\triangle PQR$, $\angle PQR = 80^\circ$ $\angle PRQ = 30^\circ$. What is the relation between the sides PR and QR ?

2. In triangle ABC, $\angle A = 55^\circ$, $\angle B = 40^\circ$. Arrange the sides of the triangle in ascending order of their length.

3. The vertex of an isosceles triangle is 70°. Prove that, the unequal side of the triangle is the greatest side.

4. In fig. no. 133, if $x = \frac{1}{2}y$, what is the relation between AB and AC ?

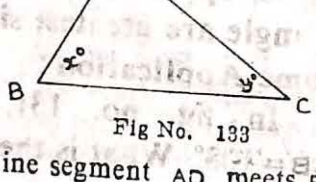


Fig No. 133

5. In $\triangle ABC$, $AB = AC$. The line segment AD meets BC at the point D.

Prove that, AD<AC.

7. In fig. no. 134, AB = CD

$\angle OCD > \angle COD$

and $\angle OAB < \angle AOB$

Prove that, OB<OD.

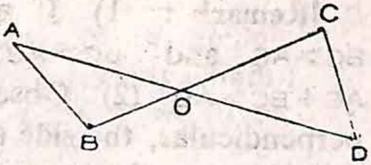


Fig No. 134

Theorem 15

The sum of two sides of a triangle is greater than the third side.

Let ABC be a triangle.

It is to be proved that, the sum of any two sides of $\triangle ABC$ is greater than the third side.

Let BC be the greatest side. Then it will be sufficient, if it is proved that $AB + AC > BC$.

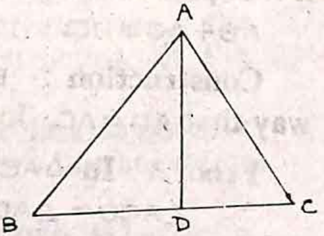


Fig No. 135

Construction : The perpendicular AD is drawn from A on BC which intersects BC at the point D.

Proof : $\because$ $AD \perp BC$

$\therefore$ Each of $\angle ADB$ and $\angle ADC$ is 1 right angle.

$\therefore$ $\triangle ADB$ and $\triangle ADC$ are right-angled triangles.

Since, only one right angle may be in a triangle,

$\therefore$ $\angle ADB > \angle BAD$ $\therefore$ $AB > BD$

in same way, $\angle ADC > \angle DAC$ $\therefore$ $AC > DC$

$\therefore$ $AB + AC > BD + DC$, $\therefore$ $AB + AC > BC$

[**Remark :** (1) If BC be the greatest side then $BC > AC$ and $BC > AB$; so $AB + BC > AC$ and $AC + BC > AB$ (2) Observe that, by drawing the perpendicular, the side BC is divided into two parts in such a way that any one part of them is less than any one of other two sides. So the perpendicular is drawn]

Alternative proof :

Let ABC be a triangle.

It is to be proved that the sum of two sides of $\triangle ABC$ is greater than the third side.

Let BC be the greatest side. Then it is sufficient, if it is proved that

$AB + AC > BC$.

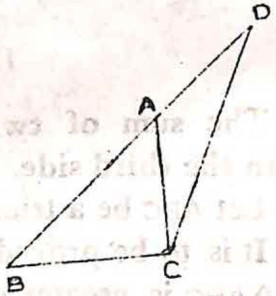


Fig No. 136

Construction : BA is produced to D in such a way that $AD = AC$, Join D, C.

Proof : In $\triangle ACD$, $AD = AC$ (by construction)

$\therefore \angle ACD = \angle ADC$

$\therefore \angle BCD = \angle ACB + \angle ACD > \angle ADC$

$\therefore$ In $\triangle BCD$, side $BD >$ side BC

that is, $BA + AD > BC$

$\therefore AB + AC > BC \quad [\because AD = AC]$

[**Remark :** Observe that, for getting a line segment of Length equal to the lengths of two line segments AB and AC, BA is produced so that the extended portion is equal in AC.]

Corollary : The difference of two sides of any triangle will be less than the third side.

Some Applications :

1. Analyse whether the construction of triangle is possible by the line segments of following length :
(i) 8 cm, 3 cm, 12 cm, (ii) 6 cm, 8 cm, 10 cm,
(iii) 3 cm, 7 cm, 4 cm.

Answer : (i) The length of the greatest line segment = 12 cm, the sum of the lengths of other two line segments = 8 cm + 3 cm = 11 cm, 11 cm < 12 cm.

∵ The sum of lengths any two sides of a triangle must be greater than the length of third side.

∴ In this case, the construction of triangle is not possible.

(ii) The length of the greatest line segment is 10 cm, the sum of the lengths of other two line segments = 6 cm + 8 cm = 14 cm, 14 cm > 10 cm.

∵ The sum of lengths of any two sides of a triangle will be greater than the length of third side and if sum of lengths of other two sides be greater than the length of the greatest side than the sum of lengths of any two sides will be greater than the third. So, in this case, the construction of the triangle is possible.

(iii) The length of the greatest line segment = 7 cm, the sum of lengths of other two line segments = 3 cm + 4 cm = 7 cm.

∵ The sum of lengths of any two sides, of a triangle must be greater than the third side.

∴ In this case, the construction of triangle is not possible.

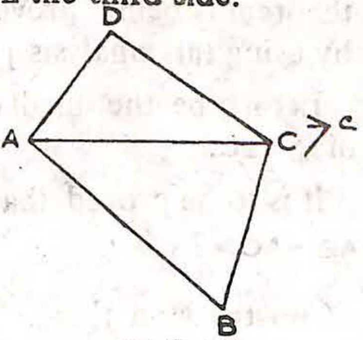


Fig No. 137

2. The sum of any three sides of any quadrilateral will be greater than the fourth side.

Suppose, ABCD is a quadrilateral. Its greatest side is AB.

It is to be proved that, the sum of any three sides is greater than the fourth side.

It will be sufficient, if it is proved that $AD+DC+CB>AB$.

Construction : A, C are joined.

Proof : In $\triangle ACD$, $AD+DC>AC$.

Again, In $\triangle ACB$, $AC+CB>AB$

$\therefore$ $AD+DC+CB>AC+CB$

i.e $AD+DC+CB>AB$

3. Prove that, if AD be a median of $\triangle ABC$ then $AB+AC>2AD$

[Let us analyse before trying to prove. We know, the sum of any two sides of a triangle is greater than the third side. Therefore, if we can construct a triangle by taking the length of the sides as AB, AC and $2AD$, then it is possible to prove the present theorem. For this, by producing AD we shall take a line segment AP of length $2AD$. Now if we can show CP equal to AB by joining C, P, then the three sides of $\triangle ACP$ will be of length AB, AC and $2AD$, or in other words, our object will be fulfilled. Now the theorem is being proved by using this analysis.]

Let AD be the median of $\triangle ABC$.

It is to be proved that, $AB+AC>2AD$.

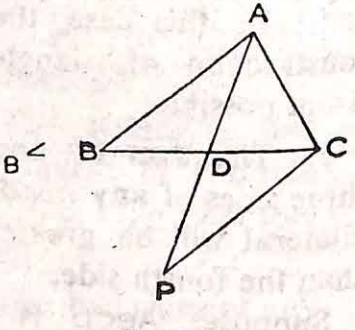


Fig No. 138

Construction :

AD is produced to the point P such that $DP=AD$. C, P are joined.

Proof : $\because$ AD is median,

$\therefore$ the point D is mid point of BC.

$\therefore$ In $\triangle ABD$ and $\triangle PCD$

$AD = DP$ (by construction)

$BD = DC$ (by hypothesis)

$\angle ADB$ = vertically opposite $\angle PDC$

$\therefore$ $\triangle ABD \cong \triangle PCD$ $\therefore$ $AB = PC$

Now, In $\triangle APC$, $AC + PC > AP$

$\therefore$ $AC + AB > AD + DP$

$\therefore$ $AB + AC > 2AD$ $[\because AD = DP]$

Exercise 18

1. In which cases, is it possible to construct triangles by the line segments of following lengths ?

(i) 3 cm, 5 cm. (ii) 8 cm, 8 cm, 10 cm.
(iii) 6 cm, 3 cm. (iv) 5 cm, 2 cm, 8 cm.
(v) 6 cm, 4 cm, 7 cm. (vi) 4 cm, 5 cm, 10 cm.

2. A, B, C are three noncollinear points. A man goes to B directly starting from A. Another man goes to B through C starting from A. If two men reach to B in same time whose velocity is greater ? Give reasons.

3. A triangle is to be formed by bending a wirc of length 12 cm. What will be the maximum and minimum lengths in centimeter within which the magnitude of the length of the greatest side lies ?

4. Prove that the perimeter of a quadrilateral is greater than the sum of lengths of two diagonals.

5. O is a point within $\triangle ABC$. Prove that

(i) $AB + AC > OB + OC$.
(ii) $AB + AC + BC > OA + OB + OC$.

[**Hint** : Let BO produced intersect AC at the point D. Analyse the relation between the sides of $\triangle ABD$ and $\triangle OCD$.]

6. Prove that, the perimeter of a quadrilateral is greater than twice the length of any diagonal.

7. P is a point within $\triangle ABC$. Prove that,

(i) $AP+BP>AB$

(ii) $AB+BC+CA>2(AP+BP+CP)$

8. Prove that, the perimeter of any triangle is greater than the sum of three medians.

9. Prove that, the sum of the distances of the vertices from any point within a quadrilateral (but not on any diagonal) is greater than the sum of the lengths of two diagonals. Now you say, what should be the position of the point so that the sum of the distances of the vertices from that point will be least.

Theorem 16

Of all the line segments that can be drawn to given straight line from a given point outside it, the perpendicular is the shortest.

Let AB be any straight line and O be a point outside it. Further, let OP be perpendicular on AB.

It is to be proved that of all other line segments that can be drawn from the point O to the straight line AB, OP is less than each of them.

Construction : Any point Q is taken on the straight line AB. O, Q are joined.

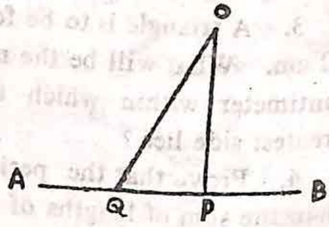


Fig No. 139

Proof : $\because OP \perp AB$

$\therefore \angle OPQ = 1$ right angle

$\therefore \angle OPQ > \angle OQP$

$\therefore$ In $\triangle OQP$, $OQ > OP$

or, in other words, $OP < OQ$

This conclusion is true for any position of O on AB.

$\therefore$ the length of OP is the shortest.

[**Remark : (1)** By the phrase the distance of any point from any straight line, we understand the perpendicular length.

(2) Of all line segments drawn to any straight line from a point outside it, other line segments except perpendicular are called obliques.

Some Application :

1. Of all the line segments drawn to any straight line from a point outside it, the shortest one will be perpendicular on that straight line.

Let AB be a straight line and O be any point outside it. Suppose, OP is the shortest of all the line segments drawn from the point O to AB.

It is to be proved that, OP is perpendicular on AB.

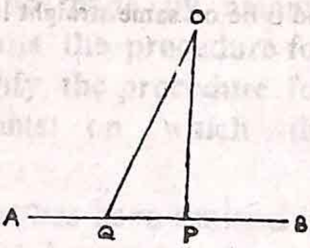


Fig No. 140

Proof : If OP be not perpendicular on AB then suppose OQ is perpendicular on AB.

∴ OQ is the shortest of all the line segments drawn from the point O to AB.

Again, by assumption, OP is the shortest.

∴ OQ = OP ∴ ∠OPQ = ∠OQP

But ∠OQP = 1 right angle

∴ ∠OPQ = 1 right angle

But it is impossible if OQ, OP do not lie on same straight line.

∴ OP is perpendicular on AB.

Exercise 19

1. For going to a road directed east-west, one of the two persons has gone in direction of south and the other has gone in the direction of north-east from any place lying in the northern side of the road. Who will reach first if they walk with same velocity ? Give reasons.

2. The shortest distance of AB from a point O, outside the straight line AB is OC ; OD and OE are two obliques such that CD>CE. Prove that, OD>OE.

3. In the quadrilateral ABCD. AB = AD and BC = DC. The shortest distance of AC from the point D is DP. Prove that B, P and D lie on same straight line.

Chapter 4

Construction

4.1. The formal construction of geometrical figures with the help of geometrical instruments is known as problem.

A problem has four parts : (i) mention of conditions given and the construction to be made, (ii) to represent the given conditions with figures and to mention the construction to be made by naming them, (iii) to describe in details the procedure for construction and (iv) to justify the procedure for construction or the reason(s) on which the construction is based.

[Remarks : The fourth part has been omitted in respect of problems you have learnt in class VI and VII because we are not aware of geometrical properties necessary for justification. You have learnt those justification from Q. No. 5 of exercise 14. In this class also you will learn some problems whose proofs will be taken up in subsequent classes.]

In the case of problem, one should keep in view the followings :

(i) The given conditions are to be represented by figures.

(ii) Wherever possible, only ruler, pencil and compasses should be used.

(iii) There should be traces of each construction.

(iv) It is desirable to have the construction neat and clean.

(v) The description of every construction should necessarily be given when the procedure for construction is written, but if the help of any construction (problem) is taken which was done earlier then no detail description of the same is necessary.

Let us now see the procedure for construction for some problems.

Problem 1

A straight line is to be drawn parallel to a given straight line passing through a given point.

First method :

Let AB be a straight line and O be a given point outside it.

A straight line is to be drawn parallel to the straight line AB passing through the point O.

Procedure for Construction : A point D is taken on the straight line AB. O, D are joined. ∠DOP is constructed at the point O on OD making equal to ∠ODB and on that side of OD which is opposite to the side in which ∠ODB lies. PO is produced and a point Q is taken on the produced part of the line segment.

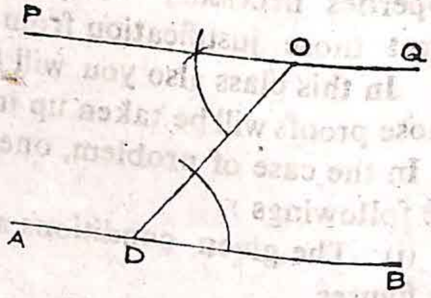


Fig No. 141

Then the straight line PQ is the required straight line which is parallel to AB and passes through the point O.

Proof : The line segment OD intersects the two straight lines AB and PQ at the points D and O respcetively.

$\angle BDO = \angle DOP$ (by construction)

But they are alternate angles.

$\therefore$ AB $\parallel$ PQ

Second Method :

Let AB be a straight line and O be a given point outside it.

A straight line is to be drawn parallel to the straight line AB passing through the point O.

Procedure for construction : A point D is taken on the straight line AB D,O are joined and DO is produced to E. $\angle EOQ$ is drawn at the point O making equal to $\angle ODB$ in the same side of OD in which $\angle ODB$ lies. QO is produced. A point P is taken on QO produced. Then the straight line PQ will be the required line which is parallel to AB and passes through the point O.

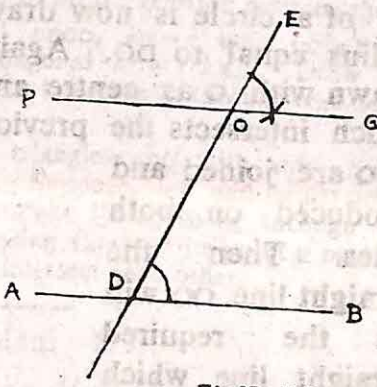


Fig No. 142

Proof : AB and PQ are two straight lines and the straight line DO intersects them at the point D and O respectively.

$\angle BDO = \angle QOE$ (by construction)

But they are corresponding angles. $\therefore$ AB $\parallel$ PQ

Alternative Method :

Let AB be a straight line and O be a given point outside it.

A straight line is to be drawn parallel to the straight line AB passing through the point O.

Procedure of Construction : A point D is taken on the straight line AB. Any line segment DC is cut off from DB. An arc of a circle is now drawn with C as centre and radius equal to DO. Again an arc of a circle is drawn with O as centre and radius equal to DC, which intersects the previous arc at the point Q. O, Q are joined and produced on both sides. Then the straight line OQ will be the required straight line which is parallel to AB and passes through the point O.

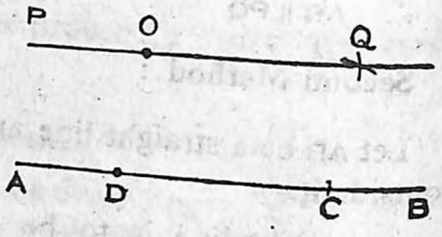


Fig No. 143

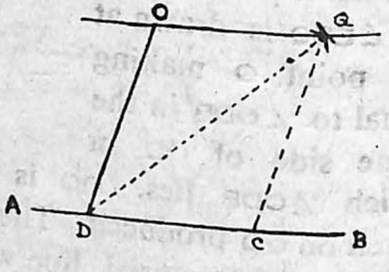


Fig No. 143(i)

Proof : D, O ; C, Q and D, Q are joined (fig. no. 143(i). In △DOQ and △DCQ, DO=CQ (by construction) OQ=DC (by construction) and DQ is common.

∴ △DOQ≅△DCQ

∴ ∠DQO=∠QDC

But they are alternate angles ∴ OQ ∥ AB

Exercise 20

1. PQ is a straight line. Draw a straight line parallel to that straight line passing through the point X outside it.

2. Construct ∠ABC making equal to 60° with help of ruler and compasses. Draw a straight line passing through the point C parallel to BA and another straight line passing through the point A parallel to BC.

3. Draw a triangle with sides equal to 7 cm, 4 cm and 5 cm. Draw the straight line passing through three vertices of the triangle parallel to the straight lines opposite to them. Measure angles and sides of the triangle formed as a result of these constructions and find the relations between those sides and angles with the sides and angles of original triangle.

4. ABC is a triangle. Bisect AB at the point D. Draw a straight line passing through the point D parallel to BC. Suppose that parallel straight line intersects AC at the point E; measure AE, CE and see what the relation between them is. Measure DE and BC and write down the relation between them.

5. In the quadrilateral ABCD, AB = 4 cm, BC = 5 cm, CD = 6 cm, DA = 4·5 cm and diagonal AC = 7·2 cm. Draw the quadrilateral, draw the straight line passing through the point B parallel to AC.

6. Draw an equilateral triangle ABC with sides 5 cm. Bisect two sides AB and BC. Suppose, two mid points are D and E respectively. Draw straight lines passing through the point D parallel to BC and passing through the point E parallel to AB. Observe, where they intersect each other.

Problem 2

A line segment is to be divided into a number of equal parts.

Let the line segment AB have to be divided into five equal parts.

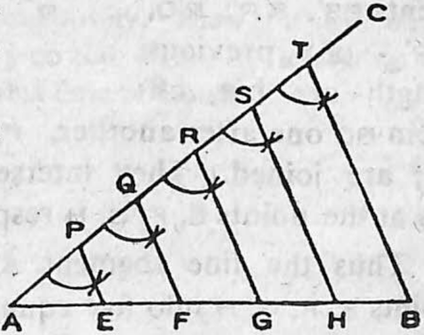


fig No 144

Procedure of Construction : The line segment AC is drawn through the point A making an angle with AB. The line segment AC is infinitely extended in direction from A to C. The line segments AP, PQ, QR, RS, ST of equal lengths are cut off from AC one after another. B, T are joined. Angles are drawn at the points S, R, Q, P making equal to ∠ ATB in same side of AC in which that angle lies. The sides of those angles intersect AB at the points H, G, F and E respectively.

Thus the line segment AB is divided at the points E, F, G and H into five equal parts.

Second Method :

Let the line segment AB have to be divided into five equal parts

Procedure of Construction : Any angle ∠BAC is drawn at the point A. ∠ABD is drawn making equal to ∠BAC on that side of AB opposite to the side in which ∠BAC lies. Four equal line segments AP, PQ, QR, RS of equal lengths are cut off from AC one after another. Four equal line segments BS′, S′R′, R′Q, O′P′ of previous length are cut off from BD one after another. P, P′ ; Q, Q′ ; R, R′ ; S, S′ ; are joined. They intersect the line segment AB at the points E, F, G, H respectively.

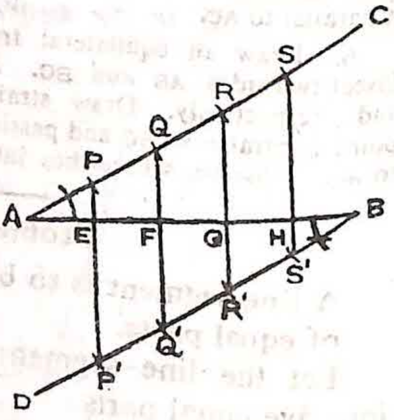


Fig No. 145

Thus the line segment AB is divided at the points E, F, G, H into five equal parts.

[Remarks : 1. In the same manner, one can divide a line segment into any number of equal parts.

2. Of course, one may adopt the method of bisection of a line segment for dividing a line segment in equal parts of two, four, eight etc.

3. You will be able to understand the justification for these two procedures of constructions in class IX ; but you may verify with the help of dividers whether the parts are equal or not.]

Exercise 21

1. Divide a line segment of length 8·2 cm into three equal parts.

2. Divide the line segment of length 10 cm into seven equal parts.

3. Divide a line segment into four equal parts by any of the above methods. Again, divide that line segment into four equal parts by method of bisection. Observe whether division points are same,

4. Draw the median AD of the triangle ABC. Divide the median into three equal parts AE, EF, FD. Join BC and produce it in such a way that it intersects AC at the point X. Find the relation between AX and CX measuring by a scale.

5. Draw three line segments AB, CD and EF whose lengths are 6 cm, 5 cm and 8 cm respectively. Draw a triangle whose sides are of length $\frac{2}{3}$ AB, $\frac{4}{5}$ CD and $\frac{1}{2}$ EF. By measuring the sides of the triangle, say what form of triangle it is.

Chapter 5

Revision of lessons of this year

Some theorems have been proved in chapter 3 and you have also solved some problems. You must have seen that in those cases you have had to use the properties obtained earlier. The main job is to understand which properties are to be used for proofs) of which theorem. Hence, we give below a list of properties to be proved and properties obtained previously which may be used for proving.

	What is to be proved	*What are to be given for proving*
1.	An angle is a right angle	(1) When a line segment stands on a straight line, the adjacent angles so formed are equal. (2) One straight line is perpendicular on another straight line. (3) The measure of one angle of a triangle is equal to the sum of measures of other two angles.
2.	Two line segments lie in a straight line or threee points are collinear	(1) The measure of the angle formed by those two line segments be 180°. (2) Those two line segments are perpendicular on a stratght line passing through the common point of those two line segments. (3) Equal angles are formed in opposite direction by two line segments with any straight line drawn through the common point of those two line segments.

	What is to be proved	*What are to be given for proving*
		(4) Any one of the above mentioned properties is obtained by joining two points with any of the three points (specially with the point between those two points.)
3.	Two angles are equal	(1) Each of the two angles is a right angle. (2) A straight line lies on another straight line perpendicularly. (3) They are vertically opposite angles. (4) A straight line intersects two parallel straight lines and the angles are corresponding angles or alternate angles. (5) Two angles are base angles of an isosceles triangle. (6) Two angles are the corresponding angles of two congruent triangles. (7) The sum of two angles of one of the two triangles is equal to the sum of two angles of another and the two angles under consideration are third angles of those two triangles.
4.	Two line segments are equal	(1) Two angles of a triangle are equal. (2) Two line segments are the corresponding sides of two congruent triangles.
5.	Two triangles are congruent	(1) Two sides and included angle are equal. (2) Two angles and a corresponding side are equal.

	What is to be proved	*What are to be given for proving*
		(3) Three sides are equal. [Remember that, if three angles be equal or two sides and angles opposite to one of the equal sides be equal, two triangles may or may not are congruent.] (4) Two triangles are right-angled triangles and hypotenuse and any one of the sides are equal.
6.	Two straight lines are parallel	(1) A straight line intersects the two straight lines and two corresponding angles or two alternate angles are equal or sum of two interior angles lying on the same side of transversal is equal to two right angles. (2) Two straight lines are perpendicular on any other straight line. (3) Each of two straight lines is parallel to another line.
7.	A straight line is perpendicular on another straight line	(1) Two adjacent angles are equal. (2) Sum of other two angles of the triangle formed by joining two points lying on two straight lines is a right angle. (3) The length of the segments drawn from any point lying on one straight line to another straight line is shortest.
8.	The sum of lengths of two line segments is equal to the length of a line segment	(1) A line segment equal to one line segment is cut off from the third line segments and the remaining part of the third line segment can be shown equal to the second.
9.	The sum of two angles is equal to another angle	(1) A part is cut off from the third angle making equal to one of the angle and if the other part is equal to the second angle.

What is to be proved	*What are to be given for proving*
	(2) If the greatest angle be the exterior angle and if first two angles be interior opposite angles of a triangle.
10. A line segment is greater than a second line segment	(1) If the second line segment be placed on the first line segment in such a way that they have a common end point and if other end point of the second line segment lies inside the first line segment. (2) If the two line segments be the sides of such a triangle that angle opposite to the first line segment is greater than the angle opposite to the second line segment.
11. An angle is greater than another angle	(1) If two angles have same vertex and have a common arm and if two angles lie on the same side of the common arm and the other arm of the second angle is nearer to common side than other arm of first angle. (2) If two angles be the angles of such a triangle whose side opposite to first angle is greater than the side opposite to second angle.
12. Three line segments will form a triangle	(1) If the sum of the lengths of two smaller line segments be greater than the length of the greatest line segment.
13. A triangle is obtused-angled, right-angled or acute-angled	(1) If the angle opposite to the greatest side of the triangle be obtuse angle, right angle or acute angle.

Miscellaneous exercise 1

1. Give definitions with figures :

(i) Right-angled triangle (ii) obtuse-angled triangle (iii) acute-angled triangle (iv) scalene triangle (v) isosceles triangle (vi) equilateral triangle (vii) hypotenuse (viii) median (ix) trapezium (x) parallelogram (xi) rectangle (xii) square (xiii) rhombus (xiv) regular polygon (xv) diagonal.

2. Write down the properties which are obtained from the definitions of the following figures :

(i) right-angled triangle (ii) obtuse-angled triangle (iii) acute-angled triangle (iv) scalene triangle (v) isosceles triangle (vi) equilateral triangle (vii) trapezium (viii) parallelogram (ix) rectangle (x) square (xi) rhombus (xii) regular polygon.

3. What is the relation between two adjacent angles which are formed when one line segment stands on a straight line ?

4. If two adjacent angles be supplementary to each other ; in which way will their two arms other than common arm lie ?

5. What is the relation between two vertically opposite angles ?

6. Under what conditions, will two triangles be congruent ?

7. Under what conditions, will two right-angled triangles be congruent ?

8. If two triangles be congruent, what are the relations between the sides and the angles of two triangles ?

9. If two sides of a triangle be equal, what is the relation between the angles opposite to them ?

10. If two sides of a triangle be unequal, what is the relation between the angle opposite to the greater side and the angle to the less ?

11. In a triangle if two angles be equal, what is the relation between the sides opposite to them ?

12. If two angles of a triangle be unequal, what is the relation between the side opposite to the greater angle and the angle opposite to the less ?

13. Under what conditions, will two straight lines be parallel ?

14. If two straight lines be parallel, what do we obtain from it ?

15. What is the sum of measures of three angles of a triangle ?

16. What is the relation between any exterior angle and two interior opposite angles of a triangle ?

17. What is the relation between any exterior angle and any one interior opposite angle of a triangle ?

18. Which is the shortest of all the line segments drawn from a point to a straight line ?

19. What is the relation between three sides of a triangle ?

20. Of two straight lines, when will one be prependicular on the other ?

21. What is the sum of measures of interior angles of any polygon with *n* sides ?

22. What is the sum of measures of exterior angles formed by producing the sides of a polygon in same direction in a definite order ?

23. What is the measure of each interior angles of a regular polygon of *n* sides ?

24. What is the measure of each exterior angle of a regular polygon of *n* sides ?

25. Fill up the table by making the table in your note book.

Serial number of all the theorems which you have proved	Mention all axioms and postulates which you have used to prove these	Mention all the theorems which you have used
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		

12		
13		
14		
15		
16		

Miscellaneous Exercise 2

1. Fill up the gaps with appropriate words :

(i) If a line segment stands on a straight line, then two adjacent angles are formed ; if the measure of one of them be 62° the measure of other will be— .

(ii) If the sum of two adjacent angles $\angle ABC$ and $\angle DBC$ be two right angles, three points A, B, D will be—.

(iii) Two straight lines intersect each other and four angles are formed. One of them is a right angle. Other three angles are—to each other and the measure of each of them is—.

(iv) The measure of one of the two interior angles on the same side of a transversal of two parallel straight lines is 132° ; the measure of the other will be—.

(v) The measure of the angle formed by the internal bisector and the external bisector of an angle is—.

(vi) The measures of angles of an equilateral triangle are —to each other and the measure of each is—

(vii) If the sum of two angles of a triangle be 90°, the triangle will be—.

(viii) The measure of each of the two angles adjacent to hypotenuse of a right-angled isosceles triangle will be—.

(ix) Measure of all angles at a point is—.

(x) In $\triangle ABC$ and $\triangle PQR$, if $AB = QR$, $\angle ACB = \angle QPP$ and $\angle ABC = \angle PRO$, then two triangles will be—and the side BC will be equal to the side—.

(xi) In $\triangle PQR$ and $\triangle XYZ$, if $PQ = ZX$, $QR = XY$ and $PR = ZY$, then two triangles will be—and $\angle PQR$ will be equal to —.

(xii) The difference of the lengths of two sides of a triangle will be—than length of the third side.

(xiii) If each of the two straight lines be parallel to a third line, first two straight lines will be—to each other.

2. Give the sign '√' against the correct statements and 'x' against the wrong statements.

(i) The lengths of two sides of a triangle are 9 cm and 6 cm. The angle opposite to the side 6 cm is 113°.

(ii) An angle of a triangle is 108° and the length of the side opposite to this angle is 5 cm and the length of another side is 8 cm.

(iii) The measure of each of the two angles adjacent to the base of a right-angled isosceles triangle is 45°.

(iv) To be a regular polygon, the number of degrees of the measure of each exterior angle will be any factor of 360 other than 1, 2, 180 and 360.

(v) The lengths of two sides of a triangle are 4 cm and 6 cm, the length of the other side will be equal to 10 cm or less than 10 cm.

(vi) If three angles of one of the two triangles be equal to three angles of the other respectively, two triangles will be congruent.

(vii) If two perpendiculars be drawn at the end points of a line segment, they will be parallel.

(viii) In a right-angled triangle, hypotenuse is the greatest side.

(ix) If two straight lines be intersected by a third straight line, two alternate angles will be equal.

(x) If two adjacent angles be equal, each of the two angles will be a right angle.

(xi) If two sides and any one angle of one of the two triangles be equal two sides and any one angle of the other, two triangles will be congruent.

(xii) If two angles and any one side of one of the two triangles be equal to two angles and any one side of the other two triangles will be congruent

(xiii) OR is the perpendicular on AB from the point O outside the straight line AB and Q is any point on that straight line. The length of OR is less than the length of OQ.

3. Write down 'yes' or 'no' against the answer of the questions :

(i) If measures of two adjacent angles ∠POQ and ∠QOR be 130° and 60° respectively, will three points P, Q, R be the vertices of a triangie ?

(ii) Will a triangle be formed by three line segments of lengths 3 cm, 5 cm, 8 cm ?

(iii) Will the sum of two angles of a triangle be 180° ?

(iv) Will a right-angled triangle be equilateral ?

(v) Can an obtuse-angled triangle be isosceles ?

(vi) An angle of any polygon is 145° ; can the polygon be regular ?

(vii) Can each of two intersecting straight lines be parallel to a third straight line ?

(viii) Can two exterior angles in same side of a transversal of two parallel straight lines be equal to each other.

(ix) Is it possible to form a triangle with three line segments of lengths 10 cm, 8 cm and 5 cm ?

(x) Will the bisector of the vertical angle of an isosceles triangle bisect also the base ?

(xi) Two sides of a triangle are equal to two sides of another triangle. Will the third side of the triangle be always equal ?

(xii) Two angles of a triangle are equal to two angles of another triangle. Will the third angles of the two triangles be always equal ?

4. Strike out the wrong expression(s) inside the bracket.

(i) (only one/more than one) perpendicular(s) can be drawn on a straight line from a point outside it.

(ii) (only one/two) perpendicular(s) can be drawn on a straight line from a point on it.

(iii) (only one/more than one/even one) straight line(s) (may/may not) be drawn parallel to a straight line through a point outside it.

(iv) (only one/more than one/even one) straight line(s) (may/may not be) drawn parallel to a straight line through a point on it.

(v) Any triangle must have (only one/two) acute angle(s).

(vi) A triangle may have (only one/more than) right angle(s).

(vii) A triangle may have (only one/more than one) obtuse angle(s).

(viii) The sum of three angles of a triangle will be (less than two right angles/equal to two right angles/more than two right angles).

(ix) The angle opposite to the smallest side of any triangle will be (obtuse angle/right angle/acute angle).

(x) If lengths of three sides of a triangle be a, b, c units, then $a+b-c$ will be (positive/negative).

(xi) In ΔABC and ΔPQR, if $AB=PQ$, $BC=QR$ and ($\angle ABC = \angle PQR$/$\angle ACB = \angle PQR$), then two triangles must be congruent.

Miscellaneous Exercise 3

1. Find the value of x in each case from the following figures :

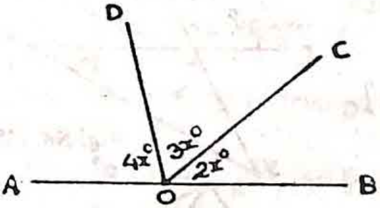


Fig. No. 146

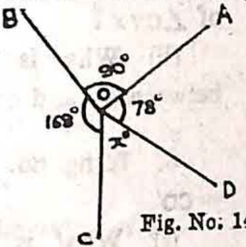


Fig. No: 147

2. Find the measures of ∠POQ, ∠QOR and ∠ROS from fig. no. 148.

Which two angles are supplementary to each other ?

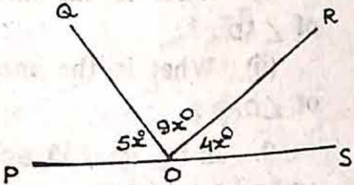


Fig No. 148

3. Answer the following questions from fig. no. 149.

(i) What is the measure of ∠PQS ?

(ii) Is POR a straight line ?

(iii) Is QOS a straight line ?

(iv) Which two angles are supplementary to each other ?

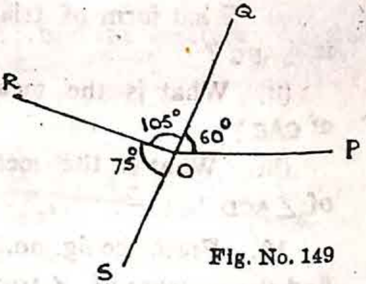


Fig. No. 149

4. AB ∥ CD in fig. no. 150, find the measure of ∠APO.

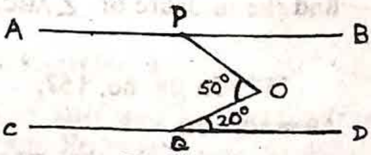


Fig. No. 150

5. Answer from fig. no. 151 :

(i) What is the relation between AB and CD ?

(ii) What form of quadrilateral is the quadrilateral ABCD ?

(iii) What is the measure of ∠DBE ?

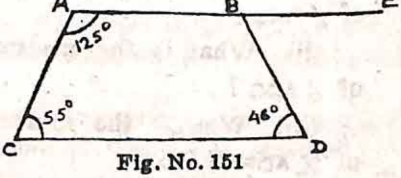


Fig. No. 151

6. In fig. no. 152 BA ∥ DC and EC = ED. What is the measure of ∠BED ?

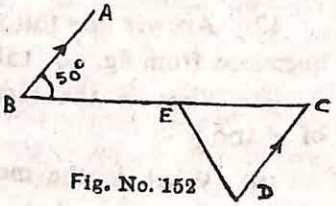


Fig. No. 152

7. In fig. no. 153, BA ∥ CD ; ∠ABC = 70°. ∠CXY = 35°.

(i) What is the measure of ∠CYX ?

(ii) What is the relation between CX and CY ?

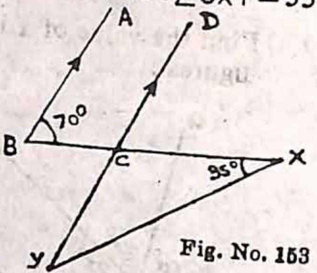


Fig. No. 153

8. In fig. no. 154, AB = AC = CD

(i) What is the measure of ∠ADC ?

(ii) What is the measure of ∠DAB ?

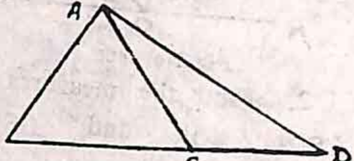


Fig. No. 154

9. In fig. no. 155, BC = AC and BC ∥ AE

(i) What form of triangle is △ABC ?

(ii) What is the measure of CAE ?

(iii) What is the measure of ∠ACD ?

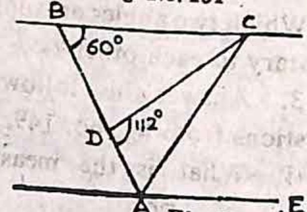


Fig. No. 155

10. From the fig. no. 156, find the measure of ∠ABC.

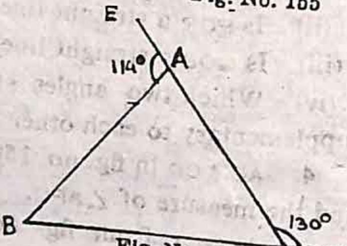


Fig. No. 156

11. In fig. no. 157, AB = AC = CD.

(i) What is the measure of ∠BAC ?

(ii) What is the measure of ∠ADC ?

(iii) What is the measure of ∠ADF ?

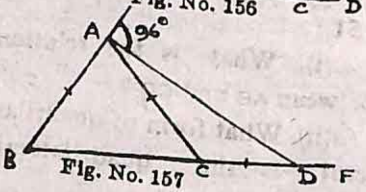


Fig. No. 157

12. Answer the following questions from fig. no. 158 :

(i) What is the measure of ∠ADC ?

(ii) What is the measure of ∠ACB ?

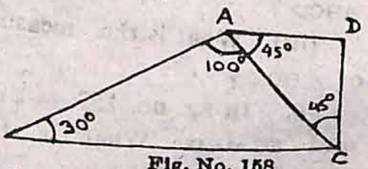


Fig. No. 158

(iii) What is the relation between AD and DC ?

(iv) What is the relation between AB and BC ?

(v) What form of triangle is $\triangle ABC$?

(vi) What form of triangle is $\triangle ADC$?

13. In fig. no. 159, $AB \parallel DE$; find the value of x.

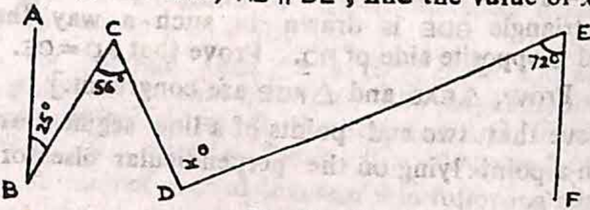


Fig. No. 159

14. In fig. no. 160, $AB \parallel EF$; find the measure of $\angle ABC$.

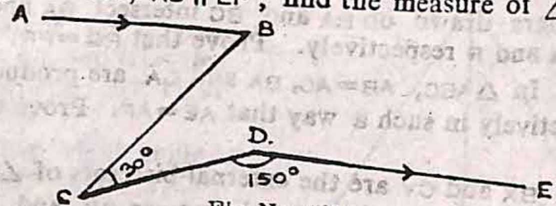


Fig. No. 160

Miscellaneous Exercise 4

1. Two isosceles triangles $\triangle ABC$ and $\triangle DBC$ are drawn on two sides of the line segment BC. Prove that, AD bisects the side BC at right angles.

2. Prove that, the straight line drawn through the vertex of an isosceles triangle parallel to the base will be external bisector of the vertical angle.

3. If the external bisector of the vertical angle of a triangle be parallel to the base, prove that the triangle is isosceles.

4. The line segments AB and CD are equal and parallel. The end points A and C are on same side. Prove that, AC and BD are equal and parallel.

5. In $\triangle ABC$, $\angle A$ is a right angle and $AC > AB$, P is any point on AC such that $CP = AB$,. Prove that $\angle BCP > \angle PBC$.

6. The side BC of the triangle ABC is trisected at the points P and Q. Prove that, if $AP = AQ$, then $AB = AC$.

7. In quadrilateral ABCD, ∠A = ∠D and AB = DC; prove that, (i) △ABD≅△ADC, (ii) △ABC≅△BCD (iii) AD ∥ BC. What is the form of this quadrilateral ?

[**Hint** : (iii) prove it by producing AB and DC.]

8. ABC is a right angled triangle whose ∠B is right angle. D is the mid point of AC, E is any point on AB. The perpendicular at the point C on BC intersects ED produced at the point F. Prove that, EF has been bisected at the point D.

9. D is any point inside the equilateral triangle ABC. The equilateral triangle BDE is drawn in such a way that the points A and E opposite side of BD. Prove that AD = CE.

[**Hint** : Prove, △EAD and △FCD are congruent.]

10. Prove that two end points of a line segment are equi distant from a point lying on the perpendicular bisector of the line segment.

11. The straight line BX is bisector of ∠ABC. Two perpendiculars drawn on BA and BC intersect BA and BC at the points Q and R respectively. Prove that PQ = PR.

12. In △ABC, AB = AC, BA and CA are produced to E and F respectively in such a way that AE = AF. Prove that ∠BFA = ∠CEA.

13, BX and CY are the external bisectors of ∠B and ∠C of △ABC. The perpendiculars drawn on BX and CY from the point A intersect CB and BC produced at the points R and S respectively. Prove that RS = AB + BC + CA.

14. ABCDEF is a regular hexagon. Prove that AD is perpendicular on CE.

15. ABC is a right angled triangle whose ∠C is right angle D is a point on that side of AC, which is opposite to the side in which B lies, in such a way that AD = AB and ∠DAC = BAC. Prove that B, C, D are collinear.

16. The diagonal AC of the quadrilateral ABCD bisects the are angles ∠BAD and ∠BCD, BO is perpendicular on AC, D, O is joined. Prove that D, O, B are collinear.

17. Two equal parts are cut off from sides BA and CA of triangle ABC, O is mid point of BC : if OD = OE, prove that the triangle is isosceles.

18. Two equal parts PX and RY are cut off from the sides PQ and RQ of the triangle PQR. If RX = PY, prove that the triangle is isosceles.

19. D and E are points on sides BA and CA respectively of triangle ABC such that $CD = BE$. Two perpendicular lengths drawn on BC from D and E are equal. Prove that $\triangle ABC$ is isosceles.

20. In $\triangle PQR$, $PQ = PR$. RP and QP are produced to S and T respectively in such a way that $PS = PT$, PL and PM are two perpendiculars on QR and ST respectively. Prove that PL and PM lie in same straight line.

21. ABC is a right angled isosceles triangle whose $\angle B$ is right angle. The bisector of $\angle BAC$ intersects BC at the point D. Prove that $AC = AB + BD$.

22. In triangle ABC, $\angle A$ is right angle and $AC > AB$, P is such a point on AC that $CP = AB$. The perpendiculars AX and PY on BC intersect BC at the points X and Y respectively. Prove that (i) $\angle BAX = \angle PCY$ (ii) $\triangle ABX = \triangle CPY$.

23. A student proved theorem 9 in following manner :

Construction : A perpendicular is drawn on BC.

Proof : Since AD is perpendicular on BC, two triangles $\triangle ABD$ and $\triangle ACD$ are right angled triangles.

Now, in two right angled triangles $\triangle ABD$ and $\triangle ACD$, hypotenuse $AB =$ hypotenuse AC and AD is common side.

$\therefore \quad \triangle ABD \cong \triangle ACD$

$\angle ABC = \angle ACD$

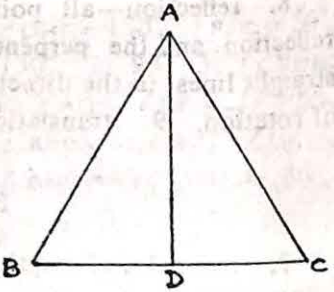


Fig No. 161

Give your opinion whether the proof is valid.

24. If the bisector of vertical angle of a triangle bisects the base, the triangle will be isosceles. To prove this theorem, the following argument is given :

In $\triangle ABD$ and $\triangle ACD$,
$BD = CD$ (by hypothesis)
AD is common side.
$\angle BAD = \angle CAD$
(by hypothesis)
$\triangle ABD \cong \triangle ACD$.

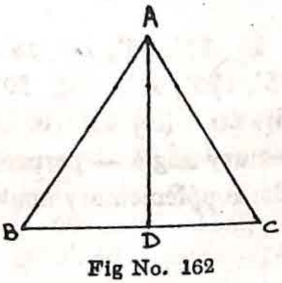


Fig No. 162

Some errors are there in this argument. What and where are the errors ? Find those. Prove the

Geometry

Answers

Exercise 1

1. (i) many (ii) many (iii) many (iv) many (v) collinear (vi) concurrent (vii) two (viii) intersecting or parallel (ix) one (x) (x) parallel 5. (a) acute angle, right angle obtuse angle, straight angle (b) acute angle—(i), (iv) ; right angle—(ii) ; obtuse angle—(iii), (vi) ; straight angle—(v) 6. magnitude of angle, betweennes, collinearity, length of a line segment or distance between two points : 7. (i) rotation (ii) reflection (iii) translation or reflection (iv) reflection (v) translation.

8. reflection—all points on the line of reflection, line of reflection and the perpendiculars on that line ; translation—straight lines in the direction of translation ; rotation—centre of rotation. 9. translation.

Exercise 2

7. two sides of lengths 8 cm and 6 cm are perpendicular to each other, the three perpendiculars will pass through the point of intersection of those two sides. 9. the perpendicular bisector will be other diagonal.

Exercise 3

3. 87°, 85°, 72°, 58°, 40°, 30°, 18° 10°, 2°. 4. 170°, 150°, 135°, 120°, 90°, 70°, 50°, 37°, 25°, 13°, 5. (i) 45°, (ii) 90° (iii) 60° (iv) 45° 6. correct sentence—(i), (vi) 7. complementary angle — perpendicular will have to be drawn on any side, supplementary angle — any side is to be produced.

Exercise 4

1. 138° 2. 50° 3. 115°, 65° 4. 120°, 60° 5. 40 6. 56, perpendicular.

Exercise 5

1. they will be on the same straight line. 2. No. 3. lying on the same straight line. 5. by producing any one side. 6. 30°.

Exercise 6

1. $\angle COA = 110^\circ$, $\angle AOD = 70^\circ$, $\angle DOB = 110^\circ$. 2. 55°, 55°, 125°, 125°. 3. (i) no (ii) no. 4. yes 5. 65, 35, 115,

Exercise 7

1. (i) parallel (ii) parallel (iii) not parallel (iv) not parallel (v) parallel 3. parallelogram.

Exercise 8

1. $\angle BGH = 130^\circ$, $\angle GHD = 50^\circ$, $\angle DHF = 130^\circ$, $\angle FHC = 50^\circ$, $\angle CHG = 130^\circ$, $\angle HGA = 50^\circ$, $\angle AGE = 130^\circ$ 2. 95° 3. ($\angle EGH$, $\angle FGK$), (HGA, $\angle KGB$, $\angle FKG$), ($\angle BGE$, $\angle AGF$, $\angle KFG$), ($\angle HGA$, $\angle KGB$, $\angle FKG$), ($\angle BGE$, $\angle AGF$, $\angle KFG$), ($\angle BGH$, $\angle DKG$), ($\angle EGA$, $\angle GFC$) 4. $\angle PAR = 110^\circ$, $\angle ARP = 40$, $\angle RAP = 30$.

Exercise 9

1. 50° 2. 50° 3. $\angle CAB = 60^\circ$, $\angle ABC = 40^\circ$, $\angle BCA = 80$.

Exercise 10

1. 61° 2. 30, 60°, 90°. 3. 30°, 60°, 90°. 4. $\angle BAD = 55^\circ$, $\angle CAD = 35^\circ$, $\angle BCA = 55^\circ$, $\angle ADB = 90^\circ$, $\angle ADC = 90^\circ$ 5. 360

Exercise 11

1. (i) 6 right angles (ii) 8 right angles (iii) 10 right angles (iv) 12 right angles (v) 16 right angles (vi) 20 right angles 2. 118° 3. 138° 4. 80°, 80°, 40°, 160° 5. no 6. no 7. 180° 8. (i) 108°, 72° (ii) 120°, 60° (iii) 135°, 45° (iv) 140°, 40° (v) 144°, 36° (vi) 150°, 30° (vii) 160°, 20° (viii) 162°, 18° 9. (i) yes, 60 (ii) yes, 36 (iii) no (iv) yes, 20 (v) yes 15, (vi) yes, 12 (vii) no (viii) yes.

9. (ix) no (x) yes, 8 (xi) no (xii) no (xiii) yes, 5
10. (i) no (ii) no (iii) yes, 6 (iv) yes, 10 (v) no (vi) yes, 18 (vii) yes, 4 (viii) yes, 30 (ix) yes, 36 (x) yes, 40 (xi) yes, 45 (xii) yes, 72, 13, 105°

Exercise 12

1. 50°, 2. 40°, 50°, 50°, 80° 4. 120°, 30°, 30°
5. (i) 36°, 72°, 72° (ii) 90°, 45°, 45° (iii) 20°, 80°, 80° (iv) 120°, 30°, 30° 6. 35°, 35° 7. 130° 8. 36°, 36°, 108°
9. 30°, 75°, 75°.

Exercise 13

1. CP = CQ

Exercise 14

1. (i) ∠BAC = ∠QPR or BC = QR (ii) BC = QR or ∠BAC = ∠QPR or ∠ACB = ∠PRQ (iii) BC = QR or AB = PQ or AC = PR

Exercise 16

1. (i)

Exercise 17

1. PR > QR 2. AB > BC > AC 4. AC > AB

Exercise 18

1. (i), (ii) (v) 2. second man who went to B via C. 3. less than 6 cm and more than 4 cm.

Exercise 19

1. the man who went to the south.

Exercise 20

3. lengths of the sides of the new triangle – 14 cm 8 cm and 10 cm, that is double the lengths of the sides of the original triangle ; the measures of the angles are same as those of the original triangle. 4. $AE = CE$, $DE = \frac{1}{2}BC$ 6. mid point of AC.

Exercise 21

4. $AX = 2CX$ 5. equilateral triangle.

Miscellaneous Exercise 2

1. (i) 118° (ii) collinear (iii) equal, one, right-angle (iv) 48° (v) 90° (vi) equal, 60° (vii) right angled (viii) 45° (ix) 360° (x) congruent, PR (xi) congruent, $\angle ZXY$ (xii) smaller (xiii) parallel 2. (i) wrong (ii) wrong (iii) right (iv) right (v) wrong (vi) wrong (vii) right (viii) right (ix) wrong (x) wrong (xi) wrong (xii) wrong (xiii) right 3. (i) no (ii) no (iii) no (iv) no (v) yes (vi) no (vii) no (viii) no (ix) yes (x) yes (xi) No (xii) yes 4. (i) more than one (ii) two (iii) more than one. even one, may not (iv) only one, more than one, may (v) only one (vi) more than one (vii) more than one (viii) less than two right angles, more than two right angles (ix) obtuse angle, right angle (x) zero negative (xi) $\angle ACB = PRQ$.

Miscellaneous Exercise 3

1. (i) 20 (ii) 24 2. 50°, 90°, 40° ; $\angle POQ$ and $\angle ROS$ 3. (i) 122° (ii) no (iii) yes (iv) $\angle POS$ and $\angle POG$, $\angle ROS$ and $\angle ROQ$ 4. 150° 5. (i) $AB \parallel CD$ (ii) trapezium (iii) 45° 6. 100° 7. (i) 35° (ii) $CX = CY$ 8. (i) 32·5° (ii) 82·5° 9. (i) isosceles (ii) 60° (iii) 80° 10. 64° 11. (i) 52° (ii) 320° (iii) 148° 12. (i) 45° (ii) 50° (iii) perpendicular to each other and equal. (iv) $BC > AB$ (v) obtuse-angled (vi) right-angled 13. 106 14. 60°.
